![pennsylvania DEPARTMENT OF EDUCATION]

# The Pennsylvania System of School Assessment

## Mathematics
## Item and Scoring Sampler



## 2022–2023
## Grade 8

# INTRODUCTION

## General Introduction

The Pennsylvania Department of Education (PDE) provides districts and schools with tools to assist in delivering focused instructional programs aligned with the Pennsylvania Core Standards (PCS).These tools include Academic Standards, Assessment Anchors and Eligible Content (AAEC) documents, assessment handbooks, and content-based item and scoring samplers. This Item and Scoring Sampler is a useful tool for Pennsylvania educators in preparing local instructional programs by providing samples of test item types and scored student responses. The item sampler is not designed to be used as a pretest, a curriculum, or any other benchmark for operational testing.

This Item and Scoring Sampler is available in Braille format. For more information regarding Braille, call (717) 901-2238.

## Pennsylvania Core Standards (PCS)

This sampler contains examples of test questions designed to assess the Pennsylvania Assessment Anchors and Eligible Content aligned to the PCS. The Mathematics, Reading, and Writing PSSA transitioned to PCS-based operational Mathematics and English Language Arts assessments starting with the spring 2015 PSSA administration.

The PCS-aligned Assessment Anchors and Eligible Content documents are posted on this portal:

> ➢ *www.education.pa.gov* [Hover over "Data and Reporting," select "Assessment and Accountability," and select "PSSA-PA System of School Assessment." Then select "Assessment Anchors/Eligible Content" on the right side of the screen.]

## What Is Included

This sampler contains test questions, or test "items," that have been written to align to the Assessment Anchors that are based on the PCS. The sample test questions model the types of items that may appear on an operational PSSA. Each sample test question has been through a rigorous review process to ensure alignment with the Assessment Anchors prior to being piloted in an embedded field test within a PSSA assessment and then used operationally on a PSSA assessment. Answer keys, scoring guidelines, and any related stimulus material are also included. Additionally, sample student responses are provided with each open-ended item to demonstrate the range of responses that students provided in response to these items.

## Purpose and Uses

The items in this sampler may be used[1] as examples for creating assessment items at the classroom level. Classroom teachers may find it beneficial to have students respond to the open-ended (OE) item in this sampler. Educators may then use the sampler as a guide to score the responses either independently or together with colleagues within a school or district. This sampler also includes the *General Description of Scoring Guidelines for Mathematics Open-Ended Items* that students will have access to during a PSSA mathematics administration. The general description of scoring guidelines may be distributed to students for use during local assessments and may also be used by educators when scoring local assessments.

## Item Format and Scoring Guidelines

The multiple-choice (MC) items have four answer choices. Each correct response to an MC item is worth one point.

Each OE item is designed to take approximately ten to fifteen minutes to complete. During the administration of the PSSA, students are given additional time as necessary to complete the test items. Each OE item in mathematics is scored using an item-specific scoring guideline based on a 0–4-point scale. In this sampler, every item-specific scoring guideline is combined with examples of student responses that represent each score point to form a practical, item-specific scoring guide.

## Item Alignment

All PSSA items are aligned to statements and specifications included in the *Assessment Anchors and Eligible Content Aligned to the Pennsylvania Core Standards.* The mathematics content, process skills, directives, and action statements included in the PSSA mathematics questions align with the Assessment Anchor Content Standards. The Eligible Content statements represent the limits of the content of the mathematics questions.

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[1]  The permission to copy and/or use these materials does not extend to commercial purposes.

## Testing Time and Mode of Testing Delivery for the PSSA

The PSSA is delivered in a traditional paper-and-pencil format as well as in an online format. The estimated time to respond to a test question is the same for both methods of test delivery. The following table shows the estimated response time for each item type.

| Mathematics Item Type | MC | OE |
|---|---|---|
| Estimated Response Time (minutes) | 2 | 10 to 15 |

During an official test administration, students are given as much additional time as is necessary to complete the test questions.

## Mathematics Reporting Categories

The Assessment Anchors are organized into four classifications as listed below.

| | |
|---|---|
| • A = Numbers and Operations | • C = Geometry |
| • B = Algebraic Concepts | • D = Data Analysis and Probability |

These four classifications are used throughout the grade levels. In addition to these classifications, there are five Reporting Categories for each grade level. The first letter of each Reporting Category represents the classification; the second letter represents the Domain as stated in the Common Core State Standards for Mathematics. Listed below are the Reporting Categories for Grade 8.

- A-N = The Number System

- B-E = Expressions and Equations

- B-F = Functions

- C-G = Geometry

- D-S = Statistics and Probability

Examples of MC and OE items assessing these categories are included in this sampler.

## Item and Scoring Sampler Format

This sampler includes the test directions and scoring guidelines that appear in the PSSA Mathematics assessments. Each MC item is followed by a table that includes the item alignment, the answer key, the depth of knowledge (DOK) level, the percentage[2] of students who chose each answer option, and a brief answer-option analysis or rationale. The OE item is followed by a table that includes the item alignment, the DOK level, and the mean student score. Additionally, each of the included item-specific scoring guidelines is combined with sample student responses representing each score point to form a practical item-specific scoring guide. The *General Description of Scoring Guidelines for Mathematics Open-Ended Items* used to develop the item-specific scoring guidelines should be used if any additional item-specific scoring guidelines are created for use within local instructional programs. The student responses in this item and scoring sampler are actual student responses; however, the handwriting has been changed to protect the students' identities and to make the item and scoring sampler accessible to as many people as possible.

### Example Multiple-Choice Item Information Table

| Item Information | |
|---|---|
| Alignment | Assigned AAEC |
| Answer Key | Correct Answer |
| Depth of Knowledge | Assigned DOK |
| *p*-value A | Percentage of students who selected option A |
| *p*-value B | Percentage of students who selected option B |
| *p*-value C | Percentage of students who selected option C |
| *p*-value D | Percentage of students who selected option D |
| Option Annotations | Brief answer-option analysis or rationale |

### Example Open-Ended Item Information Table

| Alignment | Assigned AAEC | **Depth of Knowledge** | Assigned DOK | **Mean Score** | Average Score |
|---|---|---|---|---|---|

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[2] All *p*-value percentages listed in the item information tables have been rounded.

# General Description of Scoring Guidelines for Mathematics Open-Ended Items

**4—The response demonstrates a *thorough* understanding of the mathematical concepts and procedures required by the task.**

The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. The response may contain a minor "blemish" or omission in work or explanation that does not detract from demonstrating a *thorough* understanding.

**3—The response demonstrates a *general* understanding of the mathematical concepts and procedures required by the task.**

The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a *general* understanding.

**2—The response demonstrates a *partial* understanding of the mathematical concepts and procedures required by the task.**

The response is somewhat correct with *partial* understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

**1—The response demonstrates a *minimal* understanding of the mathematical concepts and procedures required by the task.**

**0—The response has no correct answer and *insufficient* evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task for that grade level.**

The response may show only information copied from the question.

Special Categories within zero reported separately:

BLK (blank).............Is blank, is entirely erased, or gives a written refusal to respond

OT..........................Is off-task

LOE........................Is in a language other than English

IL............................Is illegible

# Grade 8 Formula Sheet

Formulas that you may need on this test are found below.
You may refer back to this page at any time during the mathematics test.
You may use calculator π or the number 3.14 as an approximation of π.

2022
Grade 8

## Exponential Properties

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-1} = \frac{1}{a}$$

## Algebraic Equations

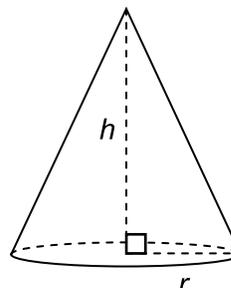**Slope:** $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

**Slope-Intercept Form:** $\quad y = mx + b$

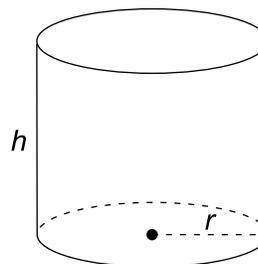## Pythagorean Theorem
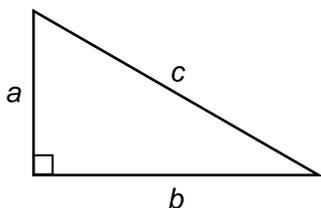


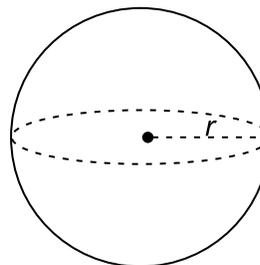$$a^2 + b^2 = c^2$$

## Cone



$$V = \frac{1}{3}\pi r^2 h$$

## Cylinder



$$V = \pi r^2 h$$

## Sphere



$$V = \frac{4}{3}\pi r^3$$

# MATHEMATICS TEST DIRECTIONS

On the following pages are the mathematics questions.

- You may <u>not</u> use a calculator for question 1. You may use a calculator for all other questions on this test.

**Directions for Multiple-Choice Questions**

Some questions will ask you to select an answer from among four choices.

For the multiple-choice questions:

- First solve the problem on scratch paper.

- Choose the correct answer and record your choice in the answer booklet.

- If none of the choices matches your answer, go back and check your work for possible errors.

- Only one of the answers provided is the correct response.

**Directions for Open-Ended Questions**

Some questions will require you to write your response.

For the open-ended questions:

- These questions have more than one part. Be sure to read the directions carefully.

- You cannot receive the highest score for an open-ended question without completing all tasks in the question. For example, if the question asks you to show your work or explain your reasoning, be sure to show your work or explain your reasoning in the space provided.

- If the question does **not** ask you to show your work or explain your reasoning, you may use the space provided, but only those parts of your response that the question specifically asks for will be scored.

- Write your response in the appropriate location within the response box in the answer booklet. Some answers may require graphing, plotting, labeling, drawing, or shading. If you use scratch paper, be sure to transfer your final response and any needed work or reasoning to the answer booklet.

Question 1 in this sampler is to be solved **without** the use of a calculator.

## MULTIPLE-CHOICE ITEMS

**1.** Which value is **closest** to $5\sqrt[3]{26}$?

   A.  5

   B.  15

   C.  25

   D.  45

| Item Information | |
|---|---|
| Alignment | A-N.1.1.3 |
| Answer Key | B |
| Depth of Knowledge | 1 |
| *p*-value A | 13% |
| *p*-value B | 46% (correct answer) |
| *p*-value C | 23% |
| *p*-value D | 18% |
| Option Annotations | A.  approximates the value as cube root of (5 • 26) <br> B.  Correct: recognizes that 26 is close to 27, identifies the cube root of 27 as 3, and then multiplies 5 by 3 <br> C.  approximates the value using $5 • \sqrt{25}$ <br> D.  approximates the value using 5(27 ÷ 3) |

**A calculator is permitted for use in solving questions 2–16 in this sampler.**

2. Which statement about non-zero real numbers is true?

    A. The sum of two rational numbers is not always rational.

    B. The product of two irrational numbers is always rational.

    C. The product of a rational number and an irrational number is always irrational.

    D. The sum of a rational number and an irrational number is sometimes irrational.

| Item Information | |
|---|---|
| Alignment | A-N.1.1 |
| Answer Key | C |
| Depth of Knowledge | 2 |
| *p*-value A | 19% |
| *p*-value B | 14% |
| *p*-value C | 41% (correct answer) |
| *p*-value D | 26% |
| Option Annotations | A. miscalculates a case where two rational numbers have a sum that is irrational OR considers the sum of a repeating decimal and a terminating decimal to be irrational<br>B. confuses the product of two irrational numbers with the product of two rational numbers OR considers only situations such as $\sqrt{2} \bullet \sqrt{2}$<br>C. Correct: recognizes that when a number that can be written as a fraction (i.e., a rational number) is multiplied by a number that cannot be written as a fraction (i.e., an irrational number), the product cannot be written as a fraction<br>D. miscalculates a case where a rational number and irrational number have a sum that is rational OR considers a negative number to be irrational and thinks the sum of a number and its opposite is 0 |

3. A city has approximately 1,000,000 residents. The city has approximately 1 library for every 5,000 residents. Which expression represents the approximate number of libraries in the city?

A. $\dfrac{5 \times 10^3}{1 \times 10^6}$

B. $\dfrac{5 \times 10^4}{1 \times 10^6}$

C. $\dfrac{1 \times 10^6}{5 \times 10^4}$

D. $\dfrac{1 \times 10^6}{5 \times 10^3}$

| Item Information | |
|---|---|
| Alignment | B-E.1.1 |
| Answer Key | D |
| Depth of Knowledge | 2 |
| *p*-value A | 26% |
| *p*-value B | 10% |
| *p*-value C | 12% |
| *p*-value D | 52% (correct answer) |
| Option Annotations | A. divides in the wrong order<br>B. misrepresents 5,000 and divides in the wrong order<br>C. misrepresents 5,000<br>D. Correct: interprets the situation as 1,000,000 ÷ 5,000, writes this division as a fraction, and converts 1,000,000 as $1 \times 10^6$ and 5,000 as $5 \times 10^3$ |

4. A cubic meter of water has a mass of $(1 \times 10^3)$ kilograms. A large swimming pool at a community center holds a mass of approximately $(3 \times 10^6)$ kilograms of water when it is completely full. What is the approximate volume, in cubic meters, of water the large swimming pool holds when it is completely full?

    A.   $2 \times 10^2$

    B.   $3 \times 10^2$

    C.   $2 \times 10^3$

    D.   $3 \times 10^3$

| Item Information | |
|---|---|
| Alignment | B-E.1.1.4 |
| Answer Key | D |
| Depth of Knowledge | 2 |
| $p$-value A | 7% |
| $p$-value B | 15% |
| $p$-value C | 19% |
| $p$-value D | 59% (correct answer) |
| Option Annotations | A.   subtracts the coefficients $(3 - 1 = 2)$ and divides the exponents $(6 \div 3 = 2)$ <br> B.   divides the coefficients $(3 \div 1 = 3)$ and exponents $(6 \div 3 = 2)$ <br> C.   subtracts the coefficients $(3 - 1 = 2)$ and exponents $(6 - 3 = 3)$ <br> D.   Correct: interprets the situation as $(3 \times 10^6) \div (1 \times 10^3)$, divides the coefficients $(3 \div 1 = 3)$, and divides the powers of 10 by subtracting the exponents $(6 - 3 = 3)$ |

**5.** Eva and Rafael are each tutors. The graph below shows the amount ($y$), in dollars, Eva charges based on the number of hours she tutors.

**Tutoring Charges**
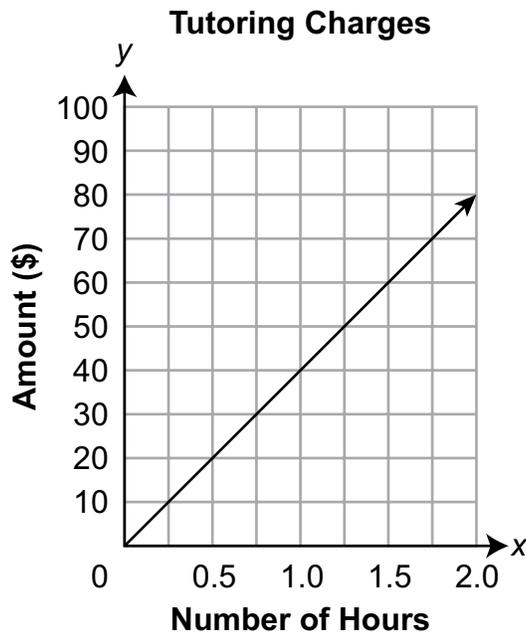


The equation $y = 35x$ represents the amount ($y$), in dollars, Rafael charges when he tutors for $x$ hours. His equation will be graphed on the same coordinate grid as Eva's graph. Based on this information, which statement is true?

A. Eva charges $5 more per hour than Rafael, and she has a higher initial fee, so their graphs will never intersect.

B. Eva and Rafael each charge the same amount per hour, but Rafael also charges an initial fee, so their graphs are parallel.

C. Eva charges $5 more per hour than Rafael, and neither charges an initial fee, so the only place their graphs will intersect is at (0, 0).

D. Eva and Rafael each charge the same amount per hour and do not charge an initial fee, so the only place their graphs will intersect is at (0, 0).

| Item Information | |
|---|---|
| Alignment | B-E.2.1.1<br>B-E.3.1.3 |
| Answer Key | C |
| Depth of Knowledge | 2 |
| *p*-value A | 17% |
| *p*-value B | 14% |
| *p*-value C | 56% (correct answer) |
| *p*-value D | 13% |
| Option Annotations | A.   finds conditions for lines that never intersect<br>B.   assumes the graphs never intersect so the lines are parallel<br>C.   Correct: interprets (1.0, 40) on the graph as Eva charging $40 per hour and the rate of change (i.e., the coefficient of *x*) in the equation as Rafael charging $35 per hour, determines that Eva charges $5 more per hour by subtracting the two rates ($40 – $35), interprets (0, 0) on the graph and the lack of a constant term in the equation as Eva and Rafael not charging an initial fee, and recognizes that linear relations with different slopes will have only one point of intersection<br>D.   does not realize the conditions that would make them have the same line OR realizes (0, 0) is a point on both lines but does not recognize that they represent different lines |

**6.** Similar triangles JKL and TSR are graphed on the coordinate grid shown below.
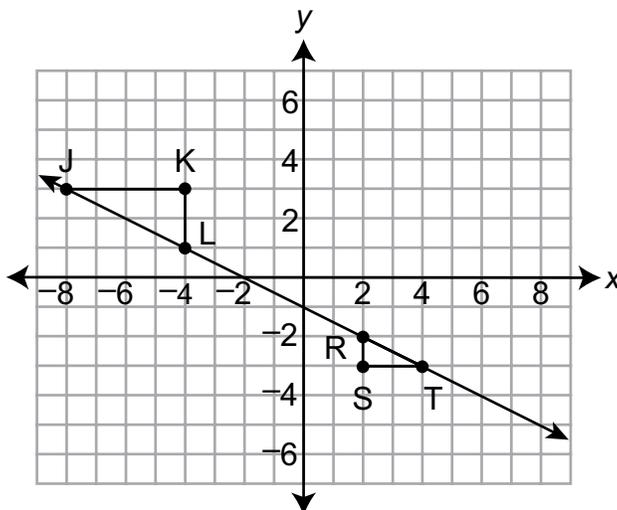


The slope of $\overline{JL}$ is $\frac{m}{2}$. The slope of $\overline{RT}$ is $q$. Which equation describes the relationship between $m$ and $q$?

A.  $m = {}^-2q$

B.  $m = \dfrac{q}{2}$

C.  $m = q$

D.  $m = 2q$

## Item Information

| | |
|---|---|
| Alignment | B-E.2.1.2<br>B-E.3.1 |
| Answer Key | D |
| Depth of Knowledge | 2 |
| *p*-value A | 18% |
| *p*-value B | 28% |
| *p*-value C | 17% |
| *p*-value D | 37% (correct answer) |
| Option Annotations | A. recognizes that the slopes are negative but does not consider that *m* and *q* can be negative numbers<br>B. thinks *q* needs to be divided by 2 since *m* was divided by 2 in the stem<br>C. understands that the slopes of line segments JL and RT must be equal but represents the slope of line segment JL as *m*<br><br>D. Correct: recognizes that the slopes are equal, sets up the equation $\frac{m}{2} = q$, and then solves for *m* by multiplying both sides of the equation by 2 |

**7.** A system of two linear equations has no solution. Which statement about the two lines representing the system of equations is true?

    A.   The two lines must have the same slope and the same *y*-intercept.

    B.   The two lines must have different slopes but the same *y*-intercept.

    C.   The two lines must have the same slope but different *y*-intercepts.

    D.   The two lines must have different slopes and different *y*-intercepts.

| Item Information | |
|---|---|
| Alignment | B-E.3 |
| Answer Key | C |
| Depth of Knowledge | 2 |
| *p*-value A | 21% |
| *p*-value B | 15% |
| *p*-value C | 44% (correct answer) |
| *p*-value D | 20% |
| Option Annotations | A.  considers only the same slope and does not recognize that the lines are colinear OR confuses "infinitely many solutions" for "no solutions"<br>B.  does not recognize that lines with the same *y*-intercept will have a solution at the *y*-intercept<br>C.  Correct: recognizes that two lines with the same slope but different *y*-intercepts will be parallel (i.e., they will never intersect) and, therefore, will have no solutions<br>D.  considers only the different *y*-intercepts and does not recognize that lines with different slopes will always have a solution |

8. A system of linear equations is shown below.

$$x + 2 = 7$$

$$2x + 3y = 8$$

Which statement describes one method to find the solution of the system of linear equations?

A. Determine the point at which $2x + 3y = 8$ crosses the vertical line $x = 5$.

B. Determine the point at which $2x + 3y = 8$ crosses the vertical line $x = 6$.

C. Determine the point at which $2x + 3y = 8$ crosses the vertical line $x = 7$.

D. Determine the point at which $2x + 3y = 8$ crosses the vertical line $x = 9$.

| Item Information | |
|---|---|
| Alignment | B-E.3.1.4 |
| Answer Key | A |
| Depth of Knowledge | 2 |
| $p$-value A | 55% (correct answer) |
| $p$-value B | 13% |
| $p$-value C | 24% |
| $p$-value D | 8% |
| Option Annotations | A. Correct: recognizes that the solution will occur at the point where the second equation crosses the first equation and solves the first equation by subtracting 2 from both sides to get $x = 5$<br>B. combines the solving of the two equations and subtracts 2 from 8<br>C. does not consider the "+ 2" in the first equation and simplifies it as $x = 7$<br>D. adds 2 to 7 in the first equation |

**9.** Ms. Martin and Mrs. Tyler each have a car. They each park at a coin-operated parking meter.

| Ms. Martin | Mrs. Tyler |
| --- | --- |
| The meter has 0 minutes of time remaining. | The meter has 35 minutes of time remaining. |
| She inserts only quarters into the meter. | She inserts only dimes into the meter. |
| Each quarter increases the time by 15 minutes. | Each dime increases the time by 8 minutes. |

Ms. Martin and Mrs. Tyler each insert the same number of coins into the meters. They are able to park their cars for the same amount of time before their meters expire. For what amount of time are Ms. Martin and Mrs. Tyler able to park their cars?

A. 75 minutes

B. 85 minutes

C. 90 minutes

D. 110 minutes

## Item Information

| | |
|---|---|
| Alignment | B-E.3.1.5 |
| Answer Key | A |
| Depth of Knowledge | 2 |
| *p*-value A | 60% (correct answer) |
| *p*-value B | 15% |
| *p*-value C | 17% |
| *p*-value D | 8% |
| Option Annotations | A. Correct: sets up Ms. Martin's equation as $y = 15x$ by using "each quarter . . . 15 minutes" as the slope, while recognizing that "0 minutes of time remaining" means the equation has no initial value; sets up Mrs. Tyler's equation as $y = 8x + 35$ by using "each dime . . . 8 minutes" as the slope and "35 minutes of time remaining" as the initial value; solves the equations by setting up $15x = 8x + 35$ (since both expressions are equal to $y$), subtracting $8x$ from each side to get $7x = 35$, dividing each side by 7 to get $x = 5$; and then substitutes $x$ into either equation to determine $y$ (either 15(5) OR 8(5) + 35)<br>B. calculates $(15 \times 8) - 35 = 85$ and uses 85 as the number of minutes<br>C. determines a possible amount of time for Ms. Martin's meter but not a possible amount of time for both meters<br>D. determines that 75 is a possible amount of time for each meter but then adds the 35 minutes that is already on Mrs. Tyler's meter |

**10.** Beth and Phil each have gift cards for a local coffee shop. Beth has $50 on her gift card, and Phil has $30 on his gift card. Every time they go to the coffee shop, each buys his or her favorite drink. The table and graph shown below model the amount remaining on each person's gift card based on the number of drinks purchased.

**Beth's Gift Card**

| Number of Drinks Purchased | Remaining Balance |
|---|---|
| 0 | $50.00 |
| 3 | $41.75 |
| 6 | $33.50 |
| 9 | $25.25 |

**Phil's Gift Card**
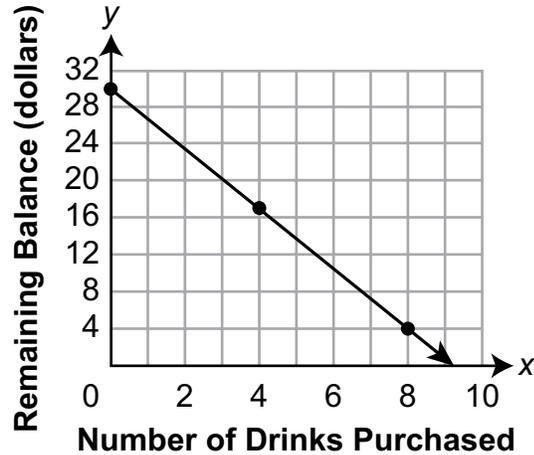


What are the prices of Beth's favorite drink and Phil's favorite drink?

A.  Beth's favorite drink: $2.75
    Phil's favorite drink: $3.75

B.  Beth's favorite drink: $2.75
    Phil's favorite drink: $3.25

C.  Beth's favorite drink: $2.81
    Phil's favorite drink: $3.25

D.  Beth's favorite drink: $2.81
    Phil's favorite drink: $3.75

## Item Information

| | |
|---|---|
| Alignment | B-F.1.1.2 |
| Answer Key | B |
| Depth of Knowledge | 2 |
| *p*-value A | 19% |
| *p*-value B | 64% (correct answer) |
| *p*-value C | 9% |
| *p*-value D | 8% |
| Option Annotations | A. calculates Beth's drink correctly but Phil's drink as $\frac{30}{8}$ <br><br> B. Correct: determines the rate of change in the table by dividing the difference between two remaining balances (e.g., \$33.50 – \$50.00 = ⁻\$16.50) by the difference between the respective number of drinks purchased (e.g., 6 – 0 = 6), determines the rate of change in the graph by dividing the difference between the *y*-coordinates of two points (e.g., \$4.00 – \$30.00 = ⁻\$26.00) by the difference between the *x*-coordinates of the same two points (e.g., 8 – 0 = 8), and interprets the negative rates as the costs of the drinks <br><br> C. calculates Beth's drink as 25.25 ÷ 9 <br><br> D. calculates Beth's drink as 25.25 ÷ 9 and Phil's drink as $\frac{30}{8}$ |

**11.** The graph below shows the total snowfall (*y*), in inches, during a day in February, based on the time, in hours, since the snowfall began.
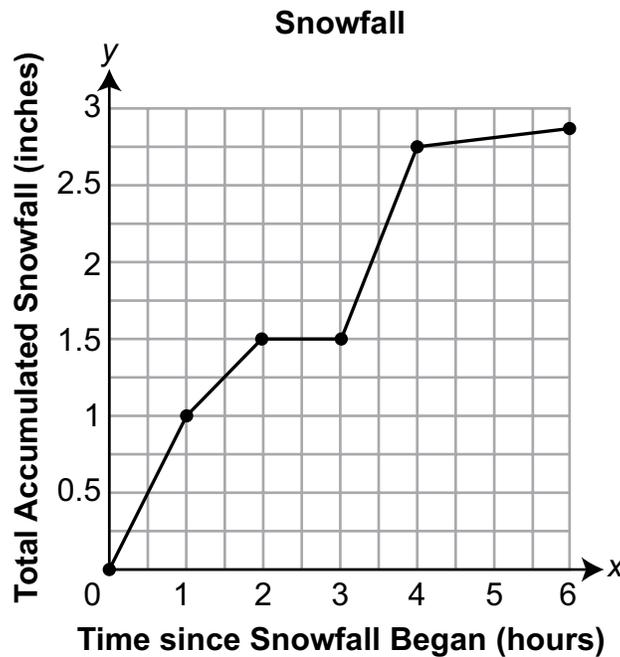
**Snowfall**



Based on the graph, which statement about the snowfall is true?

A.  The snowfall increased the fastest from hour 3 to hour 4.

B.  The snowfall increased the fastest from hour 4 to hour 6.

C.  From hour 2 to hour 3, the snowfall increased by 1.5 inches.

D.  From hour 0 to hour 6, the snowfall increased at a constant rate.

| Item Information | |
|---|---|
| Alignment | B-F.2.1.2 |
| Answer Key | A |
| Depth of Knowledge | 2 |
| *p*-value A | 76% (correct answer) |
| *p*-value B | 9% |
| *p*-value C | 9% |
| *p*-value D | 6% |
| Option Annotations | A. Correct: interprets "increased the fastest" as the interval with the steepest slope and recognizes that the interval with the steepest slope is from hour 3 to hour 4<br>B. selects the interval that ends with the greatest amount of snowfall<br>C. incorrectly considers that the *y*-value represents the hourly rate<br>D. considers "constant rate" to mean "constantly increasing" instead of "increasing at a steady rate" |

**12.** Congruent quadrilaterals PRST and P'R'S'T' are graphed on the coordinate grid shown below.
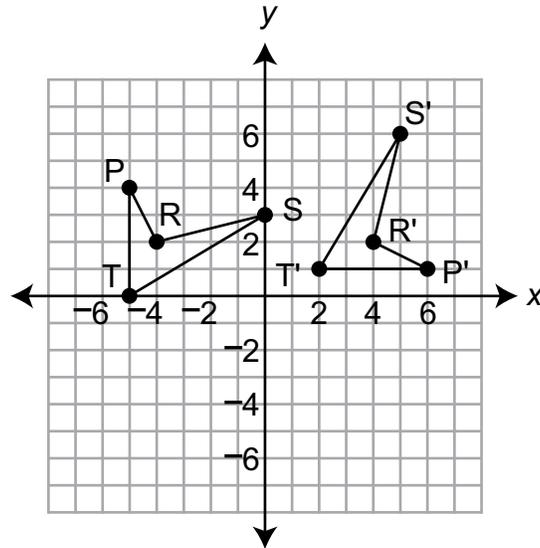


Which transformation or sequence of transformations could be used to show the congruence between the quadrilaterals?

A. a 180° clockwise rotation about point S

B. a reflection across the line $y = x$, followed by a reflection across the $x$-axis

C. a translation 8 units right, followed by a 90° counterclockwise rotation about the origin

D. a 90° counterclockwise rotation about point R, followed by a reflection across the $y$-axis

| Item Information | |
|---|---|
| Alignment | C-G.1.1.2 |
| Answer Key | D |
| Depth of Knowledge | 2 |
| *p*-value A | 14% |
| *p*-value B | 16% |
| *p*-value C | 20% |
| *p*-value D | 50% (correct answer) |
| Option Annotations | A.  produces a figure that appears in quadrant I but has an incorrect orientation <br> B.  knows that two reflections result in a rotation and uses a pair of reflections that will produce a figure that appears in quadrant I but has an incorrect orientation <br> C.  uses a translation to map point R to point R', recognizes that a rotation occurred, but considers that all rotations need to be centered at the origin <br> D.  Correct: identifies a set of transformations that moves point P to point P' by rotating point P from ($^-$5, 4) to ($^-$6, 1) and then reflecting it from 6 units to the left of the *y*-axis to 6 units to the right of the *y*-axis, moves point R to point R' by rotating point R onto itself (since it is the center of rotation) and then reflecting it from 4 units to the left of the *y*-axis to 4 units to the right of the *y*-axis, moves point S to point S' by rotating point S from (0, 4) to ($^-$5, 6) and then reflecting it from 5 units to the left of the *y*-axis to 5 units to the right of the *y*-axis, moves point T to point T' by rotating point T from ($^-$5, 0) to ($^-$2, 1) and then reflecting it from 2 units to the left of the *y*-axis to 2 units to the right of the *y*-axis |

**13.** A random sample of students who ride the bus or walk from home to school are surveyed. Two scatter plots are created from the data. The conclusions below are based on those scatter plots.

- As the distance from home increases, the number of times a week a student rides the bus to school increases.
- As the distance from home increases, the number of times a week a student walks to school decreases.

Which set of statements about one of the scatter plots is **most likely** true?

A. The *x*-axis represents the number of times a week a student walks to school, and the *y*-axis represents the distance from home. The line of best fit has a positive slope.

B. The *x*-axis represents the number of times a week a student rides the bus to school, and the *y*-axis represents the distance from home. The line of best fit has a negative slope.

C. The *x*-axis represents the distance from home, and the *y*-axis represents the number of times a week a student walks to school. The line of best fit has a negative slope.

D. The *x*-axis represents the distance from home, and the *y*-axis represents the number of times a week a student rides the bus to school. The line of best fit has a negative slope.

| Item Information | |
|---|---|
| Alignment | D-S.1 |
| Answer Key | C |
| Depth of Knowledge | 2 |
| *p*-value A | 24% |
| *p*-value B | 20% |
| *p*-value C | 42% (correct answer) |
| *p*-value D | 14% |
| Option Annotations | A. uses a relation in which the distance from home is dependent upon the number of times a week a student walks to school |
| | B. uses a relation in which the distance from home is dependent upon the number of times a week a student rides the bus to school |
| | C. Correct: recognizes that the *x*-axis should represent the distance from home since it is the independent variable and recognizes that the slope would be negative (as the distance from home increases, the number of times walking should decrease) |
| | D. does not recognize that this situation would have a positive slope (as the distance from home increases, the number of times riding the bus should also increase) |

**14.** A researcher analyzes a bivariate data set and determines that the data set is closely, but not exactly, modeled by a function. The researcher's description of the function is given below.

Each *y*-value in the data set is about 3 more than 4 times the corresponding *x*-value in the data set.

Based on the description, which table **most likely** contains values in the data set?

A.

| *x* | 12 | 18 | 21 | 28 | 43 | 62 |
|-----|------|------|------|------|-------|-------|
| *y* | 3.00 | 4.50 | 5.25 | 7.00 | 10.75 | 15.50 |

B.

| *x* | 2 | 6 | 7 | 13 | 25 | 32 |
|-----|------|-------|-------|-------|-------|--------|
| *y* | 8.15 | 23.60 | 28.25 | 51.85 | 99.75 | 128.40 |

C.

| *x* | 5 | 14 | 19 | 26 | 34 | 47 |
|-----|-------|-------|-------|--------|--------|--------|
| *y* | 23.60 | 59.25 | 78.75 | 107.45 | 138.50 | 190.85 |

D.

| *x* | 8 | 11 | 22 | 36 | 45 | 54 |
|-----|------|------|------|-------|-------|-------|
| *y* | 5.00 | 5.75 | 8.50 | 12.00 | 14.25 | 16.50 |

| Item Information | |
|---|---|
| Alignment | D-S.1.1 |
| Answer Key | C |
| Depth of Knowledge | 2 |
| *p*-value A | 22% |
| *p*-value B | 17% |
| *p*-value C | 51% (correct answer) |
| *p*-value D | 10% |
| Option Annotations | A.  identifies a relationship that models $y = \frac{1}{4}x$ exactly <br><br> B.  identifies a relationship that is close to, but not exactly, $y = 4x$ (i.e., does not apply the "3 more") <br><br> C.  Correct: identifies a relationship that is close to, but not exactly, $y = 4x + 3$ by multiplying each *x*-value by 4 before adding 3, and then comparing the result to the *y*-value (e.g., $4(5) + 3 = 23$, which is close to, but not exactly, 23.60) <br><br> D.  identifies a relationship that models $y = \frac{1}{4}x + 3$ exactly |

**15.** Sandra interviewed 250 adults for a survey. She asked each adult the two questions listed below.

- What is your age?
- Do you rent or own your home?

Her survey results are shown in the frequency table below.

**Sandra's Survey Results**

| Age Span (years) | Rent Home | Own Home |
|---|---|---|
| 18–35 | 56 | 23 |
| 36–55 | 18 | 85 |
| 56+ | 30 | 38 |

Based on the information in the frequency table, which statement is true?

A. There is no linear association between adults who rent a home and their age.

B. As age increases, adults typically move from renting to owning a home.

C. Adults aged 36 to 55 are 7.2% less likely to rent a home than any other age group.

D. Adults aged 18 to 35 are more likely to rent a home than those aged 36 to 55.

## Item Information

| | |
|---|---|
| Alignment | D-S.1.2.1 |
| Answer Key | D |
| Depth of Knowledge | 2 |
| *p*-value A | 9% |
| *p*-value B | 14% |
| *p*-value C | 12% |
| *p*-value D | 65% (correct answer) |
| Option Annotations | A.  does not interpret the differences in relative frequencies |
| | B.  only compares the first two age groups |
| | C.  determines the relative frequency of people aged 36–55 who rent a home $\left(\frac{18}{250} = 7.2\%\right)$ but interprets this incorrectly |
| | D.  Correct: determines that about 70.9% of people aged 18–35 rent a home $\left(\frac{56}{56 + 23} = \frac{56}{79}\right)$, while only about 17.5% of people aged 36–55 rent a home $\left(\frac{18}{18 + 85} = \frac{18}{103}\right)$ |

## OPEN-ENDED QUESTION

**16.** Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

---

The discounted price, in dollars, the gym charges can be represented by the equation $y = 10x + 5$.

**A.** What are the slope and the *y*-intercept of the equation? What do the slope and the *y*-intercept each represent in this situation?

---

The regular price, in dollars, the gym charges can be represented by the equation $y = 15x + 20$.

**B.** How much money, in dollars, does Justin save the first month by joining the gym at the discounted price rather than at the regular price?

---

**Go to the next page to finish question 16.**

GO ON

**16.** *Continued.* Please refer to the previous page for task explanation.

Justin creates a system of equations based on the equation from **part A** and the equation from **part B**. The solution to the system of equations is ($^-$3, $^-$25).

**C.** Why is the point ($^-$3, $^-$25) **not** a possible solution in this situation?

**After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.**

STOP

## Item-Specific Scoring Guideline

## #16 Item Information

| Alignment | B-F.2.1<br>B-E.3.1.3 | Depth of<br>Knowledge | 2 | Mean Score | 1.78 |
|---|---|---|---|---|---|

## Assessment Anchor this item will be reported under:

M08.B-F.2—Use functions to model relationships between quantities.

## Specific Anchor Descriptor addressed by this item:

M08.B-F.2.1—Represent or interpret functional relationships between quantities using tables, graphs, and descriptions.

M08.B-E.3.1—Write, solve, graph, and interpret linear equations in one or two variables, using various methods.

## Scoring Guide

| Score | In this item, the student . . . |
|---|---|
| 4 | Demonstrates a thorough understanding of using functions to model relationships between quantities by correctly solving problems and clearly explaining procedures. |
| 3 | Demonstrates a general understanding of using functions to model relationships between quantities by correctly solving problems and clearly explaining procedures with only minor errors or omissions. |
| 2 | Demonstrates a partial understanding of using functions to model relationships between quantities by correctly performing a significant portion of the required task. |
| 1 | Demonstrates minimal understanding of using functions to model relationships between quantities. |
| 0 | The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures as required by the task. The response may show only information copied from the question. |

**Top-Scoring Student Response and Training Notes**

| Score | Description |
|-------|-------------|
| 4 | Student earns 4 points. |
| 3 | Student earns 3.0–3.5 points. |
| 2 | Student earns 2.0–2.5 points. |
| 1 | Student earns 0.5–1.5 points.<br><br>    OR<br><br>Student demonstrates minimal understanding of using functions to model relationships between quantities. |
| 0 | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |

## Top-Scoring Response

**Part A (2 points):**

$\frac{1}{2}$ point for <u>each</u> correct answer

$\frac{1}{2}$ point for <u>each</u> correct explanation

| What? | Why? |
|---|---|
| slope: 10<br>$y$-intercept: 5 | **Sample Explanation:**<br><br>The slope represents the monthly charge (of $10).<br><br>**AND**<br><br>The $y$-intercept represents the joining fee (of $5).<br><br>**OR equivalent** |

**Part B (1 point):**

1 point for correct answer

| What? | Why? |
|---|---|
| ($)20 | |

**Part C (1 point):**

1 point for correct and complete explanation

OR $\frac{1}{2}$ point for correct but incomplete explanation

| What? | Why? |
|---|---|
| | **Sample Explanation:**<br><br>The point ($^-$3, $^-$25) implies going back in time and receiving money from the gym at the point where the two functions intersect. That is why the point ($^-$3, $^-$25) is not possible in terms of this context.<br><br>**OR equivalent** |

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INTENTIONALLY BLANK.**

## STUDENT RESPONSE

## Response Score: 4 points

**16.** Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

---

The discounted price, in dollars, the gym charges can be represented by the equation $y = 10x + 5$.

**A.** What are the slope and the *y*-intercept of the equation? What do the slope and the *y*-intercept each represent in this situation?

Slope is 10      *y*-intercept is 5

The slope represents the monthly rate Justin has to pay.

the y-intercept represents the discounted fee to join.

> The student correctly identified the slope (*Slope is 10*) and what the slope represents (*the monthly rate Justin has to pay*). The student also correctly identified the *y*-intercept (*y-intercept is 5*) and what the *y*-intercept represents (*the discounted fee to join*). [2 points]

---

The regular price, in dollars, the gym charges can be represented by the equation $y = 15x + 20$.

**B.** How much money, in dollars, does Justin save the first month by joining the gym at the discounted price rather than at the regular price?

regular price

$y = 15x + 20$
$y = 15(1) + 20$
$y = 15 + 20$
$y = 35$

$\dfrac{\begin{array}{r} 35 \\ -15 \end{array}}{20}$

discounted price

$y = 10x + 5$
$y = 10(1) + 5$
$y = 10 + 5$
$y = 15$

∴ Justin Saves $20 the first month

> The student provided the correct answer (*Justin saves $20 the first month*). The work shown is correct, though not necessary for credit. The student showed how each first month's cost is calculated by starting with the equation for the regular price ($y = 15x + 20$), substituting 1 in for *x*, and then solving it to get $y = 35$. The student then took the discounted price equation ($y = 10x + 5$), substituted 1 in for *x*, and then solved it to get $y = 15$. By subtracting the two *y*-values ($35 - 15$), the student determined that *Justin saves $20*. [1 point]

**16.** *Continued.* Please refer to the previous page for task explanation.

Justin creates a system of equations based on the equation from **part A** and the equation from **part B**. The solution to the system of equations is ($^-$3, $^-$25).

**C.** Why is the point ($^-$3, $^-$25) **not** a possible solution in this situation?

The point (-3, -25) is not a possible solution because both of the numbers are negative. In a real life situation, you can not go a negative amount of months or pay a negative amount of money. The solution set is saying he is going negative -3 months and paying $-25. It is just not possible.

The student provided a correct and complete explanation as to why the point ($^-$3, $^-$25) is not a possible solution (*In a real life situation, you can not go a negative amount of months or pay a negative amount of money*). The student correctly interprets that negative numbers in this context are not realistic. [1 point]

**After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.**

STOP

## STUDENT RESPONSE

## Response Score: 3 points

## PARTS A and B

Question 16
Page 1 of 2

Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

The discounted price, in dollars, the gym charges can be represented by the equation $y = 10x + 5$.

**A.** What are the slope and the $y$-intercept of the equation? What do the slope and the $y$-intercept each represent in this situation?

> The slope an $y$-intercept represent the monthly rate and the joining fee. The $y$-intercept is the joining fee. The $y$-intercept is the joining fee and the slope is the monthly fee.

141 / 1000

The student correctly identified what the slope represents (*the slope is the monthly fee*) and what the $y$-intercept represents (*The y-intercept is the joining fee*). However, the student did not identify that the 10 is the slope and the 5 is the $y$-intercept, so no further credit is awarded. [1 point]

The regular price, in dollars, the gym charges can be represented by the equation $y = 15x + 20$.

**B.** How much money, in dollars, does Justin save the first month by joining the gym at the discounted price rather than at the regular price?

> He saves 20 dollars.

The student provided the correct answer (*He saves 20 dollars*). While support is not required for Part B, the student likely calculated the regular price and the discounted price for the first month by substituting 1 in for $x$ into both equations, solving each equation for $y$, and then subtracting the two $y$-values. [1 point]

## PART C

Question 16
Page 2 of 2

Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

Justin creates a system of equations based on the equation from **part A** and the equation from **part B**. The solution to the system of equations is (−3, −25).

**C.** Why is the point (−3, −25) **not** a possible solution in this situation?

The point (-3, -25) is not a solution because a negative would mean the gym is paying him to go there.

The student provided a correct and complete explanation as to why the point (−3, −25) is not a possible solution (*a negative would mean the gym is paying him to go there*). The student correctly interpreted that negative numbers in this context are not realistic. [1 point]

## STUDENT RESPONSE

## Response Score: 2 points

**16.** Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

---

The discounted price, in dollars, the gym charges can be represented by the equation $y = 10x + 5$.

**A.** What are the slope and the $y$-intercept of the equation? What do the slope and the $y$-intercept each represent in this situation?

The y represents the amount of time (Months) that has passed. The slope represents the relationship between the monthly rate and the fee to join.

> The student incorrectly identified what both the slope and $y$-intercept represent (*The y represents the amount of time (months) that has passed. The slope represents the relationship between the monthly rate and the fee to join*). Additionally, the 10 was not identified as the slope and the 5 was not identified as the $y$-intercept. [0 points]

The regular price, in dollars, the gym charges can be represented by the equation $y = 15x + 20$.

**B.** How much money, in dollars, does Justin save the first month by joining the gym at the discounted price rather than at the regular price?

| | discount | regular | amount saved |
|---|---|---|---|
| 1 | $15 | $35 | $20 |

Justin would save $20 on the discount Price.

> The student provided the correct answer (*Justin would save $20 on the discount Price*). The work shown is correct, though not necessary for credit. The student calculated the first month's discounted price of $15 (likely by substituting 1 in for $x$ into the equation $y = 10x + 5$ and solving for $y$), the first month's regular price of $35 (likely by substituting 1 in for $x$ into the equation $y = 15x + 20$ and solving for $y$), and the amount saved of $20 (likely by subtracting $15 from $35). [1 point]

---

**16.** *Continued.* Please refer to the previous page for task explanation.

> Justin creates a system of equations based on the equation from **part A** and the equation from **part B**. The solution to the system of equations is (⁻3, ⁻25).
>
> **C.** Why is the point (⁻3, ⁻25) **not** a possible solution in this situation?
>
> (-3, -25) can not be a solution to this situation beecuse the numbers ore negative and Justin con not pay his gym a negotive emount of money.
>
> > The student provided a correct and complete explanation as to why the point (⁻3, ⁻25) is not a possible solution (*Justin con not pay his gym a negotive emount of money*). The student correctly interpreted that negative numbers in this context are not realistic. [1 point]

**After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.**

STOP

## STUDENT RESPONSE

**Response Score: 1 point**

## PARTS A and B

Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

The discounted price, in dollars, the gym charges can be represented by the equation $y = 10x + 5$.

A.  What are the slope and the $y$-intercept of the equation? What do the slope and the $y$-intercept each represent in this situation?

The slope represents the initial cost and the $y$-intercept is the extra cost per month

> The student incorrectly identified what both the slope and $y$-intercept represent (*The slope represents the initial cost and the $y$-intercept is the extra cost per month*). The student has confused the two values, as the slope is the monthly charge and the $y$-intercept is the joining fee. Additionally, the 10 was not identified as the slope and the 5 was not identified as the $y$-intercept. [0 points]

84 / 1000

The regular price, in dollars, the gym charges can be represented by the equation $y = 15x + 20$.

B.  How much money, in dollars, does Justin save the first month by joining the gym at the discounted price rather than at the regular price?

twenty
dollars

> The student provided the correct answer (*twenty dollars*). While support is not required for Part B, the student likely calculated the regular price and the discounted price for the first month by substituting 1 in for $x$ into both equations, solving each equation for $y$, and then subtracting the two $y$-values. [1 point]

## PART C

Question 16
Page 2 of 2

Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

Justin creates a system of equations based on the equation from **part A** and the equation from **part B**. The solution to the system of equations is (−3, −25).

**C.** Why is the point (−3, −25) **not** a possible solution in this situation?

Point (-3, -25) isn't possible because when the two equations are graphed they form parallel lines

The student provided an incorrect explanation as to why the point (−3, −25) is not a possible solution (*when the two equations are graphed they form parallel lines*). The response does not recognize that negative numbers in this context are not realistic. [0 points]

## STUDENT RESPONSE

## Response Score: 0 points

**16.** Justin is joining a gym. The gym is currently offering a discount on the fee to join and on the monthly rate.

---

The discounted price, in dollars, the gym charges can be represented by the equation $y = 10x + 5$.

**A.** What are the slope and the *y*-intercept of the equation? What do the slope and the *y*-intercept each represent in this situation?

10 means how much it costs

5 means how many months he will go

> The student incorrectly identified what the 10 represents (*means how much it costs*) and what the 5 represents (*means how many months he will go*). The 10 is not identified as either the slope or the monthly rate AND the 5 is not identified as either the *y*-intercept or the initial joining fee. [0 points]

---

The regular price, in dollars, the gym charges can be represented by the equation $y = 15x + 20$.

**B.** How much money, in dollars, does Justin save the first month by joining the gym at the discounted price rather than at the regular price?

He saved $10 the first time.

> The student provided an incorrect answer (*He saved $10 the first time*). No support (work or explanation) is required, so it is unclear where an error was made. [0 points]

---

**Go to the next page to finish question 16.**

**GO ON** ⟩

**16.** *Continued.* Please refer to the previous page for task explanation.

Justin creates a system of equations based on the equation from **part A** and the equation from **part B**. The solution to the system of equations is (⁻3, ⁻25).

**C.** Why is the point (⁻3, ⁻25) **not** a possible solution in this situation?

Because -25 is not on the graph.

The student provided an incorrect explanation as to why the point (⁻3, ⁻25) is not a possible solution (*Because –25 is not on the graph*). The response does not recognize that negative numbers in this context are not realistic. [0 points]

**After you have checked your work, close your answer booklet and test booklet so your teacher will know you are finished.**

**STOP**

# MATHEMATICS—SUMMARY DATA

## Multiple-Choice

| Sample Number | Alignment | Answer Key | Depth of Knowledge | *p*-value A | *p*-value B | *p*-value C | *p*-value D |
|---|---|---|---|---|---|---|---|
| 1 | A-N.1.1.3 | B | 1 | 13% | 46% | 23% | 18% |
| 2 | A-N.1.1 | C | 2 | 19% | 14% | 41% | 26% |
| 3 | B-E.1.1 | D | 2 | 26% | 10% | 12% | 52% |
| 4 | B-E.1.1.4 | D | 2 | 7% | 15% | 19% | 59% |
| 5 | B-E.2.1.1 B-E.3.1.3 | C | 2 | 17% | 14% | 56% | 13% |
| 6 | B-E.2.1.2 B-E.3.1 | D | 2 | 18% | 28% | 17% | 37% |
| 7 | B-E.3 | C | 2 | 21% | 15% | 44% | 20% |
| 8 | B-E.3.1.4 | A | 2 | 55% | 13% | 24% | 8% |
| 9 | B-E.3.1.5 | A | 2 | 60% | 15% | 17% | 8% |
| 10 | B-F.1.1.2 | B | 2 | 19% | 64% | 9% | 8% |
| 11 | B-F.2.1.2 | A | 2 | 76% | 9% | 9% | 6% |
| 12 | C-G.1.1.2 | D | 2 | 14% | 16% | 20% | 50% |
| 13 | D-S.1 | C | 2 | 24% | 20% | 42% | 14% |
| 14 | D-S.1.1 | C | 2 | 22% | 17% | 51% | 10% |
| 15 | D-S.1.2.1 | D | 2 | 9% | 14% | 12% | 65% |

## Open-Ended

| Sample Number | Alignment | Points | Depth of Knowledge | Mean Score |
|---|---|---|---|---|
| 16 | B-F.2.1 B-E.3.1.3 | 4 | 2 | 1.78 |

# PSSA Grade 8 Mathematics
# Item and Scoring Sampler