Automotive Technology (47.0604) T-Chart

Shop operation =	Recognize and evaluate random processes underlying statistical experiments

Program Task: Complete calculations for more efficient shop operation.

Program Associated Vocabulary:
COMBINATIONS, FUNDAMENTAL COUNTING PRINCIPLE

Program Formulas and Procedures:
It is important for automotive technicians to understand that math takes many shapes and forms. Although this math concept may not be directly related to a mechanical repair to help diagnose a drivability problem, this math concept is directly connected to real life and job related situations.

Example 1: Fundamental Counting Principle
How many vehicles would a dealer have to stock to have one of each of a particular model if the following specifications were available?
# of different vehicles = (4 trim levels) x (7 exterior colors) x (3 wheel configurations) x (2 engines options)
= 168 vehicles of the same model

Example 2: Combinations
There are 5 vehicles in for repair. You are assigned to complete repairs on three. How many different combinations of 3 vehicles can you choose out of the 5?

$$\text{Total \# of Combinations} = \frac{n!}{r!(n-r)!}$$
$$\text{where } n = 5 \text{ and } r = 3$$
$$= \frac{5!}{3!(5-3)!} = \frac{5!}{3!(2!)} = \frac{120}{6(2)} = \frac{120}{12} = 10$$

Another way to calculate if you do not have a calculator that computes factorials

$$\text{Total \# of Combinations} = \frac{n!}{r!(n-r)!}$$
$$\text{where } n = 5 \text{ and } r = 3$$
$$= \frac{5!}{3!(5-3)!} = \frac{5!}{3!(2!)} = \frac{5x4x3!}{3!(2!)} = \frac{5x4}{(2!)} = \frac{20}{2} = 10$$

PA Core Standard: CC.2.4.HS.B.4

Description: Recognize and evaluate random processes underlying statistical experiments.

Math Associated Vocabulary:
FACTORIAL, COMBINATIONS, FUNDAMENTAL COUNTING PRINCIPLE, PERMUTATIONS, EVENT, PROBABILITY

Formulas and Procedures:
The Fundamental Counting Principle, Permutation, and Combinations formulas are used to determine the number of sets of outcomes when multiple options are available.

Fundamental Counting Principle – This principle can be used to count the number of ways two or more sets of options can be combined:
(num. option A) x (num. option B) x (num. option C) x …

Example 1:
Combining 3 appetizers, 4 entrees, and 3 desserts into a meal:
(3 possible appetizers) x (4 possible entrees) x (3 possible desserts) = 36 possible meals

Permutation – Choosing **r** items from a list of **n** options when the order of the chosen items **does** matter (A-B-C is different than B-A-C)

$$\text{Permutations} = {}_nP_r = \frac{n!}{(n-r)!}$$

! is **factorial:** n! = n (n - 1)(n - 2) ...(2)(1)
6! = (6)(5)(4)(3)(2)(1) = 720

Example 2:
There are 12 members on a committee. In how ways can a president, secretary, and treasurer be elected—if everyone is willing to serve in each position?

$${}_{12}P_3 = \frac{12!}{(12-3)!} = \frac{12!}{9!} = 12x11x10 = 1320$$

Combination – Choosing **r** items from a list of **n** options - the order of the chosen items does **not** matter (A-B-C and B-A-C are the same).

$$\text{Combinations} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

Example 3:
Picking 3 essays from a list of 9 total and the order does not matter:

$${}_9C_3 = \frac{9!}{3!(9-3)!} = \frac{9!}{3!6!} = 84$$

Instructor's Script – Comparing and Contrasting

Choices often involve picking from a list of options and the number of possible choices may get quite large as more options become available. Understanding the number of possible choices is useful in finding the optimal or most desirable choice. In addition, this type of math can be very useful in calculating probabilities that particular outcomes may occur in the future.

The term "event" is used in mathematics to represent one of the possible choices or outcomes in the described situation. For example, if we consider a vehicle and all the options available, **one event** (out of 168 total combinations) could be a vehicle with a 3.6L engine, 4-speed transmission, yellow paint, and leather interior.

Difference between permutations and combinations:

- Permutations apply in situations where the order in which the possible events appear is important.
  Consider the example of the 3 digits 1, 2, 3. They can be arranged as 123, 231, 312, 132, 213, and 321. This can be viewed as 6 completely different numbers or **permutations** created with 1, 2, and 3.

- Combinations are useful in situations where you are not concerned with the order that the events, simply the group of events or outcomes that may result.
  Consider the example of the 3 digits 1, 2, 3. I can rearrange them to make 123, 231, 312, 132, 213, and 321. However, all of these may be considered as the same single **combination** of 1, 2, and 3 because these numbers are simply the same 3 numbers placed in different order.

- Given the same options, the number of permutations is usually higher than the number of combinations.

Many scientific calculators have factorial, combination, and permutation functions. To focus on understanding the concept of combination and permutation, use these function keys when available.

Common Mistakes Made By Students

Distinguishing between combinations and permutations: Permutations are associated with problems where the order matters. Combinations involve problems where the order does not matter. In addition, there are usually more permutations than combinations because differently ordered duplicate sets are not counted in combination problems.

Unfamiliar with the calculator: Students who borrow calculators or keep switching between styles and models have to continually determine how find the combination, permutation, and factorial functions.

CTE Instructor's Extended Discussion

Technical tasks are usually not presented using this model. Therefore, it is important that technical instructors demonstrate to students how these math concepts link to and are relevant in their technical training and that the math is presented in a way which shows a relationship to the math which CTE students use in their academic school settings.

Automotive technology instructors should help their students become proficient in working with numbers in as many aspects as possible. The most proficient and successful technicians, engineers, and automotive service and repair business owners will be those who are comfortable working with and communicating using numbers and formulas. Furthermore, make the effort and help your students become competent professionals by spending the time necessary to become comfortable understanding and interpreting data, and identifying important number relationships hidden in everyday activities that can be applied to and within the work place.

Problems	Career and Technical Math Concepts	Solutions
1. The 20 technicians in a dealership are voting for officers of their United Auto Works (UAW) local union. How many ways are the president, secretary and treasurer elected from the 20 technicians? In this problem, indicating the order in which the technicians are placed in specific offices matters.		
2. How many ways is a negotiating committee of three technicians selected from the 20 technicians? Since all the committee persons are equal, the order the technicians are chosen does not matter.		
3. You are going to purchase a new roller cabinet. These are your options; 12-drawer styles, 7-exterior colors (including stainless steel) from 4-different manufacturers. From how many different combinations of roller cabinets can you choose?		
Problems	**Related, Generic Math Concepts**	**Solutions**
4. Twelve teams enter a tournament. How many arrangements of third, second, and first place are possible? (Order **does** matter, n = 12 and r = 3).		
5. Five essay questions appear on a test. You must choose three to answer. In how many ways can this be done? (Order **does** not matter, n = 5 and r = 3).		
6. The developer of the facial recognition software states her product can identify over 10 billion different faces based on the following 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheek. Is this claim correct? Part 2 - A witness clearly remembers the hairline and eyes and eyebrows of a suspect. How many different faces can be produced by the software with this information?		
Problems	**PA Core Math Look**	**Solutions**
7. Find the value of the following expression: $\frac{10!}{2!(10-2)!}$ Is this the appropriate formula to use when the order does not matter?		
8. Jenny and her father want to visit all 6 Northeast Major League baseball stadiums over the summer, but only have time to visit 3. Since the cost is dependent on how far they have to travel between stadiums, the order matters. How many different permutations of stadiums visits could they consider?		
9. Every registered taxi cab in New Your City must be issued a medallion plaque. Each medallion consists of 3 numbers (0-9) and one letter (A-Z). How many taxi cabs are can be registered at one time?		

Problems	Career and Technical Math Concepts Solutions
1. The 20 technicians in a dealership are voting for officers of their United Auto Works (UAW) local union. How many ways are the president, secretary and treasurer elected from the 20 technicians? In this problem, indicating the order in which the technicians are placed in specific offices matters.	$\text{Total \# of Permutations} = \frac{n!}{(n-r)!}$ n = 20 and r = 3 $= \frac{20!}{(20-3)!} = \frac{20!}{17!} = 6840$
2. How many ways is a negotiating committee of three technicians selected from the 20 technicians? Since all the committee persons are equal, the order the technicians are chosen does not matter.	$\text{Total \# of Combinations} = \frac{n!}{r!(n-r)!}$ n = 20 and r = 3 $= \frac{20!}{3!(20-3)!} = \frac{20!}{3!(17!)} = 1140$
3. You are going to purchase a new roller cabinet. These are your options; 12-drawer styles, 7-exterior colors (including stainless steel) from 4-different manufacturers. From how many different combinations of roller cabinets can you choose?	12 **x** 7 **x** 4 = 336 different roller cabinet choices.

Problems	Related, Generic Math Concepts Solutions
4. Twelve teams enter a tournament. How many arrangements of third, second, and first place are possible? (Order **does** matter, n = 12 and r = 3).	$\text{Total \# of Permutations} = \frac{n!}{(n-r)!}$ n = 12 and r = 3 $= \frac{12!}{(12-3)!} = \frac{12!}{9!} = \frac{4790016000}{362880} = 1320$
5. Five essay questions appear on a test. You must choose three to answer. In how many ways can this be done? (Order **does not** matter, n = 5 and r = 3).	$\text{Total \# of Combinations} = \frac{n!}{r!(n-r)!}$ n = 5 and r = 3 $= \frac{5!}{3!(5-3)!} = \frac{5!}{3!(2)!} = \frac{120}{12} = 10$
6. The developer of the facial recognition software states her product can identify over 10 billion different faces based on the following 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheek. Is this claim correct? Part 2 - A witness clearly remembers the hairline and eyes and eyebrows of a suspect. How many different faces can be produced by the software with this information?	195 x 99 x 89 x 105 x 74 = 13,349,986,650 or… $1.334998665 \times 10^{10}$ YES, over 13 billion! Part 2 - 1 x 1 x 89 x 105 x 74 = 691530

Problems	PA Core Math Look Solutions
7. Find the value of the following expression: $\frac{10!}{2!(10-2)!}$ Is this the appropriate formula to use when the order does not matter?	$\frac{10!}{2!(10-2)!} = \frac{3628800}{2!(8)!} = \frac{3628800}{2(40320)} = 45$ ${}_{10}C_2 = 45$ Yes, this is a combination.
8. Jenny and her father want to visit all 6 Northeast Major League baseball stadiums over the summer, but only have time to visit 3. Since the cost is dependent on how far they have to travel between stadiums, the order matters. How many different permutations of stadiums visits could they consider?	$\frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{720}{6} = 120$ or ${}_6P_3 = 120$
9. Every registered taxi cab in New Your City must be issued a medallion plaque. Each medallion consists of 3 numbers (0-9) and one letter (A-Z). How many taxi cabs are can be registered at one time?	$10 \times 10 \times 10 \times 26 = 26000$ or $10^3 \times 26 = 26000$