Building Property Maintenance (46.0401) T-Chart

Identify belt and pulley ratios	=	**Use reasoning to solve equations and justify the solution method**

Program Task: Demonstrate how to lift and move heavy objects carefully.

PA Core Standard: CC.2.2.HS.D.9

Description: Use reasoning to solve equations and justify the solution method.

Program Associated Vocabulary:
PULLEY, RATIO, RPM (REVOLUTIONS PER MINUTE), DIAMETER, ANGULAR VELOCITY

Math Associated Vocabulary:
INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

Program Formulas and Procedures:
Combination pulleys are used to make the job of lifting heavy objects easier. In the past, workers needed to use their own force to move objects. The use of pulleys, and now motors to run the pulleys, has made work easier for all maintenance professionals.

Pulleys work on a ratio and proportion concept. Looking at the ratio of RPM (revolutions per minute) in relation to diameter shows students that if the original pulley diameter and rpms stay the same, as the RPM's increase on the second pulley, the diameter of the second pulley will decrease.

The formula is:

$$\text{Belt \& Pulley Ratio} \rightarrow \frac{\text{Angular Velocity (Pulley 1)}}{\text{Angular Velocity (Pulley 2)}} = \frac{\text{Diameter (Pulley 2)}}{\text{Diameter (Pulley 1)}}$$

This ratio is the basis for our example below.

Example: A maintenance supervisor must set up a pulley system in order to lift an item from the ground floor of the apartment complex to the fourth floor. He has a system that looks like this one in the diagram. If Pulley A is moving at 3,000 revolutions per minute (rpm) and has a diameter of 20", what diameter of a secondary pulley would be needed to have that pulley run at 4,000 rpm?
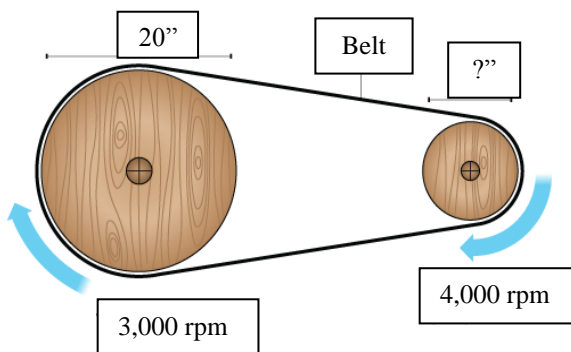
Step 1: Set up the formula for Belt and Pulley Ratio.

$$\text{Belt \& Pulley Ratio} \rightarrow \frac{\text{Angular Velocity (Pulley 1)}}{\text{Angular Velocity (Pulley 2)}} = \frac{\text{Diameter (Pulley 2)}}{\text{Diameter (Pulley 1)}}$$

Step 2: Define the variables in the equation and solve.

$$\text{Belt \& Pulley Ratio} \rightarrow \frac{3,000 \text{ rpm}}{4,000 \text{ rpm}} = \frac{x"}{20"}$$

$$3,000(20) = 4,000x$$

$$\frac{60,000}{4,000} = \frac{4,000x}{4,000}$$

$$15" = x \qquad \text{Pulley 2 needs to have a 15" diameter.}$$

Formulas and Procedures:
Direct Proportions:
Two quantities, A and B, are directly proportional if by whatever factor A changes, B changes by the same factor.

Example 1: Take the formula, distance = rate x time. If the rate remains constant, at 30 miles per hour, then the time and distance are directly proportional.

$d = 30t$
when t = 2, d = 60
when t = 4, d = 120

> *Note that when the time doubles, so does the distance.

Example 2: If speed is directly proportional to distance, and a car can travel 100 miles at 50 miles per hour, how far can that car travel during the same time if it travels at 70 mph?

Step 1: Set up proportion.

$$\frac{50 \text{ mph}}{70 \text{ mph}} = \frac{100 \text{ mi.}}{x}$$

Step 2: Cross multiply and divide to solve.

$$50x = 70(100) \rightarrow 50x = 7000 \rightarrow x = 140 \text{ miles}$$

Inverse Proportions:
Two quantities, A and B, are inversely proportional if by whatever factor A changes, B changes by the multiplicative inverse, or reciprocal of that factor.

Example 1: Take the formula, distance = rate x time. If the distance, 100 miles is constant, then as the rate increases, the time decreases.

$100 = rt$
When r = 100, t = 1
When r = 50, t = 2

> *Note that when the rate doubles, the time is halved.

Example 2: The time needed to complete a job is inversely proportional to the number of people working. If it takes one person 8 hours to pain the room alone, how long would it take 4 people to paint a room?

Step 1: Set up the proportion.   Step 2: Invert (flip) one ratio.

$$\frac{1 \text{ person}}{4 \text{ people}} = \frac{8 \text{ hours}}{x \text{ hours}} \qquad \frac{1 \text{ person}}{4 \text{ people}} = \frac{x \text{ hours}}{8 \text{ hours}}$$

Step 3: Cross-multiply and divide to solve.
$$4x = 8, \ x = 2$$
4 people can paint the room in 2 hours.

Instructor's Script - Comparing and Contrasting

Inverse and direct proportional relationships are important when working with all forms of equations, both in the maintenance environment and in other real world applications. This combination pulley problem is a great practical use of ratios and proportions in a real world situation. It is important to know that there is an inverse relation between the two pulleys. If the rpms and the pulley diameter of the first pulley remain constant, any increase in the rpms of the second pulley will result in a decrease in the diameter.

Direct proportions basically mean that when one factor changes (increases) the other factor does the same (increases). Inverse proportions basically mean that when one factor changes (increases) the other factor does the opposite (decreases).

Common Mistakes Made By Students

When students compare Direct and Inverse Proportional relationships, they may become confused and have difficulty differentiating one from the other.

Students do not set up the relation correctly and do not compare and analyze the results:

Example: If you have a maintenance person working on a job for 6 hours, how long will it take for 2 workers to complete the same job?

You choose: $\dfrac{\text{1 mtc. worker}}{\text{2 mtc. worker}} = \dfrac{\text{6 hours}}{\text{x hours}}$ $12 = \text{x hours}$ $x = 12 \text{ hours}$

Now – ask yourself, "Does this make sense?" The answer should be, "If I add a maintenance person to help work at this job, then the hours should <u>decrease</u>. In this problem they <u>increased</u>. It is an indirect proportion, so I need to flip and solve:

$\dfrac{\text{1 mtc. worker}}{\text{2 mtc. worker}} = \dfrac{\text{x hours}}{\text{6 hours}}$ $6 = 2\text{x hours}$ $x = 3 \text{ hours}$ Now say, "1 maintenance person takes 6 hours, if I add a second

maintenance person, it takes 3 hours." This makes sense.

Choosing information from a written problem:

Many students do not read thoroughly and "pull out" information from the problem correctly. Many students know how to "do the math", but if the problem is not set up properly or the information is not correct, the math will be calculated incorrectly.

CTE Instructor's Extended Discussion

It is important for students to understand the relationship between pulleys and drive belts. There are times when you are on the job where machines break and you need to do things manually. It is a real world problem that gets overlooked due to today's automation. A mechanic may need to do calculations and replace the part within an hour; therefore, knowing how to calculate a diameter based upon the rpms needed would be imperative.

Many students have trouble with ratios and proportions. They do not understand the common sense behind the fact that the faster the rpms in a combination pulley system, the smaller the diameter of the pulley. It is of value to set up a lab to demonstrate how this ratio actually works between two pulleys.

	Problems	Career and Technical Math Concepts		Solutions
1.	If pulley A is moving at 3,000 revolutions per minute (rpm) and has a diameter of 20", what is the diameter of a secondary pulley running at 7,000 rpm?			
2.	The maintenance department has two pulleys. The larger one runs at 3,000 rpms and has a diameter of 22 ½". If the other one has a diameter of 12", what are the rpm's for this pulley?			
3.	A maintenance apprentice was sent to pick up some pulleys for a job. The store has 3 pulleys for $ 145.59. If the apprentice needs 8 pulleys and can buy them for the same price as the 3 for $145.59, what is the cost of 8 pulleys?			
	Problems	Related Generic Math Concepts		Solutions
4.	If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?			
5.	The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m^3 is 0.5 atm, what would the pressure be of 0.060 m^3 of the same gas at the same temperature?			
6.	If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?			
	Problems	PA Core Math Look		Solutions
7.	Given that y and x are **directly** proportional, and y = 2 when x = 5, find the value of y when x = 15.			
8.	Given that y and x are **inversely** proportional, and y = 2 when x = 5, find the value of y when x = 15.			
9.	If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?			

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Problems	Career and Technical Math Concepts	Solutions
1. If pulley A is moving at 3,000 revolutions per minute (rpm) and has a diameter of 20", what is the diameter of a secondary pulley running at 7,000 rpm?	**(Inverse)** Belt & Pulley Ratio $\rightarrow \dfrac{3,000 \text{ rpm}}{7,000 \text{ rpm}} = \dfrac{x"}{20"}$ $3,000(20) = 7,000x$ $60,000 = 7,000x$ $8.57" = x$ Pulley 2 needs to have a 8.57" diameter.	
2. The maintenance department has two pulleys. The larger one runs at 3,000 rpms and has a diameter of 22 ½". If the other one has a diameter of 12", what are the rpm for this pulley?	**(Inverse)** Belt & Pulley Ratio $\rightarrow \dfrac{3,000 \text{ rpm}}{x \text{ rpm}} = \dfrac{12"}{22.5"}$ $3,000(22.5) = 12x$ $67,500 = 12x$ $5,625 \text{ rpm} = x$ Pulley 2 would be running at 5,625 rpms.	
3. A maintenance apprentice was sent to pick up some pulleys for a job. The store has 3 pulleys for $145.59. If the apprentice needs 8 pulleys and can buy them for the same price as the 3 for $145.59, what is the cost of 8 pulleys?	**(Direct)** Belt & Pulley Purchase $\rightarrow \dfrac{\$145.59}{3 \text{ pulleys}} = \dfrac{\$ x}{8 \text{ pulleys}}$ $\$145.59(8) = 3x$ $1,164.72 = 3x$ $\$388.24 = x$ It will cost $388.24 for 8 pulleys.	

Problems	Related, Generic Math Concepts	Solutions
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?	**(Direct)** $\dfrac{5 \text{ pounds}}{20 \text{ people}} = \dfrac{x \text{ pounds}}{50 \text{ people}} \rightarrow 20x = 5(50) \rightarrow 20x = 250$ $x = 12.5 \text{ pounds}$	
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m^3 is 0.5 atm, what would the pressure be of 0.060 m^3 of the same gas at the same temperature?	**(Inverse)** $\dfrac{0.24 \text{ m}^3}{0.060 \text{ m}^3} = \dfrac{0.5 \text{ atm}}{x \text{ atm}}$ Invert one ratio since it is an inverse proportion. $\dfrac{0.24 \text{ m}^3}{0.060 \text{ m}^3} = \dfrac{x \text{ atm}}{0.5 \text{ atm}}$ $.24 \times 0.5 = .060x$ $x = 2 \text{ atm}$	
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?	**(Direct)** $\dfrac{10 \text{ castings}}{14 \text{ castings}} = \dfrac{26 \text{ lbs.}}{x \text{ lbs.}} \rightarrow 10x = 26(14) \rightarrow x = 36.4 \text{ lbs.}$	

Problems	PA Core Math Look	Solutions
7. Given that y and x are **directly** proportional, and $y = 2$ when $x = 5$, find the value of y when $x = 15$.	**(Direct)** $\dfrac{5}{15} = \dfrac{2}{y} \rightarrow 5y = 2(15) \rightarrow y = 6$	
8. Given that y and x are **inversely** proportional, and $y = 2$ when $x = 5$, find the value of y when $x = 15$.	**(Inverse)** $\dfrac{5}{15} = \dfrac{y}{2} \rightarrow 15y = 2(5) \rightarrow y = 0.667$	
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?	**(Inverse)** $\dfrac{1}{5} = \dfrac{x}{15} \rightarrow 5x = 1(15) \rightarrow x = 3 \text{ hours}$	