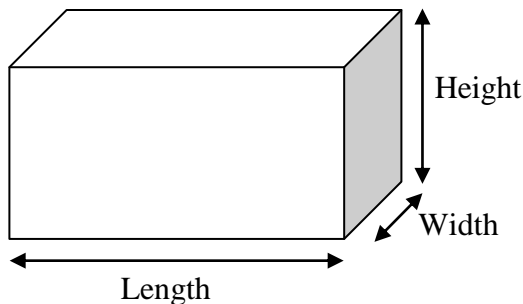| **Estimate concrete volume** | **=** | **Explain volume formulas and use them to solve problems** |

Program Task: Estimate cubic yards of concrete for a foundation wall.

PA Core Standard: CC.2.3.HS.A.12

Description: Explain volume formulas and use them to solve problems.

Program Associated Vocabulary:
DIMENSION, ESTIMATE, CUBIC FEET, CUBIC VOLUME, CUBIC YARDS

Math Associated Vocabulary:
AREA, VOLUME, LENGTH, WIDTH, HEIGHT, RECTANGULAR, CYLINDER, BASE, RADIUS, RECTANGULAR PRISM

Program Formulas and Procedures:
Concrete is estimated and ordered in cubic yards. 1 cubic yard contains 27 cubic feet. A house foundation wall is related to a rectangular prism. A foundation wall has a length, width and height. When determining how much volume of cement is needed it is important to have the length, width and height in decimal feet.

Formulas and Procedures:
Volume:

Cylinder:
$V = \pi r^2 h$
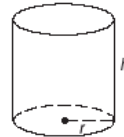
Cone:
$V = \frac{1}{3}\pi r^2 h$

Rectangular Prism:
$V = lwh$

Sphere:
$V = \frac{4}{3}\pi r^3$

Pyramid:
$V = \frac{1}{3}$ (area of the base)h
h = height   b = base
ℓ = slant length or slant height

To determine the volume needed in cubic yards the following formula is used:

Cubic yards= $\dfrac{L' \times W' \times H'}{27}$

A foundation wall measures 27'5" long, 8" wide and 9' high.

The inches in the length needs to be converted to decimal feet. 5" ÷ 12 = .417'. So 27'5" ≈ 27.417'.

The width needs to be converted to decimal feet.
8" ÷ 12 = .667'

Volume in cubic yards = $\dfrac{27.417' \times .667' \times 9'}{27}$ = $\dfrac{164.58'}{27}$
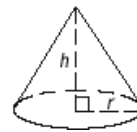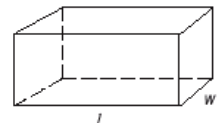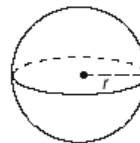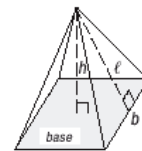
Volume = 6.10 yd.^{3}

Example:
How many cubic inches of air can a beach ball hold if it has a diameter of 14 inches? Round to the nearest whole number.

Steps to finding volume:
1. Identify the solid. (sphere)
2. Write the formula for calculating the volume of that solid using the formula sheet.
   $V = \frac{4}{3}\pi r^3$
3. Identify what information you are given in the example.
   Given: diameter (d) = 14"
4. Solve for radius using the formula radius (r) = ½ (diameter).
   r = ½ × 14 = 7
5. Perform the necessary mathematical operations to obtain your answer.
   $V = \frac{4}{3}\pi r^3 = \frac{4}{3}(3.14)(7^3) = 1{,}436$ in.^{3}
6. Write the appropriate unit after your answer.
   1,436 in.^{3}

Instructor's Script - Comparing and Contrasting

The carpentry example on page one shows how volumes must be calculated for rectangular prisms to pour foundation walls. Carpenters also work with cylinders when they pour footers for posts as presented in problem #2 on page three.

The mathematical formulas for volume indicate a certain type of orientation that may not match the application in question. For example, h will designate height of a cylinder, but if the cylinder is horizontal, h will appear as the length! The height of a cylinder can be determined if the student remembers that the height of a cylinder is always the distance between the two bases. See illustrations below.
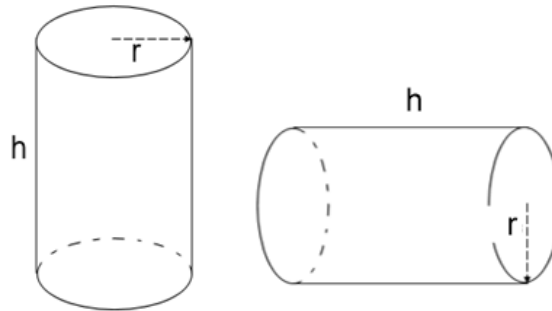

Note: Answers to the problems on page 4 will be different if students use the π key on a calculator. The author of this t-chart used 3.14 as the value of π, rather than the π key on a calculator. Answers will be different, not by much, but enough that when some students are presented with the answer key they might not recognize that they did get the correct answers!

Common Mistakes Made By Students

Students may use an incorrect formula to solve a problem: To rectify these errors have the students correctly identify the type of object they are dealing with and use the appropriate formula. Frequently two formulas may be needed for complex problems.

Students need to watch for the slant height (*l*) when working with cones and pyramids. While this dimension is often given, it is not used when calculating the volume. See diagrams.
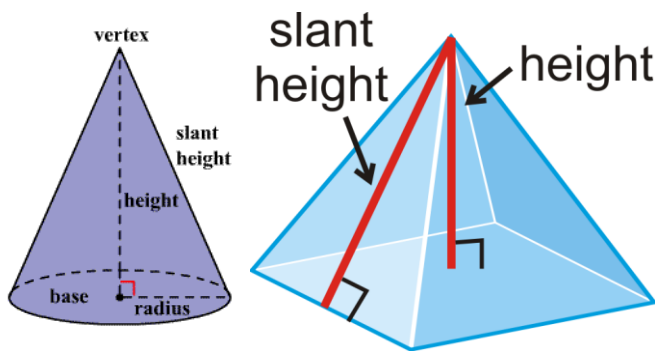
CTE Instructor's Extended Discussion

The first formula to determine cubic yards is $\underline{\text{L'} \times \text{W'} \times \text{H'}}$; it is important that all numbers are in decimal feet.
$$\frac{}{27}$$

Another formula for determining cubic yards of concrete is Length' $\times$ Width' $\times$ Thickness" $\times$ .0031 =. Notice that the thickness is in inches. It is good practice to solve for cubic yards using both formulas and compare the information.

	Problems	Career and Technical Math Concepts	Solutions
1.	A customer has asked you to construct an above ground, rain water holding tank with r = 12 feet and h = 25 feet. What will be the total volume of the water tank?		
2.	You need to set 3 concrete piers to support an above ground deck. Each pier has the following dimensions: d = 12 inches and h = 60 inches. Find the volume of one pier in in.^{3}, ft.^{3} & yd.^{3}?		
3.	A carpenter is pouring a cement walkway in front of a customer's house; the walkway is 35 feet long, 4 feet wide and 5 inches thick. How many cubic yards of cement should the carpenter order? Round up to the closest ½ yard.		

	Problems	Related, Generic Math Concepts	Solutions
4.	One soup can has a d = 3 inches and h = 4 inches; another soup can has a d = 4 inches and h = 3 inches. Which can holds more soup?		
5.	A size 7 regulation basketball has a d = 9.39 inches. What is the volume of the basketball?		
6.	How much water would you need to fill a rectangular fish tank with a height of 16.5 inches, a length of 32 inches, and a width of 8.5 inches?		

	Problems	PA Core Math Look	Solutions
7.	Find the volume of a cylinder if d = 12.5 feet and h = 28.45 feet.		
8.	Find the volume of a sphere if d = 27.75 inches.		
9.	Find the volume of a 4-sided pyramid with a square base side of 10 inches, and a height of 25 inches.		

Carpentry (46.0201) T-Chart

Problems	Career and Technical Math Concepts	Solutions
1. A customer has asked you to construct an above ground, rain water holding tank with r = 12 feet and h = 25 feet. What will be the total volume of the water tank?	$V = \pi(12^2)25$ $V = \pi \times (144) \times 25$ or $V = 3.14 \times (12^2) \times 25$ $V = 11310 \text{ ft.}^3$ (rounded from 11309.73355)	
2. You need to set 3 concrete piers to support an above ground deck. Each pier has the following dimensions: d = 12 inches and h = 60 inches. Find the volume of one pier in in.³, ft.³ & yd.³?	$V = \pi 12^2 60$ $V = \pi \times 144 \times 60$ $V = 27143 \text{ in.}^3$ $V = 27143 \div 1725$ $V = 15.7 \text{ ft.}^3$ $V = 16 \div 27$ $V = .6 \text{ yd.}^3$ (Rounded from 0.59)	
3. A carpenter is pouring a cement walkway in front of a customer's house; the walkway is 35 feet long, 4 feet wide and 5 inches thick. How much cubic yards of cement should the carpenter order? Round up to the closest ½ yard.	Cubic yards= $\dfrac{\text{L' } \times \text{ W' } \times \text{ H'}}{27}$ Cubic yards = $\dfrac{35 \times 4 \times .417}{27}$ = 2.16 Cubic yards = 2.16 $\longrightarrow$ 2.5 cubic yards	

Problems	Related, Generic Math Concepts	Solutions
4. One soup can has a d = 3 inches and h = 4 inches; another soup can has a d = 4 inches and h = 3 inches. Which can holds more soup?	$V = \pi r^2 h$ Can 1 : Can 2 : $r = \dfrac{1}{2}(3)$ $r = \dfrac{1}{2}(4)$ $V = \pi(1.5)^2 4$ $V = \pi(2)^2 3$ $V \approx 28.26 \text{ in.}^3$ $V \approx 37.68 \text{ in.}^3$ Can 2 holds more soup.	
5. A size 7 regulation basketball has a d = 9.39 inches. What is the volume of the basketball?	Radius (r) = $\dfrac{9.39}{2}$ = 4.695 $V = \dfrac{4}{3} \times \pi \times r^3$ $V = 1.333 \times \pi \times 4.695^3$ $V \approx 1.333 \times \pi \times 103.5$ $V \approx 433.21 \text{in}^3$	
6. How much water would you need to fill a rectangular fish tank with a height of 16.5 inches, a length of 32 inches, and a width of 8.5 inches?	$V = lwh$ $V = (32)(8.5)(16.5) = 4,488 \text{ in.}^3$	

Problems	PA Core Math Look	Solutions
7. Find the volume of a cylinder if d = 12.5 feet and h = 28.45 feet.	$V = \pi r^2 h$ $r = \dfrac{1}{2}(12.5) = 6.25$ $V = \pi \times 6.25^2 \times 28.75$ $V \approx 3,526.37 \text{ ft.}^3$	
8. Find the volume of a sphere if d = 27.75 inches.	$V = \dfrac{4}{3} \times \pi \times r^3$ $V = 1.333 \times \pi \times 13.875^3$ $V \approx 11,180.44 \text{ in.}^3$	
9. Find the volume of a 4-sided pyramid with a square base side of 10 inches, and a height of 25 inches.	$V = \dfrac{1}{3}(\text{area of base})h \rightarrow V = \dfrac{1}{3}(10)(10)(25) \approx 833.33 \text{ in.}^3$	