Drafting & Design Technology/Technician (15.1301) T-Chart

Calculate distance between two points = Apply coordinate geometry to prove simple geometric theorems algebraically

Program Task: Perform point to point locations.

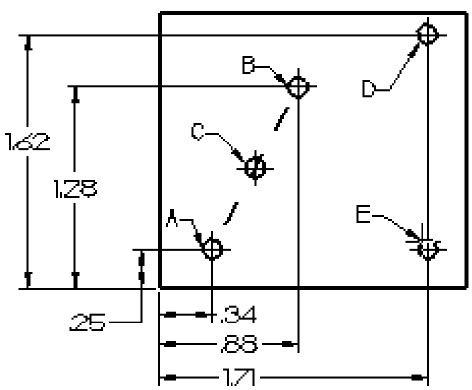
Program Associated Vocabulary:
MIDPOINT, MIDWAY

Program Formulas and Procedures:

Sometimes a machinist needs to calculate a straight line distance between two points that are positioned at angles to one another. While machinists are highly skilled in math, a drafter should take the responsibility to dimension a drawing as accurately as possible. This will result in fewer errors during the manufacturing process and saves the machinist the time it would take to perform the calculations.

These positions can be calculated using the distance and midpoint formulas.

Example:
An engineering error *may* have resulted in holes A and B being more than a specified 1.100" between centers. First determine if they are. Then if they are, determine location C, midway between A and B so an additional hole may be added.



Use the distance formula first.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(.88 - .34)^2 + (1.28 - .25)^2}$$

$$d = \sqrt{0.54^2 + 1.03^2} \rightarrow \sqrt{0.2916 + 1.0609}$$

$$d = \sqrt{1.3525} = 1.1629$$

Since A and B are too far apart, use the midpoint formula to determine the position for the additional hole at location C.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow \left(\frac{.34 + .88}{2}, \frac{.25 + 1.28}{2}\right)$$

$$\left(\frac{1.22}{2}, \frac{1.54}{2}\right) \rightarrow (0.61, 0.77)$$

PA Core Standard: CC.2.3.HS.A.11

Description: Apply coordinate geometry to prove simple geometric theorems algebraically.

Math Associated Vocabulary:
DISTANCE, MIDPOINT, SQUARE ROOT, RADICAL, ORDERED PAIR

Formulas and Procedures:

Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example:
The given points are (5,-3) and (3,-1).

Using (x_1,y_1) (x_2,y_2) $x_1 = 5$ $x_2 = 3$ $y_1 = -3$ $y_2 = -1$

Find the distance between the two points and the midpoint of the two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3-5)^2 + (-1-(-3))^2}$$

$$d = \sqrt{(-2)^2 + (2)^2}$$

$$d = \sqrt{4+4}$$

$$d = \sqrt{8}$$

$$d = \sqrt{4x2}$$

$$d = 2\sqrt{2} \approx 2.828$$

The distance between the two points is $2\sqrt{2}$ or about 2.828 units.

Midpoint Formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{5+3}{2}, \frac{-3+(-1)}{2}\right)$$

$$\left(\frac{8}{2}, \frac{-4}{2}\right)$$

$$(4, -2)$$

(4, -2) is the midpoint between these two points.

Instructor's Script – Comparing and Contrasting

Students who use the midpoint or distance formula in a math class are usually provided with coordinates or ordered pairs. They must then take these coordinates and substitute the corresponding values into the midpoint or distance formulas. When drawing machining parts, the coordinates or ordered pairs must be established before the substitution can take place. This requires the ability to establish an origin or (0, 0) point on the drawing. Keep in mind that the distance formula finds its basis in the Pythagorean theorem. The distance represents the diagonal, C, and by finding the change in x-values and y-values, one can find A and B.

Common Mistakes Made By Students

Students often write the answer to the midpoint of an ordered pair as a number and not as an ordered pair. Often students get into the habit of thinking that an answer in mathematics should be a single number and get confused when the answer has an x-coordinate and a y-coordinate, or if in other situations in math where the answer is a polynomial or has more than one solution.

Students also make the mistake in the distance formula of either forgetting about the square root or computing the square root but leaving the radical sign over the number. To keep the accuracy of the number the answer is sometimes left in simplified radical form and this is another area where student sometimes make a mistake or get confused. When using a calculator to square numbers some student make the mistake of entering -3^2 and not $(-3)^2$. If you enter -3^2 into the calculator you will get -9 since the calculator interprets this as the negative of 3^2. When you are dealing with real numbers, the numbers under the radical sign will always be positive. When you square any number it will be positive, since a negative squared is positive and a positive squared is positive. Check your work and be sure that all of your numbers are positive after you square them. Distances will always be positive. On a number line the distance between -2 and 5 is seven units.

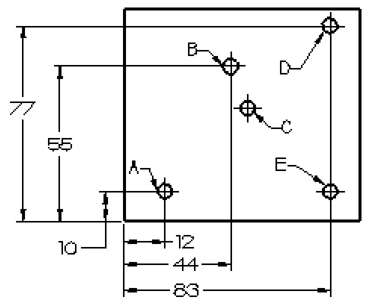
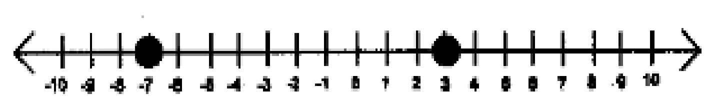
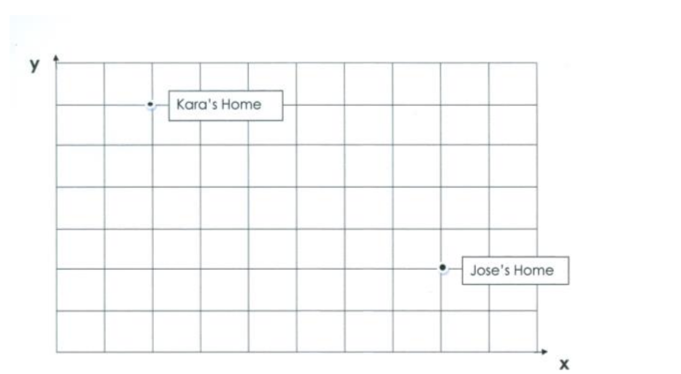
Example:

(-2-5) = 7 units or (2 – (-5)) = 7 units

CTE Instructor's Extended Discussion

As discussed on other T-Charts, the introduction of AutoCAD has placed the burden of dimensioning on the software. It is important that students understand the math involved so they can perform these calculations in instances when they do not have access to AutoCAD. Again, it is quite embarrassing to tell a client you can't do the calculation without benefit of using the software. Math skills are very important in the world of drafting, and the ability to do the math can make an employee more valuable than an employee who solely relies on the computer.

Since all drawings include tolerances, an error may be compounded if not dimensioned properly. The student should be familiar with the "True Position Dimensioning System." The example problem on page one does not give a dimension between holes A and B in either direction. Assume that these hole locations are critical to the part mating with another part, so the student should dimension the drawing accordingly.

Problems	**Career and Technical Math Concepts** / **Solutions**
1. Determine the distance between holes A and B. All dimensions are in millimeters. Round to the nearest .1mm. 	
2. Calculate the distance between holes A and D.	
3. Calculate the dimensions (coordinates) for hole C, which is midway between A and D.	
Problems	**Related, Generic Math Concepts** / **Solutions**
4. Find the distance between points (1, 5) and (9, 11).	
5. Find the midpoint of points (9, 3) and (-5, -3).	
6. Find the distance between the two points on the number line?	
Problems	**PA Core Math Look** / **Solutions**
7. Kara and Jose are working on a school project together. They would like to meet half way between their homes. Use the grid at the right to determine the midpoint between their homes.	
8. If each unit on the grid graph is 1 mile, then about how far must Kara and Jose travel to reach the meeting point?	
9. What would be the distance from Kara's home to Jose's home if the scale was 1 mile = ¾ mile?	

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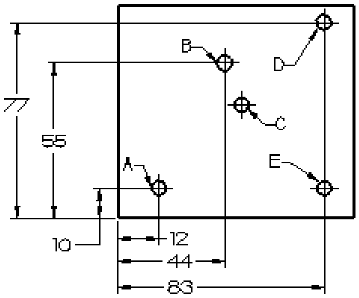
Problems	Career and Technical Math Concepts	Solutions
1. Determine the distance between holes A and B. All dimensions are in millimeters. Round to the nearest .1mm. 		$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow \sqrt{(44-12)^2 + (55-10)^2}$ $\sqrt{32^2 + 45^2} \rightarrow \sqrt{1024 + 2025}$ $\sqrt{3049} = 55.2\text{mm}$
2. Calculate the distance between holes A and D.		$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow \sqrt{(83-12)^2 + (77-10)^2}$ $\sqrt{71^2 + 67^2} \rightarrow \sqrt{5041 + 4489}$ $\sqrt{9530} = 97.6\text{mm}$
3. Calculate the dimensions (coordinates) for hole C, which is midway between A and D.		$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}) \rightarrow (\frac{12+83}{2}, \frac{10+77}{2})$ $(\frac{95}{2}, \frac{87}{2}) \rightarrow (47.5, 43.5)$ are the new hole coordinates

Problems	Related, Generic Math Concepts	Solutions
4. Find the distance between points (1, 5) and (9, 11).		$\sqrt{(9-1)^2 + (11-5)^2} \rightarrow \sqrt{8^2 + 6^2} \rightarrow \sqrt{64 + 36}$ $\sqrt{100} \rightarrow 10$
5. Find the midpoint of points (9, 3) and (-5, -3).		$(\frac{9+(-5)}{2}, \frac{3+(-3)}{2}) \rightarrow (\frac{4}{2}, \frac{0}{2}) \rightarrow (2, 0)$
6. Find the distance between the two points on the number line?		(-7 – 3) = 10 units or (3 – (-7)) = 10 units

Problems	PA Core Math Look	Solutions
7. Kara and Jose are working on a school project together. They would like to meet half way between their homes. Use the grid at the right to determine the midpoint between their homes.		$(\frac{2+8}{2}, \frac{6+2}{2}) \rightarrow (\frac{10}{2}, \frac{8}{2}) \rightarrow (5, 4)$ (5, 4) would be the coordinates of the midpoint between their homes.
8. If each unit on the grid graph is 1 mile, then about how far must Kara and Jose travel to reach the meeting point?		$\sqrt{(5-2)^2 + (4-6)^2} \rightarrow \sqrt{3^2 + (-2)^2}$ $\sqrt{13} \approx 3.61\text{miles}$
9. What would be the distance from Kara's home to Jose's home if the scale was 1 unit = ¾ mile?		$\sqrt{(8-2)^2 + (2-6)^2} \rightarrow \sqrt{6^2 + (-4)^2} \rightarrow \sqrt{36 + 16}$ $\sqrt{52} \rightarrow \sqrt{4 \times 13} \rightarrow 2\sqrt{13} \approx 7.21\text{units}$ $7.21\,\text{units} \times 3/4\,\text{miles} = 5.41\,\text{miles}$