

Construct circles, arcs and tangent arcs and lines = Apply geometric theorems to verify properties of circles

Program Task: Design machine parts.

Program Associated Vocabulary:
DIAMETER, RADIUS, CIRCUMFERENCE

Program Formulas and Procedures:
When designing machine parts, the drafter must frequently design holes, circles and partial circles (arcs). Understanding the relationship between radius and diameter is critical so that drafting is performed accurately, thus reducing the time the machinist must spend in performing calculations needed to set up for the particular task.

Diameter = 2 x Radius

Radius = 1/2 Diameter

Example:
You design a part that required a hole with a 3/8" radius. Since drill bit sizes are stated in diameter, what is the correct diameter drill bit to use for the hole?

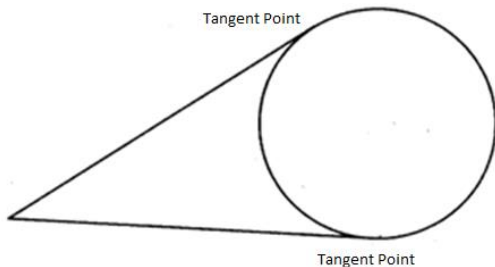
Since a hole's (circle's) radius is 1/2 its diameter, the 3/8" radius must be multiplied by two.

$$\frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$$

A 3/4" diameter hole must be drilled. By stating the correct size, you have taken that burden off the machinist and also reduced the probability of an error.

The concept of tangent is also vital to the drafter because a design often requires arcs to be connected to other arcs or to lines at tangent points.

Example:
The sketch below shows two lines that are tangent to an arc and their tangent points.



PA Core Standard: CC.2.3.HS.A.8
Description: Apply geometric theorems to verify properties of circles.

Math Associated Vocabulary:
RADIUS, DIAMETER, TANGENT, CIRCLE, CONGRUENT, PERPENDICULAR, POINT OF TANGENCY, EXTERNAL, INTERNAL, CIRCUMFERENCE, AREA, VOLUME

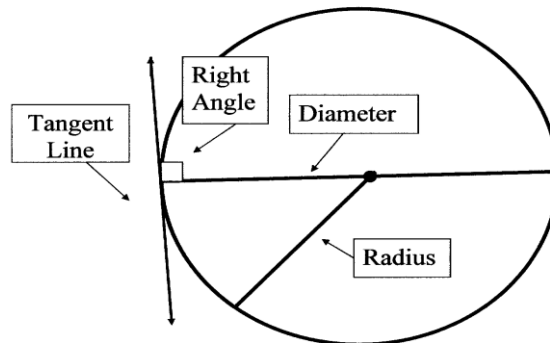
Formulas and Procedures:
Area of a Circle: $A = \pi r^2$
Circumference of a Circle: $C = 2\pi r$

Cylinder: $SA = 2\pi r^2 + 2\pi rh$ $V = \pi r^2 h$

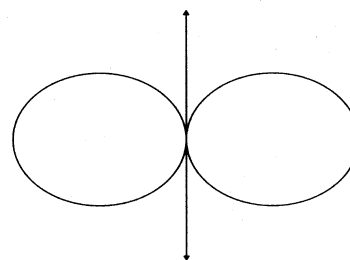
Sphere: $SA = 4\pi r^2$ $V = (4/3) \times \pi \times r^3$

Cone: $SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}$ $V = 1/3\pi r^2 h$

- Two tangents to the same circle from the same external point will be congruent.
- A radius always meets a tangent to the circle at a right angle and at a point that is called the point of tangency.



The picture below shows tangent circles with a common interior tangent line.



Instructor's Script – Comparing and Contrasting

The first example on the drafting side of the T-Chart uses the property that the diameter is two times the radius. This is the simplest application of the PA Core Standard. The second example shows how a curve in a drawing is actually made by using ¼ of a circle and two lines tangent to the circle.

Common Mistakes Made By Students

Students do not know the concepts or how to apply them.

Example: Find the value of x

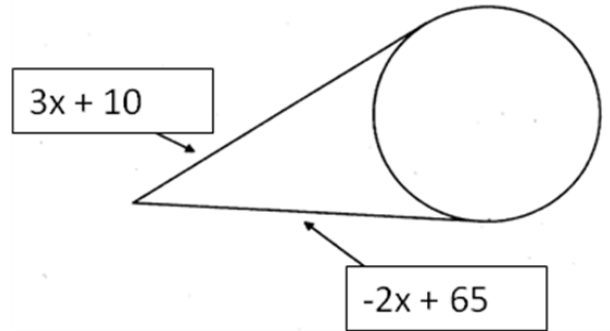
$3x + 10 = -2x + 65$ Set up the equation.

$3x = -2x + 55$ Subtract 10 from each side of the equation.

$5x = 55$ Add 2x to each side of the equation.

$x = 11$ Divide by 5.

The value of x is 11.



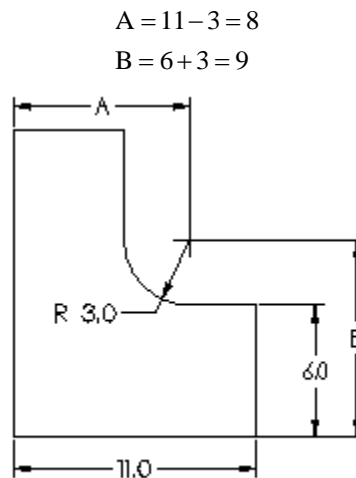
CTE Instructor's Extended Discussion

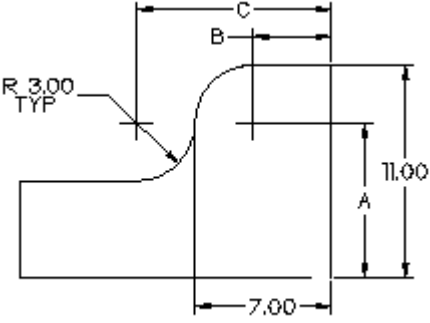
There are other situations that involve diameter/radius and tangent relationships. One situation involves machining an inside corner radius or fillet. If a fillet is specified on a blueprint, the machinist must choose the correct diameter tool to produce that radius since cutting tools are defined by diameter, not radius.

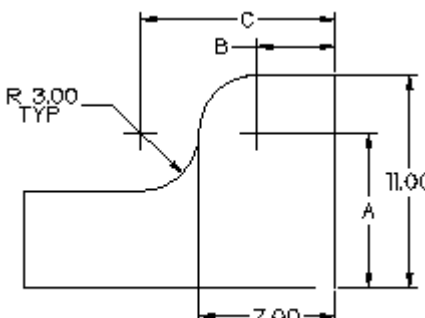
Example: What diameter endmill must be used to create a 10 mm. fillet radius? Since diameter = 2 x radius, the correct end mill diameter would be found by multiplying 10 by 2, so a 20 mm. diameter endmill would be needed.

Another instance is when two linear distances and a corner radius are specified. The machinist needs to subtract the size of the radius from the linear dimensional locations to determine the location of the center point of the radius. The corner radius will then be positioned correctly and tangent to each line.

Example: The 6" and the 11" dimensions are given, but "A" and "B" need to be determined to locate the center point of the radius for layout.



Problems	Career and Technical Math Concepts	Solutions
1. A family raises horses and asks you to design a 120' diameter fenced in corral. How much fencing is required for the corral?		
2. How much fencing would be required to enclose an area consisting of a 100' x 50' rectangle, with a semicircle at each end to make it rounded?		
3. A layout needs to be performed according to the blueprint shown. Determine distances A, B, and C to locate the center points of the radii. All dimensions are in inches.		
Problems	Related, Generic Math Concepts	Solutions
4. What must the radius be of a cylinder whose height is 3" and whose volume is 35 in ³ of fluid?		
5. A family would like to build a fence in their backyard for a riding ring for their horse. How much fence should they buy if the ring covers ½ an acre of land? (1 acre = 4840 yards ²)		
6. A family has 75 feet of fencing to fence in their circular garden. What is the approximate radius of the garden?		
Problems	PA Core Math Look	Solutions
7. Find the radius of a circle whose circumference is 20'.		
8. Find the radius of a circle whose area is 45 in ² .		
9. Find the height of a cylinder whose volume is 300 ft ³ and whose radius is 6 ft.		

Problems	Career and Technical Math Concepts	Solutions
1. A family raises horses and asks you to design a 120' diameter fenced in corral. How much fencing is required for the corral?	$C = \pi D$ $C = 3.14 \times 120 = 376.8'$ of fencing required.	
2. How much fencing would be required to enclose an area consisting of a 100' x 50' rectangle, with a semicircle at each end to make it rounded?	$100' + 100' = 200'$ (length of the two sides) The two semicircles added together equals one full circle: $C = 3.14 \times 50 = 157$ $200 + 157 = 357$ 357' of fencing is required.	
3. A layout needs to be performed according to the blueprint shown. Determine distances A, B, and C to locate the center points of the radii. All dimensions are in inches.	 <p>$A = 11 - 3 = 8''$ $B = 7 - 3 = 4''$ $C = 7 + 3 = 10''$</p>	
Problems	Related, Generic Math Concepts	Solutions
4. What must the radius be of a cylinder whose height is 3" and whose volume is 35 in ³ of fluid?	$v = \pi r^2 h$ $35 = \pi r^2 3$ $\frac{35}{3\pi} = r^2$ $r = \sqrt{\frac{35}{3\pi}} \approx 1.93$ inches	The radius of the cylinder must be about 1.93 inches.
5. A family would like to build a fence in their backyard for a riding ring for their horse. How much fence should they buy if the ring covers ½ an acre of land? (1 acre = 4840 yards ²)	$\frac{1}{2}$ acre = 2420 yds. ² $2420 = \pi r^2$ $r = \sqrt{\frac{2420}{\pi}}$ $r \approx 27.75$ yds. $C = 2\pi r$ $C = 2\pi(27.75)$ $C = 174.36$ yds.	They should buy about 175 yards of fencing.
6. A family has 75 feet of fencing to fence in their circular garden. What is the approximate radius of the garden?	$C = 2\pi r$ $75 = 2\pi r$ $r = \frac{75}{(2\pi)}$ $r \approx 11.94$ feet	The radius is approximately 11.94 feet.
Problems	PA Core Math Look	Solutions
7. Find the radius of a circle whose circumference is 20'.	$C = 2\pi r$ $20 = 2\pi r$ $r = \frac{20}{(2\pi)}$ $r \approx 3.18$ feet	
8. Find the radius of a circle whose area is 45 in ² .	$A = \pi r^2$ $45 = \pi r^2$ $r = \sqrt{\frac{45}{\pi}}$ $r \approx 3.78$ inches	
9. Find the height of a cylinder whose volume is 300 ft ³ and whose radius is 6 ft.	$V = \pi r^2 h$ $300 = \pi 6^2 h$ $300 = 36\pi h = \frac{300}{(36\pi)} = h$ $h \approx 2.65$ feet	