

**Calculate the length of a rafter using the Pythagorean Theorem and Order of Operations**

**Apply and extend the properties of exponents to solve problems with rational exponents**

**Program Task:** Calculate the length of a rafter using the Pythagorean Theorem.

**PA Core Standard:** CC.2.1.HS.F.1

**Program Associated Vocabulary:**  
 FORMULA, ORDER OF OPERATIONS, PYTHAGOREAN THEOREM

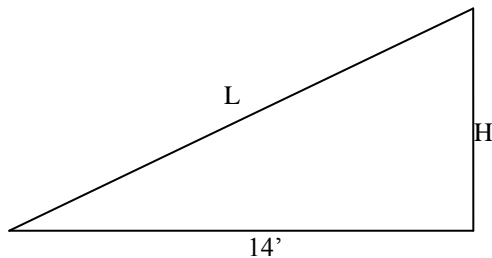
**Description:** Apply and extend the properties of exponents to solve problems with rational exponents.

**Math Associated Vocabulary:**  
 SIMPLIFY NUMERICAL EXPRESSION, TERM

**Program Formulas and Procedures:**  
 Understanding how to use the order of operations is critical for all drafters. If the order is not followed precisely, the answer will ultimately be wrong, resulting not only in an incorrect drawing, but potentially an error found in the field during construction. This will most likely result in a back charge to you or your company; something you should always avoid.

**Formulas and Procedures:**  
**P** Do all operations in **PARENTHESIS**. Start with the innermost set.  
**E** Evaluate all **EXPONENTS**.  
**M** Do **MULTIPLICATION** and **DIVISION** in order from left to right.  
**D**  
**A** Do **ADDITION** and **SUBTRACTION** in order from left to right.  
**S**

Roof slope, as mentioned on other T-Charts is expressed as rise over run. In order to calculate the length of a rafter, you must be able to calculate the rise from the slope indicated on the drawings. In order to do this, you will need the Pythagorean Theorem, and understand the Order of Operations.



One way to remember the order of operations is:

Please Excuse My Dear Aunt Sally.

The above sketch is one half of a roof (since most roofs are sloped from the center of the building), with a slope of 6:12.

Remembering that my and dear go together since they both describe Aunt Sally who is one person.

Since 6" = 0.5' and 12" = 1' set up a proportion comparing rise and run.

**Example:**

$$\begin{aligned} (7 + 3)^2 - 21 \div 7 + 10(2) &= \\ 10^2 - 21 \div 7 + 10(2) &\text{ P}arenteses \\ 100 - 21 \div 7 + 10(2) &\text{ E}xponents \\ 100 - 3 + 20 &\text{ M}ultiplication and \text{ D}ivision \\ 97 + 20 &\text{ A}ddition and \text{ S}ubtraction \\ = 117 & \end{aligned}$$

$$\frac{\text{rise}}{\text{run}} = \frac{0.5'}{1'} = \frac{H}{14'}$$

Cross multiply and divide to solve.

$$H = (0.5 \times 14) \div 1 = 7'$$

Now that we have solved for "H", we can use the Pythagorean Theorem to solve for length (L).

$$\begin{aligned} L^2 &= 14^2 + 7^2 \\ L^2 &= 196 + 49 \\ L^2 &= 245 \\ L &= \sqrt{245} \\ L &= 15.65' \end{aligned}$$

Since lumber comes in even increments of 2', you will need to order 16' lengths of lumber for the rafters.

**Instructor's Script – Comparing and Contrasting**

Order of operations is a vital skill for any student in a career and technical area that uses formulas. In addition to substituting values into the formula, the student must also apply the order of operations to reach a solution. Before teaching the Pythagorean Theorem, or any surface area or volume formula, make sure you review the order of operations with your students!

**Common Mistakes Made By Students**

**Improper use of calculators:** Students are usually very quick to use calculators when faced with formulas but if they are not proficient in using the order of operations, they will not insert parentheses where needed or press “=” at the wrong points and arrive at incorrect answers.

**Familiarity with the calculator:** In some calculators, you must enter the radical sign first and in some calculators the radical sign is entered after the number is entered. Some calculators automatically do some of the correct order of operations. You need to know your calculator. Calculators are great tools, but you need to know the correct way to use them.

**When entering the square of a negative number in a calculator it is important to put it in parentheses.** You need to enter  $(-2)^2$  not  $-2^2$ . For the latter the calculator thinks you are saying the negative of 2 squared or -4, and not  $(-2)(-2) = 4$ .

**When dealing with fractions students often will forget to put the numerator of the fraction and the denominator of the fraction in parentheses.** If you enter  $(3 + 6)/9$  into the scientific calculator, it recognizes that  $3 + 6$  is in the numerator and does this operation first, giving the answer  $9/9$  or 1. If you put  $3 + 6/9$  (without the parentheses) into a scientific calculator, it will give you an answer of 3.66...

**Students will often also forget that Multiplication and Division are done together and left to right. Especially in examples like the one below.**

$$20 - 12 \div 3 \cdot 2 + 14$$

$$20 - 12 \div 3 \cdot 2 + 14$$

$$20 - 4 \times 2 + 14$$

$$20 - 12 \div 6 + 14$$

Correct →  $20 - 8 + 14$

Incorrect →  $20 - 2 + 14$

$$12 + 14$$

$$18 + 14$$

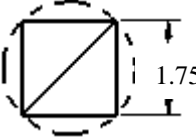
$$26$$

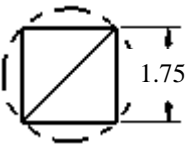
$$32$$

**CTE Instructor's Extended Discussion**

While roof slope and horizontal building dimensions are always given on a set of drawings, the actual length of the rafter (the hypotenuse) is never given. In order to determine the proper length of lumber needed for a given slope, it is imperative that the student understands how to calculate it.

In addition, note that stock lumber comes in even increments of 2'. Sometimes it may be better to change the design by a few inches so that waste is kept to a minimum. Also note that stock lumber always comes a few inches longer than the stated length. For instance, a 20' length of 2 x 6 could be as long as 20'-3". It is important that the student understands this in a case where your calculated length is 20'-1". You would have enough material in a standard 20' length, negating the necessity of ordering 22' lengths of lumber.

Problems	Career and Technical Math Concepts	Solutions
1. Using the sample problem, calculate the length of the rafter if the base is 16'-6" and slope is 9:12.		
2. Determine what diameter round stock is required to machine a 1.75" square. Use the Pythagorean Theorem in the form $c = \sqrt{a^2 + b^2}$ to solve for the length of side c. (a and b are equal.)		
3. Using the sample problem, calculate the length of the rafter if the base is 14'-0" and slope is 12:12.		
Problems	Related, Generic Math Concepts	Solutions
4. Simplify $3(5 + 7)^2 - 10/5$		
5. Simplify $5(8 + 2) + (-5 + (2 + 3)(7 - 4))$		
6. Simplify $\sqrt{(5 + 8)^2 - (7 + 5)^2}$		
Problems	PA Core Math Look	Solutions
7. Simplify $(5 + 7 + 3) \div (3 + 2)$		
7. Simplify $5 + 7 + 3 \div 3 + 2$		
9. Compare problem #7 with problem #8. Explain how someone may make the mistake of thinking they are the same problem.		

Problems	Career and Technical Math Concepts	Solutions
1. Using the sample problem, calculate the length of the rafter if the base is 16'-6" and slope is 9:12.	Since $9'' = 0.75'$ , $h = .75 \times 16.5' = 12.38'$ $L^2 = 16.5^2 + 12.38^2$ $L^2 = 272.25 + 153.26$ $L^2 = 425.51$ $L = \sqrt{425.51}$ $L = 20.63'$ A 22' length of lumber must be ordered.	
2. Determine what diameter round stock is required to machine a 1.75" square. Use the Pythagorean Theorem in the form $c = \sqrt{a^2 + b^2}$ to solve for the length of side c. (a and b are equal.)		$c = \sqrt{a^2 + b^2}$ $c = \sqrt{1.75^2 + 1.75^2}$ $c = \sqrt{3.0625 + 3.0625}$ $c = \sqrt{6.125}$ $c = 2.4748''$
3. Using the sample problem, calculate the length of the rafter if the base is 14'-0" and slope is 12:12.	Since $12'' = 1.0'$ , $h = 1.0 \times 14' = 14'$ $L^2 = 14^2 + 14^2$ $L^2 = 196 + 196$ $L^2 = 392$ $L = \sqrt{392}$ $L = 19.79'$ A 20' length of lumber must be ordered	
Problems	Related, Generic Math Concepts	Solutions
4. Simplify $3(5 + 7)^2 - 10/5$		$3(5 + 7)^2 - 10/5 = 3(12)^2 - 10/5 = 3(144) - 10/5 = 432 - 2 = 430$
5. Simplify $5(8 + 2) + (-5 + (2 + 3)(7 - 4))$		$5(8 + 2) + (-5 + (2 + 3)(7 - 4)) =$ $5(8 + 2) + (-5 + (5)(3)) =$ $5(10) + (-5 + 15) =$ $5(10) + (10) = 50 + 10 = 60$
6. Simplify $\sqrt{(5 + 8)^2 - (7 + 5)^2}$		$\sqrt{(5 + 8)^2 - (7 + 5)^2}$ $\sqrt{13^2 - 12^2}$ $\sqrt{169 - 144}$ $\sqrt{25}$ 5
Problems	PA Core Math Look	Solutions
7. Simplify $(5 + 7 + 3) \div (3 + 2)$		Following the order of operations, $(5 + 7 + 3) \div (3 + 2) =$ Parenthesis $15 \div 5 =$ Division 3
8. Simplify $5 + 7 + 3 \div 3 + 2$		Following the order of operations, $5 + 7 + (3 \div 3) + 2 =$ Division $5 + 7 + 1 + 2 =$ Addition 14
9. Compare problem #7 with problem #8. Explain how someone may make the mistake of thinking they are the same problem.		In problem #7 you are asked to add $5 + 7 + 3$ first, then add $3 + 2$ , and finally divide the two answers $(5 + 7 + 3)/(3 + 2)$ . In problem #8, the first thing to do is divide 3 by 3 and then add $5 + 7 + 1 + 2$ .