

**Calculate angles**

**Program Task:** Calculate angles without using AutoCAD.

**Program Associated Vocabulary:**

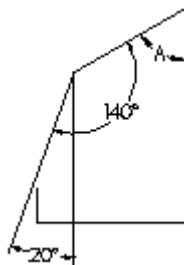
ANGLE, COMPLEMENTARY ANGLE, QUADRILATERAL, SUPPLEMENTARY ANGLE

**Program Formulas and Procedures:**

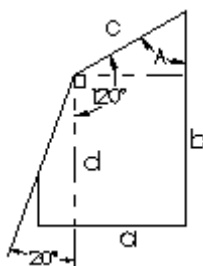
Engineering drawings frequently show parts with angled sides. With the advent of computer-aided drafting, it is a simple matter of using the angle dimensioning tool to determine the angle. As you may not always have access to a PC, it is important to understand how to calculate the angles without benefit of the software.

Sometimes complementary (90 degrees – given angle) and supplementary (180 degrees – given angle) angles can also be used during the process to find the desired angle(s).

**Example:** Angle “A” is missing from a drawing you are checking. You are at home, and do not have access to AutoCAD. Calculate the angle.



Adding the broken horizontal and vertical lines breaks the part profile into triangles and a quadrilateral with sides a, b, c, and d.



Then we can subtract  $20^\circ$  from the original  $140^\circ$  to get the  $120^\circ$  shown. Since all four sided figures contain  $360^\circ$ , angle A can be found by:

$$360 - (90 + 90 + 120) = 60$$

Angle A is  $60^\circ$

**Verify and apply geometric theorems as they relate to geometric figures**

**PA Core Standard:** CC.2.3.HS.A.3

**Description:** Verify and apply geometric theorems as they relate to geometric figures.

**Math Associated Vocabulary:**

CONGRUENT FIGURES, POLYGON, QUADRILATERAL, PARALLELOGRAM, PARALLEL, RECTANGLE, RHOMBUS, SQUARE, KITE, TRAPEZOID, ISOSCELES TRAPEZOID, SUPPLEMENTARY ANGLES

**Formulas and Procedures:**

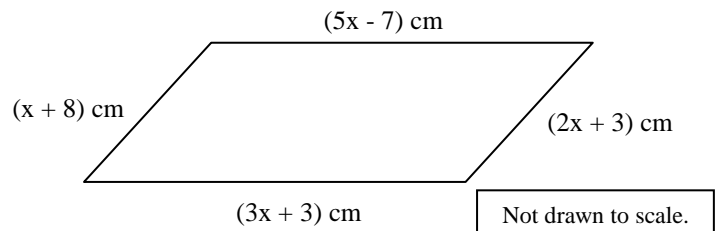
The following are the ways to prove a quadrilateral is a parallelogram:

1. Both pairs of opposite sides are parallel.
2. Both pairs of opposite sides are congruent.
3. Both pairs of opposite angles are congruent.
4. One pair of opposite sides are both congruent and parallel.
5. The diagonals bisect each other.
6. An angle is supplementary to both of its consecutive angles.

In order to prove that a parallelogram has right angles, you can measure the diagonals. If the diagonals are equal in length then it has right angles.

If the diagonals of a parallelogram are perpendicular then it has equal side lengths.

**Example:**



The figure above is a parallelogram. What are its side lengths?

$(5x - 7) = (3x + 3)$  Set opposite sides equal.

$2x - 7 = 3$  Subtract 2x.

$2x = 10$  Add 7.

$x = 5$  Divide by 2.

$3x + 3 = 3(5) + 3 = 18$  Substitute.

$x + 8 = 5 + 8 = 13$  Substitute.

The side lengths are 18 cm., 18 cm., 13 cm. and 13 cm.

**Instructor’s Script – Comparing and Contrasting**

Quadrilaterals, four sided, enclosed figures, have four interior angles. Although the student does not need to classify this quadrilateral as a right trapezoid, he/she must know that the all interior angle measurements add up to  $360^\circ$ .

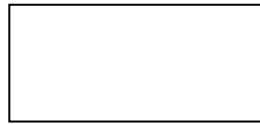
**Common Mistakes Made By Students**

A common mistake made by students is to confuse the properties of different quadrilaterals. A rectangle and square have congruent diagonals, but not every parallelogram has that property. Another common mistake is looking at the picture that may not be drawn to scale and making assumptions about the figure.

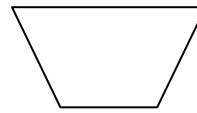
**Examples of Quadrilaterals:**



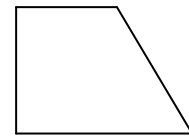
Square



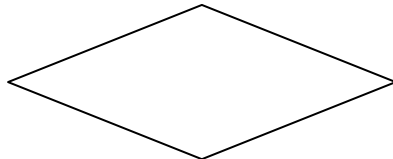
Rectangle



Trapezoid



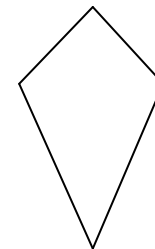
Right Trapezoid



Rhombus



Parallelogram

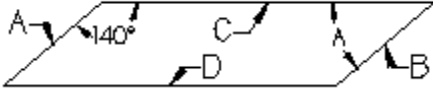

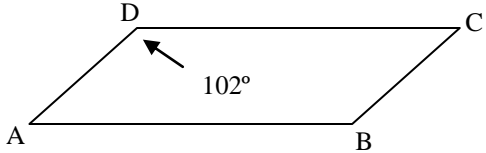


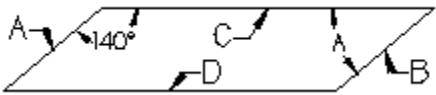


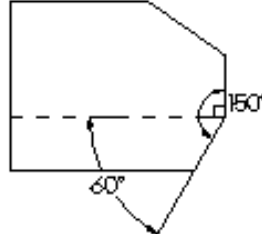
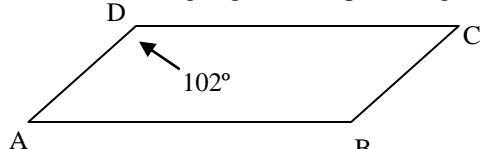
Kite

**CTE Instructor’s Extended Discussion**

There is often more than one way to break a complex shape with angular sides into quadrilaterals and instructors should take time to show different ways during examples. Depending on how the shape is “broken”, complementary or supplementary angle use might be required to find the desired angle.

With the advent of computer-aided drafting, many drafters today totally rely on the computer to do the work for them, negating the need to understand the math required to find the solution. It is embarrassing when a drafter forgets to dimension an angle, is on a site visit at the machine shop, and is asked to correct the drawing before s/he leaves, especially if they can’t perform the basic math involved due to their reliance on modern technology.

Problems	Career and Technical Math Concepts	Solutions
<p>1. You need to fill in the missing angle indicated as A. Sides A and B are parallel. Sides C and D are also parallel. What is angle A?</p>		
<p>2. Determine angle A. Then find its supplement as a setting for an angle vise to grind the part.</p> 		
<p>3. Determine angle B in the sketch above and then find its supplement for the angle vise setting.</p>		
Problems	Related, Generic Math Concepts	Solutions
<p>4. If the angles of a quadrilateral are <math>(x+7)^\circ</math>, <math>(5x-7)^\circ</math>, <math>(2x+2)^\circ</math> and <math>10^\circ</math>, what is each angle?</p>		
<p>5. If one side of a parallelogram measures <math>(2x+10)</math> in. and the opposite side measures <math>(3x-2)</math> in., what is the measure of these sides?</p>		
<p>6. If one diagonal of a rectangle is <math>(2x + 3)</math> cm and the other is <math>(x + 8)</math> cm, what is length of the diagonal?</p>		
Problems	PA Core Math Look	Solutions
<p>7. You have drawn a rhombus in a coordinate plane. The slope of one of the diagonals is <math>\frac{1}{4}</math>. What is the slope of the other diagonal?</p>		
<p>8. What are the missing angles in the parallelogram?</p> 		
<p>9. Which type of quadrilateral has exactly one pair of opposite sides parallel?</p>		

Problems	Career and Technical Math Concepts	Solutions
<p>1. You need to fill in the missing angle indicated as A. Sides A and B are parallel. Sides C and D are also parallel. What is angle A?</p>		<p>Since a parallelogram has equal opposite angles and all four angles add up to 360, Angle A = <math>[360 - (2 \times 140)] \div 2 = 40</math></p>
<p>2. Determine angle A. Then find its supplement as a setting for an angle vise to grind the part.</p> 		<p>Break the shape into a quadrilateral as shown. The <math>55^\circ</math> is found by subtracting <math>90^\circ</math> from the original <math>145^\circ</math>. Then find A by:                      Angle A = <math>360 - (90 + 90 + 55) = 125</math>                      Supplement of A = <math>180 - 125 = 55</math></p>
<p>3. Determine angle B in the sketch above and then find its supplement for the angle vise setting.</p>		<p>Again, break the profile into a quadrilateral. Subtract <math>90^\circ</math> from the original <math>150^\circ</math> to get the <math>60^\circ</math> shown. Then:                      Angle B = <math>360 - (90 + 90 + 60) = 120</math>                      Supplement of B = <math>180 - 120 = 60</math></p>
Problems	Related, Generic Math Concepts	Solutions
<p>4. If the angles of a quadrilateral are <math>(x+7)^\circ</math>, <math>(5x-7)^\circ</math>, <math>(2x+2)^\circ</math> and <math>10^\circ</math>, what is each angle?</p>		<p><math>(x+7) + (5x-7) + (2x+2) + 10 = 360</math>  <math>\rightarrow 8x+12=360 \rightarrow 8x=348 \rightarrow x=43.5</math>                      Substitute to find the angles. <math>(x+7)=50.5</math>, <math>(5x-7)=210.5</math>, <math>(2x+2)=89</math>                      The angles of the quadrilateral are <math>50.5^\circ</math>, <math>210.5^\circ</math>, <math>89^\circ</math> and <math>10^\circ</math>.</p>
<p>5. If one side of a parallelogram measures <math>(2x+10)</math> in. and the opposite side measures <math>(3x-2)</math> in., what is the measure of these sides?</p>		<p>Since opposite sides of a parallelogram are congruent, they are also equal in length. <math>3x-2=2x+10 \rightarrow x=12</math> Substitute to find the side lengths. <math>3(12) - 2 = 34</math> You can check your work by substituting into the expression for the other side length. <math>2(12) + 10 = 34</math> The side length is 34 inches.</p>
<p>6. If one diagonal of a rectangle is <math>(2x + 3)</math> cm and the other is <math>(x + 8)</math> cm, what is length of the diagonal?</p>		<p>Since the diagonals of a rectangle are congruent, <math>2x + 3 = x + 8</math>  <math>\rightarrow x + 3 = 8 \rightarrow x = 5</math> Substitute to find the length of the diagonal <math>2(5) + 3 = 13</math> The diagonal has a length of 13cm.</p>
Problems	PA Core Math Look	Solutions
<p>7. You have drawn a rhombus in a coordinate plane. The slope of one of the diagonals is <math>\frac{1}{4}</math>. What is the slope of the other diagonal?</p>		<p>Since the diagonals of a rhombus are perpendicular the other diagonal would have a slope that is the negative reciprocal of the given slope. The slope of the other diagonal is -4.</p>
<p>8. What are the missing angles in the parallelogram?</p> 		<p>Since opposite angles are congruent, the measure of angle B is <math>102^\circ</math>. Since consecutive angles are supplementary angles A and C are <math>78^\circ</math>.</p>
<p>9. Which type of quadrilateral has exactly one pair of opposite sides parallel?</p>		<p>A trapezoid is a quadrilateral with exactly one pair of opposite sides parallel.</p>