

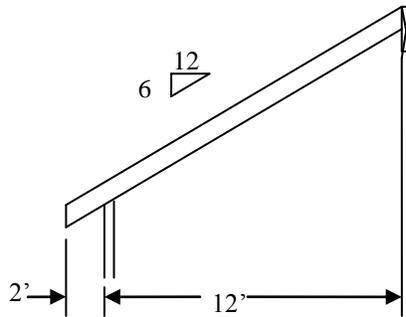
**Find the lengths of roof rafters = Understand and apply the Pythagorean theorem to solve problems**

**Program Task:** Determine the length of a roof rafter.

**Program Associated Vocabulary:**  
 HYPOTENUSE, DIAGONAL, LEG, RIGHT ANGLE, RIGHT TRIANGLE, PYTHAGOREAN THEOREM, ROOT, SQUARE

**Program Formulas and Procedures:**

Below is a sketch of a section through a roof of a framed house. Determine the length of the roof rafter. Note that a 2 foot eave is called for and that the top of the rafter connects to a ridge beam that is 1 1/2" wide. For convenience, the 12' dimension is given to the face of the ridge beam.



As we have an overhang (eave), we need to add that to the 12' dimension to calculate our length. Since this is 14', we can now determine our height. Since the slope indicated is 6:12, the rafter rises 6" for every foot of run. Since the run is 14', the rise (height) is 7' (.5 x 14').

$$C^2 = a^2 + b^2$$

$$C^2 = 14^2 + 7^2$$

$$C^2 = 196 + 49$$

$$C^2 = 245$$

$$C = \sqrt{245}$$

$$C = 15.65'$$

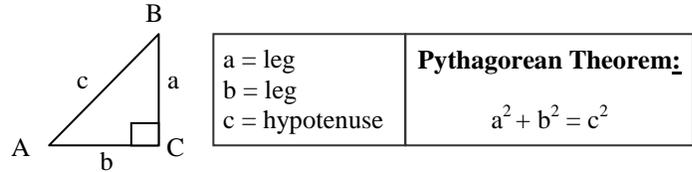
Since lumber comes in 2 foot increments, a 16 foot piece of lumber is required.

**PA Core Standard:** CC.2.3.8.A.3

**Description:** Understand and apply the Pythagorean theorem to solve problems.

**Math Associated Vocabulary:**  
 HYPOTENUSE, DIAGONAL, LEG, RIGHT ANGLE, RIGHT TRIANGLE, PYTHAGOREAN THEOREM, ROOT, SQUARE

**Formulas and Procedures:**



Solving for the hypotenuse, c, when given both legs.

**Example 1:**

A rectangle has side measurements of 8 inches and 12 inches. Find the length of the diagonal.

Step 1: Substitute known values into the Pythagorean theorem.

$$a^2 + b^2 = c^2 \rightarrow 8^2 + 12^2 = c^2$$

Step 2: Square and add each number as directed by the theorem.

$$64 + 144 = c^2 \rightarrow 208 = c^2$$

Step 3: Take the square root of each side to solve for c.

$$\sqrt{208} = \sqrt{c^2} \rightarrow 14.4 = c$$

Solving for a leg when given the hypotenuse and the other leg.

**Example 2:**

A right triangle has a hypotenuse that measures 10 inches and one of the legs measures 6 inches. Find the length of the other leg.

Step 1: Substitute known values into Pythagorean theorem.

$$a^2 + b^2 = c^2 \rightarrow 6^2 + b^2 = 10^2$$

Step 2: Square each number as directed by the theorem.

$$6^2 + b^2 = 10^2 \rightarrow 36 + b^2 = 100$$

Step 3: Subtract from both sides to isolate the variable.

$$36 - 36 + b^2 = 100 - 36 \rightarrow b^2 = 64$$

Step 4: Take the square root of each side to solve for the variable.

$$b^2 = 64 \rightarrow \sqrt{b^2} = \sqrt{64} \rightarrow b = 8$$

**Instructor’s Script – Comparing and Contrasting**

In the example shown on the drafting side of the T-chart, the student must be able to use the Pythagorean Theorem to solve for the diagonal, c. In many CTE applications the diagonal is the missing dimension of the triangle. It is also important to show students how to solve for one of the legs of the right triangle. The computation is slightly different and more complex and this knowledge will provide them with the ability to use the Pythagorean Theorem in other settings and situations.

**Common Mistakes Made By Students**

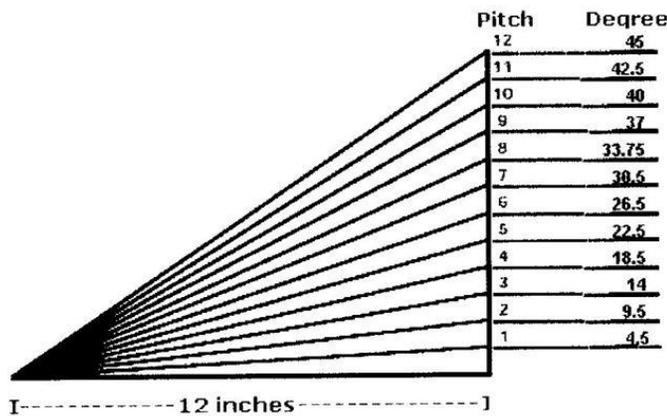
**Incorrectly identifying a, b, and c:** Students often confuse the hypotenuse with one of the legs or incorrectly substitute values into the equation. To avoid this problem recognize that the diagonal often is used to describe a hypotenuse. Label your hypotenuse right away by quickly identifying the right angle and marking the side opposite the right angle as the hypotenuse.

**Inability to manipulate the equation to solve for a or b:** Solving for the hypotenuse is much simpler than solving for a leg of a right triangle. Students need to be given many opportunities to solve for all the variables in the Pythagorean Theorem.

**Inability to recognize the Pythagorean Theorem in multiple contexts:** The Pythagorean Theorem appears in many contexts in standardized testing. Sometimes a test question will describe a right triangle and ask the student to solve for the missing side. Other times, the right triangle is drawn and the student must solve for the missing side. In many cases, a more complex picture is drawn and the student must use the Pythagorean Theorem to solve part of the problem. In these cases, it is not obvious that the Pythagorean Theorem is needed and the student must be able to select and use the theorem.

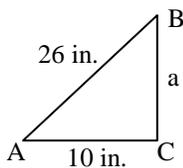
**CTE Instructor’s Extended Discussion**

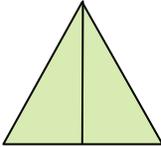
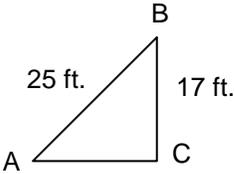
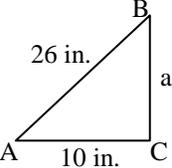
Below is a graphic to assist the student when drafting roof plans. It equates roof slope with angle, making it simpler for them to create their drawings.



While “slope” and “pitch” are often used interchangeably, the following is an explanation of the difference. In the United States, slope is typically given in inches per 1 foot or as a ratio of inches per 12 inches; and commonly referred to with units of “pitch” (e.g., for a slope of 1:3, “4 pitch (es)” is 4 inches of rise over 1 foot of run; “4:12 slope is 4 inches of rise over 12 inches of run. In the U.K., Australia and many other places, roof pitches given in degrees (<sup>0</sup>) are inclinations.

Problems	Career and Technical Math Concepts	Solutions
1. What is the length of a rafter if the base is 14' and the slope is 8/12?		
2. What is the length of a rafter if the base is 16' and the slope is 4/12?		
3. What is the length of a rafter if the base is 18' and the slope is 9/12?		
Problems	Related, Generic Math Concepts	Solutions
4. A tent has two slanted sides that are both 5 feet long and the bottom is 6 feet across. What is the height of the tent in feet at the tallest point?		
5. Three sides of a triangle measure 9 feet, 16 feet and 20 feet. Determine if this triangle is a right triangle.		
6. On a baseball diamond, the bases are 90 feet apart. What is the distance from home plate to second base using a straight line?		
Problems	PA Core Math Look	Solutions
7. The lengths of the legs of a right triangle measure 12 m. and 15 m. What is the length of the hypotenuse to the nearest whole meter?		
8. In a right triangle ABC, where angle C is the right angle, side AB is 25 feet and side BC is 17 feet. Find the length of side AC to the nearest tenth of a foot.		
9. In the given triangle, find the length of a.		



Problems	Career and Technical Math Concepts	Solutions
1. What is the length of a rafter if the base is 14' and the slope is 8/12?	Height = .67' x 14' = 9.38' $C^2 = 9.38^2 + 14^2$ $C^2 = 87.98 + 196$ $C^2 = 283.98$ $C = \sqrt{283.98}$ $C = 16.85'$	An 18 foot piece of lumber is required.
2. What is the length of a rafter if the base is 16' and the slope is 4/12?	Height = .33' x 16' = 5.28' $C^2 = 5.28^2 + 16^2$ $C^2 = 27.88 + 256$ $C^2 = 283.88$ $C = \sqrt{283.88}$ $C = 16.85'$	An 18 foot piece of lumber is required.
3. What is the length of a rafter if the base is 18' and the slope is 9/12?	Height = .75' x 18' = 13.5' $C^2 = 13.5^2 + 18^2$ $C^2 = 182.25 + 324$ $C^2 = 506.25$ $C = \sqrt{506.25}$ $C = 22.50'$	A 24 foot piece of lumber is required.
Problems	Related, Generic Math Concepts	Solutions
4. A tent has two slanted sides that are both 5 feet long and the bottom is 6 feet across. What is the height of the tent in feet at the tallest point?	$a^2 + b^2 = c^2$ $a^2 + 3^2 = 5^2$ $a^2 + 9 = 25$ $a^2 = 16$ $a = 4'$	
5. Three sides of a triangle measure 9 feet, 16 feet and 20 feet. Determine if this triangle is a right triangle.	$a^2 + b^2 = c^2$ $16^2 + 9^2 = 20^2$ $256 + 81 \neq 400$ Therefore, it is not a right triangle.	
6. On a baseball diamond, the bases are 90 feet apart. What is the distance from home plate to second base using a straight line?	$90^2 + 90^2 = c^2$ $8100 + 8100 = c^2$ $16200 = c^2$ $\sqrt{16200} = c$ $127.28\text{ft.} = c$	
Problems	PA Core Math Look	Solutions
7. The lengths of the legs of a right triangle measure 12 m. and 15 m. What is the length of the hypotenuse to the nearest whole meter?	$a^2 + b^2 = c^2$ $12^2 + 15^2 = c^2$ $144 + 225 = c^2$ $369 = c^2$ $\sqrt{369} = c$ $19 = c$ $c = 19 \text{ m.}$	
8. In a right triangle ABC, where angle C is the right angle, side AB is 25 feet and side BC is 17feet. Find the length of side AC to the nearest tenth of a foot.	$a^2 + b^2 = c^2$ $17^2 + b^2 = 25^2$ $289 + b^2 = 625$ $b^2 = 336$ $\sqrt{b^2} = \sqrt{336}$ $b = 18.3 \text{ ft}$	
9. In the given triangle, find the length of a.		$a^2 + b^2 = c^2$ $a^2 + 10^2 = 26^2$ $a^2 + 100 = 676$ $a^2 = 576$ $a = \sqrt{576}$ $a = 24 \text{ in.}$