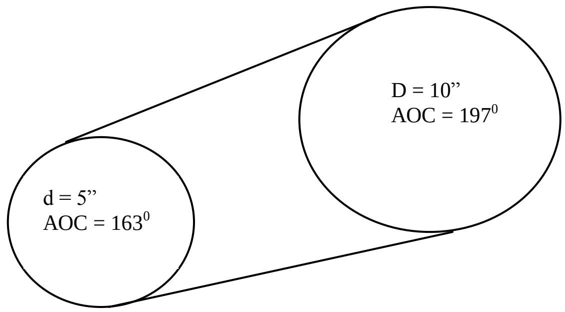
Drafting & Design Technology/Technician (15.1301) T-Chart

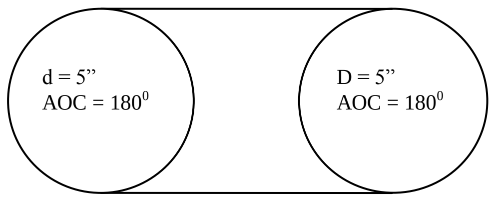
Determine Arc of Contact	=	Extend the concept of similarity to determine arc lengths and areas of sectors of circles

Program Task: Design a sheave and belt system.

Program Associated Vocabulary:
ARC OF CONTACT/MINIMUM BELT WRAP ANGLE, SHEAVE (PULLEY) CIRCUMFERENCE, DIAMETER, RATIO, CENTER TO CENTER DISTANCE

Program Formulas and Procedures:





Formula: d’s AOC^0 = 180 – (((D-d) x 60) / C)
D’s AOC^0 = 180 – (((d-D) x 60) / C)

Where:
D = larger sheave diameter
d = smaller sheave diameter
C = distance between sheaves, center to center
(18” center to center distance in this example)

Figure 2 used in this example:
d’s AOC^0 = 180 – (((D-d) x 60) / C)
d’s AOC^0 = 180 – (((10-5) x 60) / 18)
d’s AOC^0 = 163^0 (rounded to nearest whole number)

D’s AOC^0 = 180 – (((d-D) x 60) / C)
D’s AOC^0 = 180 – (((5-10) x 60) / 18)
D’s AOC^0 = 197^0 (rounded to nearest whole number)

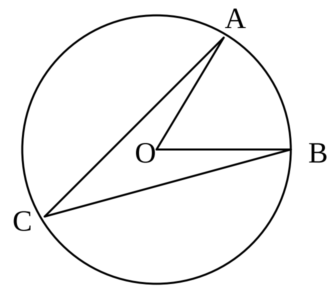
Note that 5 - 10 results in a negative number (- 5).
When subtracting the result from 180, you must add since subtracting a negative number requires changing the sign to a positive sign.

PA Core Standard: CC.2.3.HS.A.9

Description: Extend the concept of similarity to determine arc lengths and areas of sectors of circles.

Math Associated Vocabulary:
ARC, MINOR ARC, MAJOR ARC, SEMICIRCLE, CENTRAL ANGLE, CHORD, INSCRIBED ANGLE

Formulas and Procedures:



Vocabulary Examples:
Minor Arc: arc AB
Major Arc: arc ACB
Central Angle: ∠ AOB
Chord: AC and BC
Inscribed Angle: ∠ ACB

m∠ ACB = ½m∠ AOB
arc AB = m∠ AOB

Arc: a part of a circle or a curve between two points.

Minor Arc: an arc of a circle that is less than 180°.

Major Arc: an arc of a circle that is more than 180°.

Semicircle: an arc of a circle that is half of the circle (180°).

Central Angle: an angle whose vertex is the center of the circle.

Chord: a segment whose endpoints lie on a circle.

Inscribed Angle: an angle whose vertex is on the circle and whose sides are chords of a circle.

When an inscribed angle has the same end points as a central angle (subtended by the same arc), then the measurement of the inscribed angle is half of the measurement of the central angle.

Examples:

If m∠ AOB = 30°, then the m∠ ACB = 15°

If m∠ ACB = 50° , then the m∠ AOB = 100°

Find the central angle measurement, given the arc length.

The length of arc AB (above) is 4 cm. and the circumference of the circle is 10 cm.

1. Find the fraction of the circle that the arc intercepts.

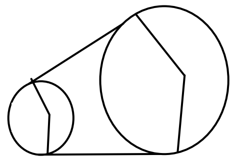
$$\frac{4}{10} = \frac{2}{5}$$

2. Multiply this fraction by 360° to get the central angle measurement:

$$\frac{2}{5} \cdot 360° = 144°$$

Instructor's Script – Comparing and Contrasting

In the example with the two equal sheaves shown on the drafting portion of the T-chart, the arcs of contact for each of the sheaves form two semicircles of 180°. In the second diagram, the arc of contact of the larger sheave forms a major arc while the arc of contact of the smaller sheave forms a minor arc. In both diagrams, the arcs of contact form central angles with the sheaves.



To extend the problem, have the students determine the arc of contact of the drive belt in inches. The procedure to do this for the problem presented in figure 2 is as follows:

1. Find the arc of contact of the smaller sheave: (197°)
2. Find the circumference of the smaller circle.

   $c = 2\pi r$ or $c = \pi d$

   $c = 3.14(5) = 15.7 \text{ in.}$
3. Then find the fraction of the circle covered by the arc of contact.

   $$\frac{197}{360}$$
4. Use this fraction to set up a proportion:

   $$\frac{AOC}{360} = \frac{\text{arc (in.)}}{c}$$

   $$\frac{197}{360} = \frac{\text{arc (in.)}}{15.7 \text{ in.}}$$
5. Cross-multiply and divide to find arc of contact length in inches:

   $$\frac{197}{360} = \frac{\text{arc (in.)}}{15.7 \text{ in.}}$$

   $$\frac{197(15.7)}{360} = 8.6 \text{ in.}$$

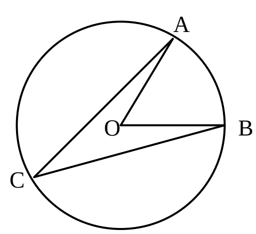
Common Mistakes Made By Students

- Lack of familiarity with and understanding of vocabulary associated with the concept.
- Setting up the proportion incorrectly to find arc lengths, given central angle measurements.

CTE Instructor's Extended Discussion

If the ratio of the driving sheave (pulley) to the driven sheave were the only important consideration when choosing sheave sizes, we would only need a minimal number of sheave sizes, hence decreasing the number of sheaves needed to be stocked. Why use a motor sheave with a 5" diameter and a blower sheave with a 10" diameter when 1" and 2" diameter sheaves respectively would provide the same ratio, hence the same speed? This would cause unnecessary wear and tear on the belt, causing frequent replacements.

Generally speaking, the area of belt sheave contact is critical to the performance of the device, and the smaller sheave is of primary concern when considering contact area. The relationship between the sheave's diameters and the center to center distance between the sheaves become critical factors in contact surface area calculations.

Problems / Career and Technical Math Concepts	Solutions
1. Determine the Arc of Contact for the smallest sheave. Sheave set A: motor sheave (d) = 5 and blower sheave (D) = 15, distance between sheave centers (C) = 18	
2. Determine the Arc of Contact for the smallest sheave. Sheave set B: motor sheave (d) = 5 and blower sheave (D) = 15, distance between sheave centers (C) = 24”	
3. Determine the Arc of Contact for the smallest sheave. Sheave set C: motor sheave (d) = 5 and blower sheave (D) = 30, distance between sheave centers (C) = 24”	
Problems / Related, Generic Math Concepts	**Solutions**
4. To find the Earth's approximate Arc of Shade, use the formula: AOS = 180 - (((S-E) x 60) / C) Where: S = Sun's diameter (870,000 miles) E = Earth's diameter (7,926 miles) C = distance between Sun and Earth (93,000,000 miles)	
5. Imagine the Earth moved into Mercury’s orbit, it would be only 28,600,000 miles from the Sun! Find its Arc of Shade, using the formula: AOS = 180 - (((S-E) x 60) / C). As a bonus, determine the difference in night hours between the two orbits. Where: S = Sun's diameter (870,000 miles) E = Earth's diameter (7,926 miles) C = fictional distance between Sun/Earth (28,600,000 miles)	
6. When a clock reads 9:00, the central angle between the two hands is 90°. If the arc length between the hands is 6 cm, how many cm. of metal would be needed to frame the clock?	
Problems / PA Core Math Look	**Solutions**
7. Use the diagram shown to answer questions 7-9: If m ∠ ACB = 21.5°, find the measurement of ∠ AOB. 	
8. If the length of arc AB = 3 cm. and the circumference of circle O = 15 cm, what is the measurement of ∠ AOB?	
9. If the measurement of ∠ AOB = 73° and segment OB = 8 cm, find the length of arc AB in centimeters.	

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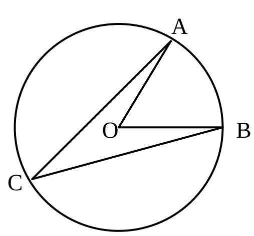
Problems	Occupational (Contextual) Math Concepts Solutions
1. Determine the Arc of Contact for the smallest sheave. Sheave set A: motor sheave (d) = 5 and blower sheave (D) = 15, distance between sheave centers (C) = 18	d's AOC^0 = 180 – (((D-d) x 60) / C) d's AOC^0 = 180 – (((15-5) x 60) / 18) d's AOC^0 = 146.7^0
2. Determine the Arc of Contact for the smallest sheave. Sheave set B: motor sheave (d) = 5 and blower sheave (D) = 15, distance between sheave centers (C) = 24"	d's AOC^0 = 180 – (((D-d) x 60) / C) d's AOC^0 = 180 – (((15-5) x 60) / 24) d's AOC^0 = 155^0
3. Determine the Arc of Contact for the smallest sheave. Sheave set C: motor sheave (d) = 5 and blower sheave (D) = 30, distance between sheave centers (C) = 24"	d's AOC^0 = 180 – (((D-d) x 60) / C) d's AOC^0 = 180 – (((30-5) x 60) / 24) d's AOC^0 = 117.5^0
Problems	**Related, Generic Math Concepts Solutions**
4. To find the Earth's approximate Arc of Shade, use the formula: AOS = 180 - (((S-E) x 60) / C) Where: S = Sun's diameter (870,000 miles) E = Earth's diameter (7,926 miles) C = distance between Sun and Earth (93,000,000 miles)	AOS = 180 - (((S-E) x 60) / C) AOS = 180 - (((870,000 - 7,926) x 60) / 93,000,000) AOS = 180 - ((862,074 x 60) / 93,000,000)) AOS = 180 - .556 AOS = 179.4°
5. Imagine the Earth moved into Mercury's orbit, it would be only 28,600,000 miles from the Sun! Find its Arc of Shade, using the formula: AOS = 180 - (((S-E) * 60) / C). As a bonus, determine the difference in night hours between the two orbits. Where: S = Sun's diameter (870,000 miles) E = Earth's diameter (7,926 miles) C = fictional distance between Sun/Earth (28,600,000 mi)	AOS = 180 - (((S-E) x 60) / C) AOS = 180 – 1.8 AOS = 178.2° Prob #4 Shade Time = (179.4 / 360) x 24 hrs. = 11.96 hrs. Prob #5 Shade Time = (178.2 / 360) x 24 hrs. = 11.88 hrs. 11.96 - 11.88 = .08 hours .08 x 60 = 4.8 less minutes night time in Mercury's orbit
6. When a clock reads 9:00, the central angle between the two hands is 90°. If the arc length between the hands is 6 cm, how many cm. of metal would be needed to frame the clock?	$\frac{90}{360} = \frac{6}{x}$ 90x = 6(360) $x = \frac{6(360)}{90} = 24$ cm.
Problems	**PA Core Math Look Solutions**
7. Use the diagram shown to answer questions 7-9: If m ∠ ACB = 21.5°, find the measurement of ∠ AOB.  	21.5 x 2 = 43°
8. If the length of arc AB = 3 cm. and the circumference of circle O = 15 cm, what is the measurement of ∠ AOB?	$\frac{3}{15} = \frac{x}{360}$ $x = \frac{3(360)}{15} = 72°$
9. If the measurement of ∠ AOB = 73° and segment OB = 8 cm, find the length of arc AB in centimeters.	C = 16(3.14) = 50.26 cm. $\frac{73}{360} = \frac{x}{50.26} \rightarrow x = \frac{73(50.26)}{360} =$ 10.2 cm.