

Identify and use isosceles and equilateral triangles

Verify and apply geometric theorems as they relate to geometric figures

Program Task: Determine the height of an existing smoke stack without measuring it.

Program Associated Vocabulary:
ISOSCELES, EQUILATERAL, BASE, LEG

Program Formulas and Procedures:

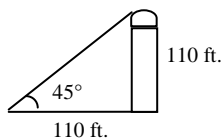
You work for a contractor who specializes in rebuilding brick smokestacks. You are asked to visit the site and get the dimensions of the existing stack so the contractor can submit a proposal. When you get there, you realize the stack is probably about 100' high and there is no ladder available to measure it, nor do you wish to climb the steel rungs attached to the smokestack since they appear to be rusty and may not be safe...maybe you are also afraid of heights! You can get the circumference easily enough with a steel tape measure...but how do you get the height?



You have with you a laser measuring device, but can't use it since it will not reflect the light

back to the device due to the angle, but you can see the laser. You also have a 45 degree carpenter's square with you. Let's assume that the ground around the smokestack is level.

1. Place the 45 degree square on the ground and rest the laser on it, pointing at the smokestack.
2. Keep moving away from the stack until the laser light is exactly at the top of the stack.
3. Now, on the ground, measure back from the point you found to the center of the smokestack. You find it is 110'.
4. Since you just created an isosceles triangle, the distance from the laser on the ground to the center of the stack is the same as the height of the stack.



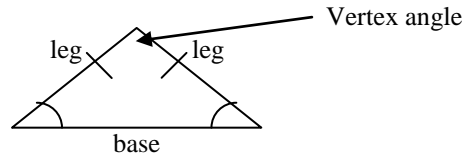
PA Cord Standard: CC.2.3.HS.A.3

Description: Verify and apply geometric theorems as they relate to geometric figures.

Math Associated Vocabulary:
ISOSCELES, EQUILATERAL, CONGRUENT, BASE, LEG, VERTEX

Formulas and Procedures:

Isosceles Triangle: a triangle with two congruent sides and therefore, two congruent angles.



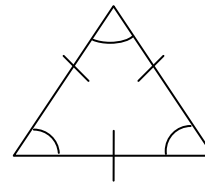
If you are given one angle of an isosceles triangle, you can find the other two missing angles using the fact that the two base angles are congruent and the sum of the angles of a triangle is 180°.

Also, if you are given the length of one leg then you can find the other leg, since the two are congruent.

Example: $\triangle JKL$ is isosceles. Base angle J is 15°. Find the $m\angle K$ if $\angle K$ is the vertex angle.

Solution: Since base angle J is 15, then base angle L is also 15. The vertex angle K must equal $180 - 30$ (since all angles in a triangle add up to 180). Vertex K measures 150°.

Equilateral Triangle: a triangle with three congruent sides and therefore, three congruent angles.



Since all the angles in any triangle add up to 180°, then each angle of an equilateral triangle measures 60°.

You can find the length of each side when given the perimeter of the triangle by dividing by 3.

You can find the length of each side when given one side, since all sides are equal.

Example: If the perimeter of an equilateral triangle equals 120 inches, what is the measure of each side?

Solution: $120/3 = 40$ inches.

Instructor's Script - Comparing and Contrasting

The drafting problem presented on page one of the T-chart is a very practical application of the properties of isosceles triangles. Students who have taken a geometry course should be able to describe why this process works without being told that an isosceles triangle is formed.

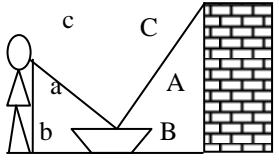
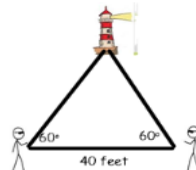
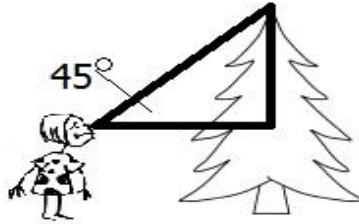
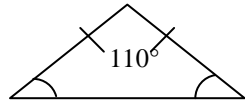
Common Mistakes Made By Students

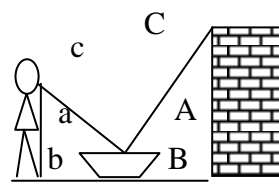
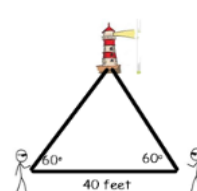
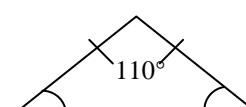
- Students may have difficulty identifying isosceles and equilateral triangles when presented in application problems.
- Students forget that all angles in a triangle add up to 180° .
- Students may have difficulty identifying the vertex angle.

CTE Instructor's Extended Discussion

Very often drafters are sent to project sites to field measure. Most of the time this is simply a matter of knowing how to use a tape measure and a sketch pad...no big deal. However, on occasions where you have to get the heights of such things as flagpoles, silos, smokestacks, etc., it is not always possible, and usually not safe, to obtain these measurements. Understanding how isosceles triangles relate to lengths will allow the drafter to complete his assignment correctly and safely!

In problem #1 on page 3, water is used to determine the height of an object. The use of a mirror could cause inaccuracy unless it was perfectly level. Since water seeks its own level, there is less chance of an error using water.

Problems	Career and Technical Math Concepts	Solutions
<p>1. An alternate solution if you do not have a laser:</p> <ol style="list-style-type: none"> Measure the distance between the worker's eyes and his feet (a). Place a saucer of water (not a mirror) between the worker and the building, making sure that the distance from his feet to the center of the water (b) is always equal to a. Move the water around the object until the top of the object is reflected in the water. Measure the distance from the center of the water (B) to the base of the object- that is the height of the object (A). 		 <p>You follow the procedure on the left. If $a = b$, and B is 80°, what is the height of the chimney?</p>
<p>2. You are asked to order guy wire that will be as far from the base of the chimney as it is high. How far from the base of the chimney will you install the guy wire?</p>		
<p>3. Use the Pythagorean Theorem to determine the length of the wire.</p>		
Problems	Related, Generic Math Concepts	Solutions
<p>4. Two people, standing 40 feet apart, are looking at the top of a lighthouse in the distance. The angle of elevation for each person to the top of the lighthouse is 60°. What is the perimeter of the triangle formed by the lighthouse and two people?</p>		
<p>5. If the vertex angle of an isosceles triangle is 90°, what are the measures of the other angles?</p>		
<p>6. A person uses a tool to look at a tree at an angle of elevation of 45°. She continues to back up until she can see the top of the tree. If she is about 5 feet tall and is standing 46 feet from the tree when she sees the top, how tall is the tree?</p>		
Problems	PA Core Math Look	Solutions
<p>7. In isosceles triangle LMN, $\angle M$ is the vertex angle. $m\angle L = 60^\circ$ and $m\angle N = 5x - 10$. Find the value of x.</p>		
<p>8. Find the measurement of the two missing angles in the triangle.</p>		
<p>9. $\triangle XYZ$ is an equilateral triangle. $XY = 2x + 10$, $YZ = 3x + 5$ and $XZ = 20$. Find the value of x.</p>		

Problems	Career and Technical Math Concepts	Solutions
<p>1. An alternate solution if you do not have a laser:</p> <p>a) Measure the distance between the worker's eyes and his feet (a).</p> <p>b) Place a saucer of water (not a mirror) between the worker and the building, making sure that the distance from his feet to the center of the water (b) is always equal to a.</p> <p>c) Move the water around the object until the top of the object is reflected in the water.</p> <p>d) Measure the distance from the center of the water (B) to the base of the object – that is the height of the object (A).</p>		 <p>You follow the procedure on the left. If $a = b$, and B is 80°, what is the height of the chimney? 80°, because B would be equal to A.</p>
<p>2. You are asked to order guy wire that will be as far from the base of the chimney as it is high. How far from the base of the chimney will you install the guy wire?</p>		<p>$80'$ from the base. You determined the height above, and were told it had to be as far from the chimney as the height of the chimney.</p>
<p>3. Use the Pythagorean Theorem to determine the length of the wire.</p>	$C^2 = 80^2 + 80^2$ $C = \sqrt{12800}$	$C^2 = 6400 + 6400$ $C^2 = 12,800$ $C = 113.13'$
Problems	Related, Generic Math Concepts	Solutions
<p>4. Two people, standing 40 feet apart, are looking at the top of a lighthouse in the distance. The angle of elevation for each person to the top of the lighthouse is 60°. What is the perimeter of the triangle formed by the lighthouse and two people?</p>		<p>Since two of the angles are 60°, then the other angle must also be 60° and therefore the triangle must be equilateral. The perimeter would equal $40 \times 3 = 120$ feet.</p> 
<p>5. If the vertex angle of an isosceles triangle is 90°, what are the measures of the other angles?</p>		<p>The other two angles are both 45°.</p>
<p>6. A person uses a tool to look at a tree at an angle of elevation of 45°. She continues to back up until she can see the top of the tree. If she is about 5 feet tall and is standing 46 feet from the tree when she sees the top, how tall is the tree?</p>		<p>An isosceles triangle is formed. The distance ($46'$) between the person and the tree is one leg of the triangle; the other leg of the triangle is the tree trunk from the person's height to the top of the tree. As each leg of an isosceles triangle is equal, the tree trunk leg is also 46 feet tall. Adding the $46'$ to the person's height of $5'$, yields the total height of the tree $51'$.</p>
Problems	PA Core Math Look	Solutions
<p>7. In isosceles triangle LMN, $\angle M$ is the vertex angle. $m \angle L = 60^\circ$ and $m \angle N = 5x - 10$. Find the value of x.</p>		<p>Angle L = 60, so angle N also equals 60 $60 = 5x - 10 \rightarrow 60 + 10 = 5x - 10 + 10$ $70 = 5x, x = 14$</p>
<p>8. Find the measurement of the two missing angles in the triangle.</p> 		<p>The sum of the base angles must equal $180 - 110 = 70$. Since each angle is equal, they must both measure 35°</p>
<p>9. ΔXYZ is an equilateral triangle. $XY = 2x + 10$, $YZ = 3x + 5$ and $XZ = 20$. Find the value of x.</p>		<p>$2x + 10 = 20 \rightarrow 2x + 10 - 10 = 20 - 10$ $2x = 10, x = 5$</p>