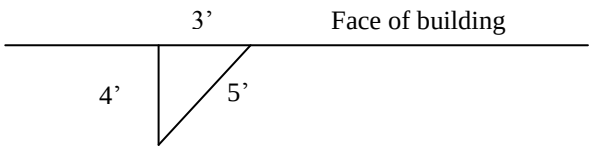
Drafting & Design Technology/Technician (15.1301) T-Chart

The "3-4-5 Rule"	=	Verify and apply geometric theorems as they relate to geometric figures

Program Task: Create a line perpendicular to a building using 2 tape measures.

Program Associated Vocabulary:
ISOSCELES, RIGHT, PYTHAGOREAN THEOREM

Program Formulas and Procedures:
While it is a simple matter to draw a line on a plain perpendicular (90^0) to a given wall by either using a 90^0 triangle if hand drafting, or setting your Ortho on a CAD drawing, it is quite another matter to be called out to the field and create the same line. This is where the 3-4-5 Rule comes into effect and is used extensively by drafters and carpenters.



1. Mark a point on the wall where you want the perpendicular line.
2. Measure 3' to the right and place another mark on the wall. This is the base of your triangle.
3. Now take one tape measure and set and lock it so it is 4' away from the building. This is the vertical leg of the triangle.
4. Next take your second tape and set it at 5'.
5. Keeping the ends of each tape against the marks on the wall, carefully move them until the 4' and 5' lines intersect. You now have a point that is perpendicular to the first mark on the building. Why does this work? It works because of the Pythagorean Theorem: $C^2 = A^2 + B^2$.

$5^2 = 3^2 + 4^2$
$25 = 9 + 16$
$25 = 25$

Although these layouts are typically created using transits and lasers for greater accuracy, this rule is often applied in the design stage of a project to determine such things as grade, location of obstructions, etc. Understanding this simple rule will prove invaluable and does not require any math.

Pa Core Standard: CC.2.3.HS.A.3

Description: Verify and apply geometric theorems as they relate to geometric figures.

Program Associated Vocabulary:
CONGRUENT, SIMILAR, EQUILATERAL, ISOSCELES, SCALENE, RIGHT, OBTUSE, ACUTE, MEDIAN, ALTITUDE, ANGLE BISECTOR, PERPENDICULAR BISECTOR, CENTROID, ORTHOCENTER, INCENTER, CIRCUMFERENCE

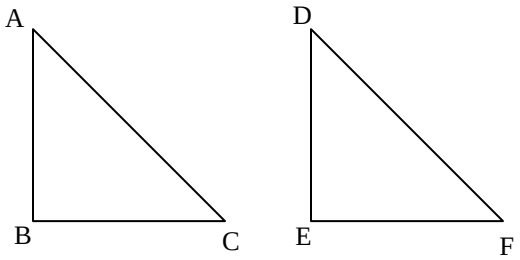
Formulas and Procedures:
Procedures for proving triangles congruent: SSS (side-side-side), SAS (side-angle-side), ASA (angle-side-angle), AAS (angle-angle-side), HL (hypotenuse-leg).

Procedures for proving triangles similar: AA (angle-angle similarity), SAS (side-angle-side similarity), SSS (side-side-side similarity).

The Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

The sum of the angles of a triangle is 180º.

Example:



$\Delta ABC \cong \Delta DEF$

What are the congruent parts?

$\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$\overline{AC} \cong \overline{DF}$

Congruent Sides

$\angle A \cong \angle D$
$\angle B \cong \angle E$
$\angle C \cong \angle F$

Congruent Angles

Instructor's Script – Comparing and Contrasting

Depending on the length of your perpendicular line, you may want a point that extends out more than 4 feet for better accuracy. Similar triangles are triangles that have congruent angles and similar sides, meaning that the corresponding sides are in proportion to each other. Multiplying each measurement in the drafting example allows us to create a similar triangle to the 3-4-5 rule being used. For instance, we can use 6-8-10, or 9-12-15. The 3-4-5 Rule used to mark off perpendicular lines is a great opportunity to teach the Pythagorean theorem, similar triangles, and ratio and proportions. You could extend this problem and ask the following: your wall must extend out perpendicular to the house. The wall will measure 20 feet. What multiple of 3-4-5 would you use to get closest to the 20 feet?

Common Mistakes Made By Students

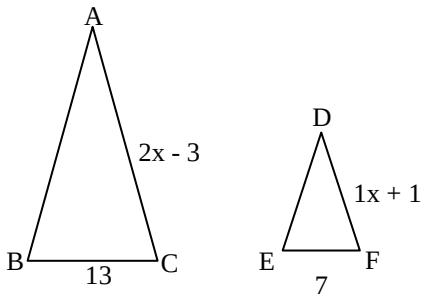
- Lack of familiarity with the concept and understanding of the vocabulary associated with the concept.
- Setting up the proportion incorrectly.
- Confusing the properties of congruent triangles with the properties of similar triangles.

Example:

$\Delta ABC \sim \Delta DEF$

Find the length of AC.

Since the triangles are similar, the ratio of the corresponding sides is proportional.

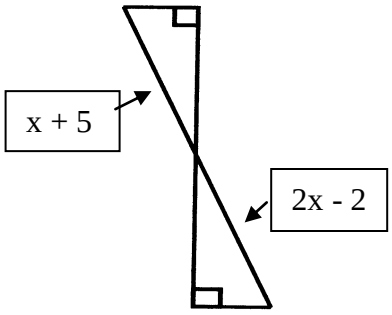


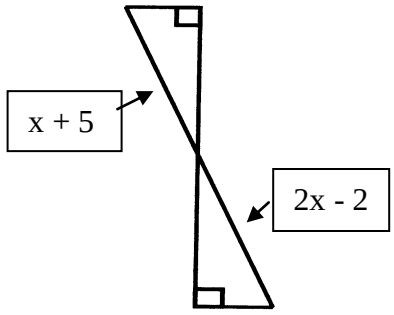
$$\frac{2x-3}{1x+1} = \frac{13}{7} \rightarrow 7(2x-3) = 13(x+1) \rightarrow 14x - 21 = 13x + 13 \rightarrow x = 34$$

Using x = 34, you can substitute to find the length of AC.
AC = 2x – 3
2(34) – 3 = 65
AC would be 65 units.

CTE Instructor's Extended Discussion

The 3-4-5 rule is used extensively in the building trades, not just by carpenters, but also by electricians laying out conduit and pipefitters laying out piping runs. Very often drafters, after gaining experience, enter into project management. That is, they will visit the site weekly to ensure that the design is being followed. Many have had the occasion to use the 3-4-5 Rule to verify that the construction is indeed in compliance with the drawings.

Problems	Career and Technical Math Concepts	Solutions
1. A series of holes in a steel plate to be machined need Ø.40" x 90° chamfers. What countersink depth must be dimensioned?		
2. You are asked to design a landscape feature that is triangular. The base is 7' and the length of the sides is 35'. The client wants the same shape, but wants the base increased to 13'. How long will the sides be?		
3. You are asked to design a landscape feature that is triangular. The base is 10' and the length of the sides is 50'. The client wants the same shape, but wants the base increased to 15'. How long will the sides be?		
Problems	**Related, Generic Math Concepts**	**Solutions**
4. Name all congruent parts. $\Delta ABC \cong \Delta XYZ$		
5. Which of the following side lengths can NOT make a triangle? a) 3, 4, 4 b) 2, 4, 5 c) 3, 7, 9 d) 10, 14, 2		
6. If an isosceles triangle has a vertex angle that is 3 times the measure of the base angles, what are the angles of the triangle?		
Problems	**PA Core Math Look**	**Solutions**
7. What value of x would make the triangles congruent?  		
8. What congruence theorem could you use to prove the triangles congruent?		
9. What would be the length of the congruent sides for which measurements are shown?		

Problems	Career and Technical Math Concepts	Solutions
1. A series of holes in a steel plate to be machined need Ø.40" x 90° chamfers. What countersink depth must be dimensioned?	One half of the Ø.40" is on each side of the hole's centerline creating two 45-45-90 triangles with leg lengths equal to ½ of the .40". The dimensioned depth should be .20".	
2. You are asked to design a landscape feature that is triangular. The base is 7' and the length of the sides is 35'. The client wants the same shape, but wants the base increased to 13'. How long will the sides be?	Since the triangles are similar, the ratio of the corresponding sides is proportional. $\frac{x}{35} = \frac{13}{7} \rightarrow 13 \times 35 = 7x \rightarrow 7x = 455 \rightarrow x = 65'$	
3. You are asked to design a landscape feature that is triangular. The base is 10' and the length of the sides is 50'. The client wants the same shape, but wants the base increased to 15'. How long will the sides be?	Since the triangles are similar, the ratio of the corresponding sides is proportional. $\frac{x}{50} = \frac{15}{10} \rightarrow 15 \times 50 = 10x \rightarrow 10x = 750 \rightarrow x = 75'$	
Problems	**Related, Generic Math Concepts**	**Solutions**
4. Name all congruent parts. $\Delta ABC \cong \Delta XYZ$	$\angle A \cong \angle X \quad \angle B \cong \angle Y \quad \angle C \cong \angle Z$ $\overline{AB} \cong \overline{XY} \quad \overline{BC} \cong \overline{YZ} \quad \overline{AC} \cong \overline{XZ}$	
5. Which of the following side lengths can NOT make a triangle? a) 3, 4, 4 b) 2, 4, 5 c) 3, 7, 9 d) 10, 14, 2	d) 10 + 2 is not greater than 14, so this can NOT make a triangle.	
6. If an isosceles triangle has a vertex angle that is 3 times the measure of the base angles, what are the angles of the triangle?	$3x + x + x = 180^o \rightarrow 5x = 180^o \rightarrow x = 36^o$ $3x = 3(36) \rightarrow 3x = 108^o$ The vertex angle is 108º, and the base angles are 36º each.	
Problems	**PA Core Math Look**	**Solutions**
7. What value of x would make the triangles congruent?  	$2x - 2 = x + 5$ $x - 2 = 5$ $x = 7$	The sides would need to be equal in length for the triangles to be congruent. Subtract "x" from each side of the Add 2 to each side of the equation.
8. What congruence theorem could you use to prove the triangles congruent?	You would use the AAS (angle-angle-side) congruence postulate.	
9. What would be the length of the congruent sides for which measurements are shown?	$x + 5 = 7 + 5 \rightarrow x = 12$ $2x - 2 = 2(7) - 2 \rightarrow x = 12$	