

Find averages to determine usages = Summarize, represent and interpret data on a single count or measurement variable

Program Task: Determine average kilowatt consumption for an average household.

Program Associated Vocabulary:
MEAN, MAXIMUM, MINIMUM, AVERAGE, HIGH

Program Formulas and Procedures:
Some electric utilities companies allow homeowners to pay their electric bills by setting up budget billing. This budget billing figure is derived by the average electric bill over a certain period, usually the prior 12 months.

The electric company, like PPL electric, will take twelve months of bills (Jan-Dec) of the prior year, add the total, and divide by 12.

For example, the usage for the prior 12 months is shown below:

January	\$135.00
February	\$145.00
March	\$112.00
April	\$101.00
May	\$98.00
June	\$87.00
July	\$116.00
August	\$121.00
September	\$113.00
October	\$107.00
November	\$126.00
December	\$131.00

What is the average of this homeowner’s electric bill for this given year? Average=Mean

$$\frac{135+145+112+101+98+87+116+121+113+107+126+131}{12} = 116$$

The average electric bill for this house is \$116.

One could assume, if electrical consumption remained the same for the following year, this household’s electrical budget amount per month would be around \$116.00.

What would happen if a heat wave hit in August and the family’s electrical bill for the month increased to \$449? By how much would this affect their budgeted amount? The charge for August would be an outlier, because it is so much higher than the other values. This outlier would increase the average for all 12 months.

$$\frac{135+145+112+101+98+87+116+449+113+107+126+131}{12} = 143$$

With a heat wave the average electrical bill for this house will increase to \$143.

PA Core Standard: CC.2.4.HS.B.1

Description: Summarize, represent and interpret data on a single count or measurement variable.

Math Associated Vocabulary:
MEAN, MEDIAN, MODE, OUTLIER

Formulas and Procedures:

Outlier	An extreme value in a set of data which is much higher or lower than the other numbers.
Mean (Average)	The average of set of data that is calculated by dividing the sum of the data by the number of items in the set.
Median	The middle value when data are arranged in numeric order or the average of the two middle numbers when the set has an even number of data.
Mode	The value that occurs most frequently in a set of data.

Measures of central tendency are mean, median and mode. Outliers affect the mean value of the data but have little effect on the median or mode of a given set of data.

Example: A student receives a zero on a quiz and subsequently has the following scores:

$$0, 70, 70, 80, 85, 90, 90, 90, 95, 100$$

Outlier: 0

Mean:

$$\frac{0 + 70 + 70 + 80 + 85 + 90 + 90 + 90 + 95 + 100}{10} = 77$$

Median: since the data set has 10 values, there are two middle numbers, so one must find the mean of these two values, 85 and 90.

$$\frac{85 + 90}{2} = 87.5$$

Mode: The score 90 occurs more frequently than the other values (three times), so 90 is the mode.

Receiving a zero on a quiz significantly affects a student’s mean, or average. Notice that the outlier had a small effect on the median and mode of the data.

It should be noted that because outliers affect the mean and have little effect on the median, the median is often used to describe “average” income. Often, one hears that the median income for a group is a certain value. Mean is not typically used because outliers, people who make significantly more or make no money at all, affect this measure.

Instructor's Script – Comparing and Contrasting

Outliers are numbers in a data set that are vastly larger or smaller than the other values in the set. Mean, median and mode are measures of central tendency. Mean is the only measure of central tendency that is always affected by an outlier. Mean, the average, is the most popular measure of central tendency.

Common Mistakes Made By Students

Calculator error when finding the mean: Students often forget to use parenthesis when finding the mean of a data set. For instance, to find the average of 40 and 50, parenthesis must be used for the sum before dividing by two. Students often enter $40 + 50/2$, which yields an answer of 65 instead of entering $(40 + 50)/2$ which yields the correct answer of 45.

Changing the divisor: When determining how an outlier affects the mean of a data set, the student must find the mean with the outlier, then find the mean again once the outlier is removed. Removing the outlier decreases the number of data by one and therefore you must decrease the divisor. For instance, when you find the mean of 0, 10, 10, 12, 12, you must divide the sum by 5, but when you remove the outlier of 0, you must then divide by 4.

When calculating the median, students must list the data need in numerical order.

Finding the median of an even set of data: Finding the median or middle number, of a set of data is simple when there is an odd number of data. When there is an even number, there are two middle numbers, and these numbers must be averaged to obtain the median. For instance, the median of 1, 1, 2, 3, 3 is 2 because 2 is the middle number. If the data set is 1, 2, 3, 3, then 2 and 3 are the middle numbers and must be averaged to obtain the median of 2.5.

CTE Instructor's Extended Discussion

Averaging is a skill utilized in many different applications. For example, teachers and students may utilize this type of problem solving technique to determine a grade for a marking period.

Problems	Career and Technical Math Concepts	Solutions
1. An electrician wants to find the average temperature in a free standing garage over the three winter months. He needs to determine the proper size heater to be installed to keep the garage at a comfortable temperature. (Dec: 38 degrees, Jan: 25 degrees, Feb: 21 degrees). What is the average temperature for these months?		
2. A homeowner wants to find out how much electricity his central air conditioner used for the summer of 2008. His (non-AC) summer average for 2007 was \$105. 2008 bills: June: \$126, July: \$132, August: \$141 What is the 2008 summer average, how much more does his central AC cost him?		
3. An electrician made \$1,236, \$1,162, \$1,329, and \$1,512 each week for 4 weeks. What was his average weekly salary for the above weeks?		
Problems	Related, Generic Math Concepts	Solutions
4. Sally earned scores of 60, 65, 65, and 80 on 4 tests. How would scoring 100 on a fifth test affect the mean?		
5. Tom recorded his daily caloric intake for 5 days. The results were as follows: 2500, 2600, 2600, 2400, and 3900. How would removing the outlier affect the mean, median and mode of the data?		
6. Angela recorded the number of hours she spent watching TV for one week. The results were as follows: 6, 2, 2, 1.5, 3, 2.5, 2. How would removing the outlier affect the mean, median, and mode of the data?		
Problems	PA Core Math Look	Solutions
7. Which of the following measures of central tendency does an outlier affect the most? a) Mean b) Median c) Mode		
8. Which measure of central tendency would best depict the following data: 10, 200, 200, 300, 325, 350 and 400? a) Mean b) Median c) Mode		
9. How would removing the outlier affect the mean of the following data: 1200, 2400, 2400, 2500 and 9000?		

Problems	Career and Technical Math Concepts	Solutions
1. An electrician wants to find the average temperature in a free standing garage over the three winter months. He needs to determine the proper size heater to be installed to keep the garage at a comfortable temperature. (Dec: 38 degrees, Jan: 25 degrees, Feb: 21 degrees). What is the average temperature for these months?	$\frac{38 + 25 + 21}{3} = 28$	The average temperature for the three months is 28 degrees.
2. A homeowner wants to find out how much electricity his central air conditioner used for the summer of 2008. His (non-AC) summer average for 2007 was: \$105 2008 bills: June: \$126, July: \$132, August: \$141 What is the 2008 summer average, how much more does his central AC cost him?	$\frac{\$126 + \$132 + \$141}{3} = \133	His average summer electric bill for 2008 is \$133. $\$133 - \$105 = \$28$ It cost \$28 per month more to run his central AC, or \$84 for the summer of 2008.
3. An electrician made \$1,236, \$1,162, \$1,329, and \$1,512 each week for 4 weeks. What was his average weekly salary for the above weeks?	$\frac{\$1,236 + \$1,162 + \$1,329 + \$1,512}{4} = \$1,309.75$	His average weekly salary was: \$1,309.75
Problems	Related, Generic Math Concepts	Solutions
4. Sally earned scores of 60, 65, 65, and 80 on 4 tests. How would scoring 100 on a fifth test affect the mean?	Initial Mean = $\frac{60 + 65 + 65 + 80}{4} = 67.5$ Mean with outlier = $\frac{60 + 65 + 65 + 80 + 100}{5} = 74$	
5. Tom recorded his daily caloric intake for 5 days. The results were as follows: 2500, 2600, 2600, 2400, and 3900. How would removing the outlier affect the mean, median and mode of the data?		The mean would decrease from 2800 to 2525. The median would decrease from 2600 to 2550. The mode would remain constant at 2600.
6. Angela recorded the number of hours she spent watching TV for one week. The results were as follows: 6, 2, 2, 1.5, 3, 2.5, 2. How would removing the outlier affect the mean, median, and mode of the data?		The mean would decrease from 2.71 to 2.17 The median would remain constant at 2. The mode would remain constant at 2.
Problems	PA Core Math Look	Solutions
7. Which of the following measures of central tendency does an outlier affect the most? a) Mean b) Median c) Mode	a) Mean	
8. Which measure of central tendency would best depict the following data: 10, 200, 200, 300, 325, 350 and 400? a) Mean b) Median c) Mode	Median, because the outlier of 10 would make the average lower, and the mode of 200 would represent the lower data. The median is 300.	
9. How would removing the outlier affect the mean of the following data: 1200, 2400, 2400, 2500 and 9000?		The mean would decrease from 3500 to 2125.