

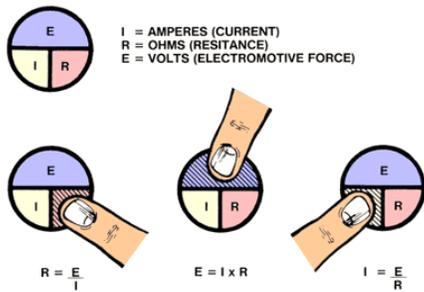
Identify direct and inverse proportions

Use reasoning to solve equations and justify the solution method

Program Task: Solve for the quantities (voltage, current, resistance) that make up an electrical circuit.

Program Associated Vocabulary:
OHM’S LAW, E, I, R, VOLTAGE, INTENSITY, ELECTROMOTIVE FORCE, CURRENT, RESISTANCE

Program Formulas and Procedures:
Ohm’s Law is a set of equations that are used to find and plan for certain electrical circuit criteria. **Ohm’s Law** defines the relationship between current (I), voltage (E), and resistance (R). The **Ohm’s Law** equation can be memorized and practiced effectively by using the **Ohm’s Law** circle.



To find the equation for E, I, or R when two quantities are known, cover the unknown third quantity. The other two

quantities in the circle will indicate how the covered quantity may be found. In addition to being able to use Ohm’s Law, students should recognize the relationship between current, voltage, and resistance. If $E = IR$ and resistance remains constant, then as the voltage increases, so must the current. Therefore, the relationship between current and voltage is directly proportional. On the other hand, when voltage remains constant, as current increases, the resistance must decrease, making the relationship between resistance and current inversely proportional.

Example 1: Let $E = 120$ V, $R = 60$ Ohms
What would happen to the current if the voltage was doubled?

$$E = IR \rightarrow 120 = I \cdot 60 \rightarrow \frac{120}{60} = \frac{60I}{60} \rightarrow 2 \text{ amps} = I$$

$$E = IR \rightarrow 240 = I \cdot 60 \rightarrow \frac{240}{60} = \frac{60I}{60} \rightarrow 4 \text{ amps} = I$$

Current would double because current and voltage are in direct proportion.

Example 2: What would happen to the resistance if the current was doubled?

$$E = IR \rightarrow 120 = 2gR \rightarrow \frac{120}{2} = \frac{2R}{2} \rightarrow 60 \text{ ohms} = R$$

$$E = IR \rightarrow 120 = 4gR \rightarrow \frac{120}{4} = \frac{4R}{4} \rightarrow 30 \text{ ohms} = R$$

Resistance would be halved because current and resistance are in inverse proportion.

PA Core Standard: CC.2.2.HS.D.9

Description: Use reasoning to solve equations and justify the solution method.

Math Associated Vocabulary:
INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

Formulas and Procedures:

Direct Proportions:
Two quantities, A and B, are directly proportional if by whatever factor A changes, B changes by the same factor.

Example 1: Take the formula, distance = rate x time. If the rate remains constant, at 30 miles per hour, then the time and distance are directly proportional.

$$d = 30t$$

when $t = 2$, $d = 60$
when $t = 4$, $d = 120$

*Note that when the time doubles, so does the distance.

Example 2: If speed is directly proportional to distance, and a car can travel 100 miles at 50 miles per hour, how far can that car travel during the same time if it travels at 70 mph?

Step 1: Set up proportion.

$$\frac{50 \text{ mph}}{70 \text{ mph}} = \frac{100 \text{ mi.}}{x}$$

Step 2: Cross multiply and divide to solve.

$$50x = 70(100) \rightarrow 50x = 7000 \rightarrow x = 140 \text{ miles}$$

Inverse Proportions:

Two quantities, A and B, are inversely proportional if by whatever factor A changes, B changes by the multiplicative inverse, or reciprocal of that factor.

Example 1: Take the formula, distance = rate x time. If the distance, 100 miles is constant, then as the rate increases, the time decreases.

$$100 = rt$$

When $r = 100$, $t = 1$
When $r = 50$, $t = 2$

*Note that when the rate doubles, the time is halved.

Example 2: The time needed to complete a job is inversely proportional to the number of people working. If it takes one person 8 hours to paint the room alone, how long would it take 4 people to paint a room?

Step 1: Set up the proportion. Step 2: Invert (flip) one ratio.

$$\frac{1 \text{ person}}{4 \text{ people}} = \frac{8 \text{ hours}}{x \text{ hours}} \qquad \frac{1 \text{ person}}{4 \text{ people}} = \frac{x \text{ hours}}{8 \text{ hours}}$$

Step 3: Cross-multiply and divide to solve.

$$4x = 8, x = 2$$

4 people can paint the room in 2 hours.

Instructor's Script – Comparing and Contrasting

Although it's important for students to be able to use Ohm's Law to solve problems, teaching direct and inverse proportional relationships help to deepen their understanding and make problem solving easier. For instance, if the problem on the Electrical Occupations side of the T-chart was rephrased to read, "When the current is equal to 20 amps, the resistance is equal to 60 ohms. What would be the resistance value be, when the current equaled 40 amps?" Students who do not understand the inverse proportional relationship between resistance and current would have to use Ohm's Law to first solve for voltage and then use the voltage to find the new resistance.

Common Mistakes Made By Students

When students compare Direct and Inverse Proportional relationships, they may become confused and have difficulty differentiating one from the other. One way to keep them straight is to:

1. Set up one pair of values on the same line, e.g., $\frac{12''}{24''} = \frac{100 \text{ lbs.}}{x \text{ lbs.}}$
2. Beneath that line, place the other pair of values, $\frac{12''}{24''} = \frac{100 \text{ lbs.}}{x \text{ lbs.}}$
3. Students need to be aware that direct proportions mean that as one variable increases so does the other variable. An inverse proportion means that one variable increases when the other one decreases. Students struggle with this concept.
4. If the problem is a direct proportion, students should cross multiply (24 times 100) and (12 times x) and then divide to solve the problem.
5. If an inverse relationship exists, then students should first invert one ratio before cross multiplying and dividing to solve the problem.
6. If need be, have the student set up the problem and do it both ways to see which answer makes sense! We know in problem #9, for example, that it won't take 5 rabbits more time than it took 1 rabbit to eat 20 carrots, so it must be an inverse proportion.

CTE Instructor's Extended Discussion

In the electrical trade, one must understand Ohm's Law theory in order to get the necessary parameters needed to calculate the appropriate unknown.

There are three ways to express Ohm's Law mathematically:

1. The applied voltage (E) to a circuit is equal to the product of the current (I) and the resistance (R) of the circuit:
 $E = I \times R = IR$
2. The current (I) in a circuit is equal to the voltage (E) applied to the circuit divided by the resistance (R) of the circuit:
 $I = E / R$
3. The resistance (R) of a circuit is equal to the voltage (E) applied to the circuit divided by the current (I) in the circuit:
 $R = E / I$

Where: **I** = current (amperage)
R = resistance (ohms)
E = voltage (volts)

Electrical (46.0399) T-Chart

Problems	Career and Technical Math Concepts	Solutions
1. If a window air conditioner that is being supplied with 240 volts of electricity is drawing 6 amps of current, how many amps of current would it draw if the voltage was reduced to 120 volts?		
2. It was estimated that three electricians should be able to rough wire an apartment in 35 hours. How long would it take to wire the apartment if five electricians were sent to do the job?		
3. If a motor has a resistance of 24 ohms, and a current draw of 5 amps, what is the applied voltage to this motor? What is the applied voltage if the current draw is 7.5 amps?		
Problems	Related, Generic Math Concepts	Solutions
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?		
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm, what would the pressure be of 0.060 m ³ of the same gas at the same temperature?		
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?		
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional, and y = 2 when x = 5, find the value of y when x = 15.		
8. Given that y and x are inversely proportional, and y = 2 when x = 5, find the value of y when x = 15.		
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?		

Problems	Career and Technical Math Concepts	Solutions
1. If a window air conditioner that is being supplied with 240 volts of electricity is drawing 6 amps of current, how many amps of current would it draw if the voltage was reduced to 120 volts?		One way to approach this problem is do some reasoning. $E = 240$ volts draws 6amps $E = 120$ volts draws _____amps We know that E and I are directly proportional so...since E is halved so will I. It will be 3 amps when $E = 120$ volts.
2. It was estimated that three electricians should be able to rough wire an apartment in 35 hours. How long would it take to wire the apartment if five electricians were sent to do the job?		One way approach this problem is to set up an inverse proportion. $\frac{3 \text{ people}}{5 \text{ people}} = \frac{35 \text{ hrs.}}{x \text{ hrs.}} \rightarrow (\text{flip}) \frac{3 \text{ people}}{5 \text{ people}} = \frac{x \text{ hrs.}}{35 \text{ hrs.}}$ $35 \times 3 \div 5 = 21$ hours
3. If a motor has a resistance of 24 ohms, and a current draw of 5 amps, what is the applied voltage to this motor? What is the applied voltage if the current draw is 7.5 amps?		One way to approach this problem $E = IR$ $E = IR$ $E = 5 \times 24$ $E = 7.5 \times 24$ $E = 120v$ $E = 180v$
Problems	Related, Generic Math Concepts	Solutions
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?		(Direct) $\frac{5 \text{ pounds}}{20 \text{ people}} = \frac{x \text{ pounds}}{50 \text{ people}} \rightarrow 20x = 5(50) \rightarrow 20x = 250$ $x = 12.5$ pounds
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm, what would the pressure be of 0.060 m ³ of the same gas at the same temperature?		(Inverse) $\frac{0.24 \text{ m}^3}{0.060 \text{ m}^3} = \frac{0.5 \text{ atm}}{x \text{ atm}}$ Invert one ratio since it is an inverse proportion. $\frac{0.24 \text{ m}^3}{0.060 \text{ m}^3} = \frac{x \text{ atm}}{.5 \text{ atm}}$ $0.24(0.5) = 0.060x$ $x = 2$ atm
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?		(Direct) $\frac{10 \text{ castings}}{14 \text{ castings}} = \frac{26 \text{ lbs.}}{x \text{ lbs.}} \rightarrow 10x = 26(14) \rightarrow x = 36.4 \text{ lbs.}$
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional, and y = 2 when x = 5, find the value of y when x = 15.		(Direct) $\frac{5}{15} = \frac{2}{y} \rightarrow 5y = 2(15) \rightarrow y = 6$
8. Given that y and x are inversely proportional, and y = 2 when x = 5, find the value of y when x = 15.		(Inverse) $\frac{5}{15} = \frac{y}{2} \rightarrow 15y = 2(5) \rightarrow y = 0.667$
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?		(Inverse) $\frac{1}{5} = \frac{x}{15} \rightarrow 5x = 1(15) \rightarrow x = 3 \text{ hours}$