

Identify the relationship between operating weight, ground contact area and ground pressure

Program Task: Identify the relationship between operating weight, ground contact area, and ground pressure.

Program Associated Vocabulary:

TRACK SHOE WIDTH, TRACK ON GROUND, OPERATING WEIGHT, GROUND CONTACT AREA, GROUND PRESSURE, PSI, PROPORTIONAL

Program Formulas and Procedures:

PSI means “pounds per square inch”. What does a ground pressure of 12.3 psi mean? The ground pressure is equal to the operating weight divided by the ground contact area.

We can represent this relationship algebraically as follows:

$$P_g = \frac{W_o}{A_c}, \text{ where}$$

P_g = Ground Pressure

W_o = Operating Weight

A_c = Ground Contact Area

Two track loaders weigh the same, but one has larger tracks (more ground contact area). Which one will have the largest ground pressure?

As ground contact area increases, the ground pressure decreases.

This means that ground contact area and pressure are inversely proportional.

Two track loaders have the same track specifications (equal contact area). One of the track loaders weighs more than the other. Which one will have the largest ground pressure?

As operating weight increases, the ground pressure increases.

This means that operating weight and ground pressure are directly proportional.

Example:

A track loader with a standard shoe has a contact area of 3565 in² and a ground pressure of 9.5 psi. If the same track loader with the optional shoe has a contact area of 2790 in², what is the ground pressure?

Step 1: Set up the proportion Step 2: Invert ratio

$$\frac{3565 \text{ in.}^2}{2790 \text{ in.}^2} = \frac{9.5 \text{ psi}}{x \text{ psi}} \qquad \frac{3565 \text{ in.}^2}{2790 \text{ in.}^2} = \frac{x \text{ psi}}{9.5 \text{ psi}}$$

Step 3: Cross-multiply and divide to solve

$$9.5(3565) = 2790x, x = 12.1 \text{ psi}$$

Use reasoning to solve equations and justify the solution method

PA Core Standard: CC.2.2.HS.D.9

Description: Use reasoning to solve equations and justify the solution method.

Math Associated Vocabulary:

INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

Formulas and Procedures:

Direct Proportions

Two quantities, A and B, are directly proportional if by whatever factor A changes, B changes by the same factor.

Example 1: Take the formula, distance = rate (r) x time (t). If the rate remains constant, 30 miles per hour, then the time and distance are directly proportional.

$$d = 30t$$

when t = 2, d = 60
when t = 4, d = 120

*Note that when the time doubles, so does the distance.

Example 2: If speed is directly proportional to distance, and a car can travel 100 miles at 50 miles per hour, how far can that car travel during the same time if it travels 70 mph?

Step 1: Set up proportion.

$$\frac{50 \text{ mph}}{70 \text{ mph}} = \frac{100 \text{ mi.}}{x}$$

Step 2: Cross multiply and divide to solve.

$$50x = 70(100) \rightarrow 50x = 7000 \rightarrow x = 140 \text{ miles}$$

Inverse Proportions

Two quantities, A and B, are inversely proportional if by whatever factor A changes, B changes by the multiplicative inverse, or reciprocal, of that factor.

Example 1: Take the formula, distance = rate (r) x time (t). If the distance is constant, 100 miles, then as the rate increases, the time decreases.

$$100 = rt$$

When r = 100, t = 1
When r = 50, t = 2

*Note that when the rate doubles, the time is halved.

Example 2: If the time needed to complete a job is inversely proportional to the number of people working, how long would it take 4 people to paint a room if 1 person needs 8 hours?

Step 1: Set up the proportion. Step 2: Invert (flip) one ratio.

$$\frac{1 \text{ person}}{4 \text{ people}} = \frac{8 \text{ hours}}{x \text{ hours}} \qquad \frac{1 \text{ person}}{4 \text{ people}} = \frac{x \text{ hours}}{8 \text{ hours}}$$

Step 3: Cross-multiply and divide to solve.

$$4x = 8, x = 2$$

4 people can paint the room in 2 hours.

Instructor's Script - Comparing and Contrasting

Take notice that although the formula for calculating ground pressure includes weight and contact area, the example problem did not include the weight of the equipment. Understanding proportional relationships allows one to make calculations even when some information is missing. The example shown on the Heavy Equipment side of the T-chart represents an inverse proportion. Decreasing the contact area of the track will increase the ground pressure. Inverse proportions are often difficult for students to understand.

It should be noted that this sample problem can be adapted to provide trade related applications of direct proportions. For instance, if the ground contact area remains the same, but the weight increases, then the ground pressure will also increase. When teaching proportional relationships, it is very important to teach students how to identify whether inverse or direct proportions exist for the situation given.

Common Mistakes Made By Students

When students compare direct and inverse proportional relationships, they may become confused and have difficulty differentiating one from the other. One way to keep them straight is to:

1. Set up one pair of values on the same line, e.g., $\frac{12}{24} = \frac{100 \text{ lbs.}}{x \text{ lbs.}}$
2. Beneath that line, place the other pair of values, $\frac{24}{24} = \frac{100 \text{ lbs.}}{x \text{ lbs.}}$
3. Students need to be aware that direct proportions mean that as one variable increases so does the other variable. An inverse proportion means that one variable increases when the other one decreases. Students struggle with this concept.
4. If the problem is a direct proportion, students should cross multiply (24 times 100) and (12 times x) and then divide to solve the problem.
5. If an inverse relationship exists, then students should first invert one ratio before cross multiplying and dividing to solve the problem.
6. If need be, have the student set up the problem and do it both ways to see which answer makes sense! We know in problem #9, for example, that it won't take 5 rabbits more time than it took 1 rabbit to eat 20 carrots, so it must be an inverse proportion.

CTE Instructor's Extended Discussion

If your students are having difficulty seeing the relationship between weight, contact area, and pressure, try demonstrating the following: Fill a bucket with mud. Find washers of varying weights and sizes. "Float" the washers on top of the mud. Have your students describe their observations.

Heavy Equipment (49.0202) T-Chart

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A small track loader has a ground pressure of 7.7 psi and an operating weight of 20,908 lbs. If the same track loader increased its operating weight to 22,605 lbs., what would be its ground pressure?		
2. A track loader with a standard shoe has a contact area of 4340 in ² and a ground pressure of 10.3 psi. If the same track loader with the optional shoe has a ground pressure of 12.4 psi, what is the contact area?		
3. A track loader with a standard shoe has a contact area of 3565 in ² and a ground pressure of 9.5 psi. If the same track loader with the optional shoe has a contact area of 2790 in ² , what is the ground pressure?		
Problems	Related, Generic Math Concepts	Solutions
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?		
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm (atmospheres), what would the pressure be of 0.060 m ³ of the same gas at the same temperature?		
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?		
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional and y = 2 when x = 5, find the value of y when x = 15.		
8. Given that y and x are inversely proportional and y = 2 when x = 5, find the value of y when x = 15.		
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?		

Heavy Equipment (49.0202) T-Chart

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A small track loader has a ground pressure of 7.7 psi and an operating weight of 20,908 lbs. If the same track loader increased its operating weight to 22,605 lbs., what would be its ground pressure?	*Ground pressure and weight are directly proportional.	Step 1: Set up the proportion. $\frac{20,908 \text{ lbs.}}{22,605 \text{ lbs.}} = \frac{7 \text{ psi}}{x \text{ psi}}$ Step 2: Cross-multiply and divide to solve. $20,908x = 7.7(22,605) \rightarrow x \approx 8.3 \text{ psi}$
2. A track loader with a standard shoe has a contact area of 4340 in ² and a ground pressure of 10.3 psi. If the same track loader with the optional shoe has a ground pressure of 12.4 psi, what is the contact area?	*Ground pressure and contact area are inversely proportional.	Step 1: Set up the proportion. Step 2: Invert one ratio. $\frac{4340 \text{ in.}^2}{x \text{ in.}^2} = \frac{10.3 \text{ psi}}{12.4 \text{ psi}} \quad \frac{x \text{ in.}^2}{4340 \text{ in.}^2} = \frac{10.3 \text{ psi}}{12.4 \text{ psi}}$ Step 3: Cross-multiply and divide to solve. $12.4x = 10.3(4340) \rightarrow x = 3605 \text{ in}^2$
3. A track loader with a standard shoe has a contact area of 3565 in ² and a ground pressure of 9.5 psi. If the same track loader with the optional shoe has a contact area of 2790 in ² , what is the ground pressure?	*Ground pressure and contact area are inversely proportional.	Step 1: Set up the proportion. Step 2: Invert one ratio. $\frac{3565 \text{ in.}^2}{2790 \text{ in.}^2} = \frac{9.5 \text{ psi}}{x \text{ psi}} \quad \frac{3565 \text{ in.}^2}{2790 \text{ in.}^2} = \frac{x \text{ psi}}{9.5 \text{ psi}}$ Step 3: Cross-multiply and divide to solve. $2790x = 9.5(3565) \rightarrow x = 12.1 \text{ psi}$
Problems	Related, Generic Math Concepts	Solutions
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?	(Direct)	$\frac{5 \text{ pounds}}{20 \text{ people}} = \frac{x \text{ pounds}}{50 \text{ people}} \rightarrow 20x = 5(50) \rightarrow 20x = 250$ $x = 12.5 \text{ pounds}$
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m ³ is 0.5 atm (atmospheres), what would the pressure be of 0.060 m ³ of the same gas at the same temperature?	(Inverse)	$\frac{0.24\text{m}^3}{0.060\text{m}^3} = \frac{0.5\text{atm}}{x\text{atm}}$ (Invert one ratio since it is an inverse proportion.) $\frac{0.24\text{m}^3}{0.060\text{m}^3} = \frac{x \text{ atm}}{0.5 \text{ atm}} \rightarrow 0.24(0.5) = 0.060x \rightarrow x = 2 \text{ atm}$
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?	(Direct)	$\frac{10 \text{ castings}}{14 \text{ castings}} = \frac{26 \text{ lbs.}}{x \text{ lbs.}} \rightarrow 10x = 26(14) \rightarrow x = 36.4 \text{ lbs.}$
Problems	PA Core Math Look	Solutions
7. Given that y and x are directly proportional and y = 2 when x = 5, find the value of y when x = 15.	(Direct)	$\frac{5}{15} = \frac{2}{y} \rightarrow 5y = 2(15) \rightarrow y = 6$
8. Given that y and x are inversely proportional and y = 2 when x = 5, find the value of y when x = 15.	(Inverse)	$\frac{5}{15} = \frac{y}{2} \rightarrow 15y = 2(5) \rightarrow y = 0.667$
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?	(Inverse)	$\frac{1}{5} = \frac{x}{15} \rightarrow 5x = 1(15) \rightarrow x = 3 \text{ hours}$