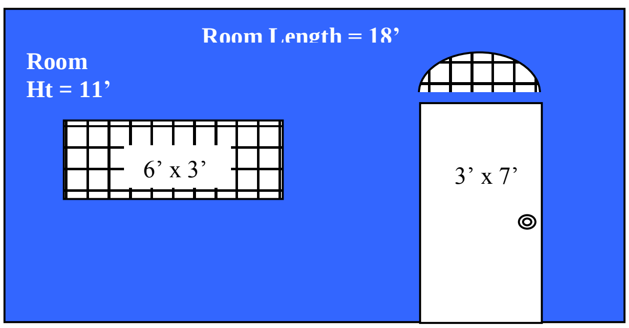
HVAC (47.0201) T-Chart

Determine complex surface areas	=	Apply geometric concepts to model and solve real-world problems

Program Task: Determine space load requirements.

Program Associated Vocabulary:
GROSS AREA, NET AREA

Program Formulas and Procedures:
When a heat loss and/or load calculation must be performed, the HVAC technician must be able to determine net wall area and the combined areas of each dissimilar construction material type. For example, in the wall represented below, separate area values are needed for the wall (blue or shaded), the combined windows (grid pattern), and the door (white).



Example: Determine the net wall area (gross wall area – non-wall areas).
Calculate the total surface area.
Rectangular Area (Length × Height) or 18' × 11' = 198 ft.^{2}

The glass (grid pattern) is calculated next.
For the 6' x 3' window, once again use the formula for rectangular area (Area = Length × Height), 6 × 3 = 18 ft.^{2}

Next consider the area of the semi-circle of glass above the door. Because the door is 3' wide, we can assume the semi-circle is about 3' wide (diameter).
We use the formula $.5 \times \pi \times r^2$ to determine that the area of the semi-circle = $.5 \times 3.14 \times 1.5^2 \approx 3.5$ ft.^{2}.

The door, another rectangle, is 3' × 7' or 21ft.^{2}.

Finally, we summarize our results before continuing with our heat loss/load project:
Combined glass area = (18 ft.^{2} + 3.5 ft.^{2}) = 21.5 ft.^{2}

Door area = 21 ft.^{2}

Sum of non-wall Areas = (21 ft.^{2} + 21.5 ft.^{2}) = 42.5 ft.^{2}
Net wall area = Gross wall Area – non-wall areas
Net wall area = 198 ft.^{2} - 42.5 ft.^{2} = 155.5 ft.^{2}

PA Core Standard: CC2.3.HS.A.14

Description: Apply geometric concepts to model and solve real-world problems.

Math Associated Vocabulary:
LENGTH, HEIGHT, BASE, WIDTH, DIAMETER, RADIUS, HYPOTENUSE, AREA, PERIMETER, CIRCUMFERENCE

Formulas and Procedures:
Rectangle: $A = lw$ $P = 2l + 2w$

Trapezoid: $A = \frac{h(a+b)}{2}$

Circle: $A = \pi r^2$ $C = 2\pi r$ or πd
(Circumference = circle perimeter)

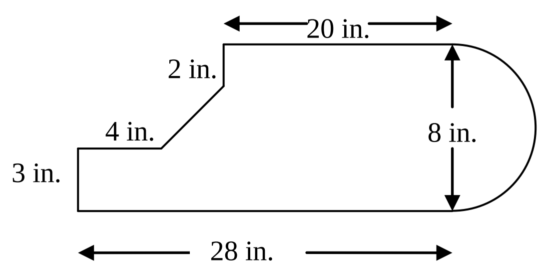
Triangle: $A = \frac{1}{2}bh$

Pythagorean Theorem: $c^2 = a^2 + b^2$

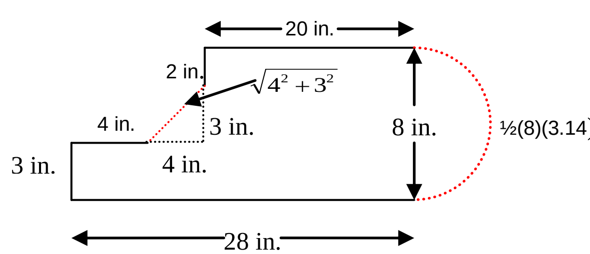
An irregular figure can be broken down into two or more regular shapes, such as triangles, circles, trapezoids or rectangles.

To find the **perimeter** around irregular figures, add the lengths of the sides. If the sides of the figures include circles, use the circumference formula to calculate the length of that portion of the figure and add it to the total of the other sides.

Example 1: To find the **area** of an irregular figure, separate the figure into shapes for which you can calculate the area. The sum of the areas of each smaller figure is the area of the irregular figure.



Example 2: To find the **perimeter** of the figure above, use the Pythagorean theorem and circumference formula to find the missing lengths:



To find the **area** of the same figure, divide the figure into one triangle, two rectangles, and one semi-circle.

Instructor's Script - Comparing and Contrasting
Area, perimeter or circumference problems use a toolbox of formulas for basic shapes. The critical step is to break down the irregular shape into these basic shapes (circle, rectangle, and/or triangle) and apply the correct formulas.

Whether trying to solve a trade application or a math problem, you should try to draw in new lines that create simple shapes within the complex shape.

Note: Answers to the problems on page 4 will be different if you are using 3.14 as π, rather than the π key on your calculator. Answers will be different, not by much, but enough that when some students are presented with the answer key they might not recognize that they did get the correct answers!

Common Mistakes Made By Students
Mixing perimeter and area formulas or calculations: Perimeter formulas calculate the length of the outside edge of an object, while area formulas calculate the space taken up by the shape. Areas and perimeters should not be compared (apples and oranges) because perimeter is measured as a unit length while area is that same unit squared.

Perimeter calculations should <u>not</u> include inner edges: The perimeter of an irregular object should follow the outer edge of the figure. If you find the perimeter for basic shapes constructed within the irregularly shaped object, be sure to eliminate the auxiliary lines (inner edges) that don't follow the outside edge.

Finding basic shapes within irregular objects can be frustrating: Some irregular objects can be broken into basic shapes with only a couple of extra lines, while others seem to take a lot more. Don't feel locked in to your first attempt if it is too messy.

Empty shapes in the figure require subtracting the area of the "hole": If your plan includes areas that create holes in the object, you will be subtracting out that area to get a final answer (e.g., a deck plan that has a spot for a hot tub).

Final answer may include multiple parts: Don't forget to total all the various areas or perimeters to get your final answer.

Be sure to find all missing lengths before calculating the perimeter.

CTE Instructor's Extended Discussion
In order to correctly size heating or cooling equipment for a given structure, HVAC professionals must first complete a detailed heat loss/gain survey of that structure. Since dissimilar materials transmit heat at different rates, building materials must be separated by insulation value before their areas can be totaled and applied to the American Society of Heating, Refrigeration, and Air Conditioning Engineers (ASHRAE) heat loss/gain formula: $BTU/h = AREA * \Delta T * U$

Where: BTU/h = heat energy (BTU's) transmitted through the material per hour
ΔT = Temp difference (indoors / outside)
U = reciprocal of material's insulating (R)-value

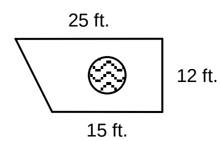
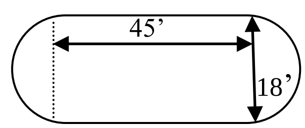
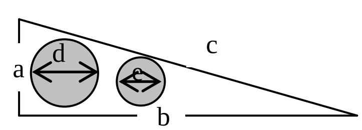
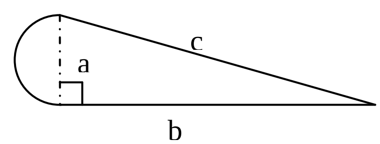
To extend the example used on the T-Chart, let's assume that the R-values and U-values for the 3 areas are as follows:
Wall = R-22 U = 1/22 = .045
Glass = R-2 U = 1/2 = .5
Door = R-11 U = 1/11 = .09

Further, let's assume a designated outdoor temperature of 5^{o} F and an indoor temperature of 70^{o} F, giving us a delta-T (ΔT) of 65^{o} F. Finally, let's plug all these results into the ASHRAE formula to determine how many BTU's per hour are passing through our wall (see HVAC example on page 1) on a 5 degree day with a 70 degree indoor temperature:

$$BTU/h = AREA \times \Delta T \times U$$

Net wall BTU/h loss = $155.5 \times 65 \times .045 \approx 455$ BTU's per hour (loss)
Combined glass area = $21.5 \times 65 \times .5 \approx 699$ BTU's per hour (loss)
Door Area = $21 \times 65 \times .09 \approx 123$ BTU's per hour (loss)

Total wall BTU/h loss = 455 + 699 + 123 = 1,277 BTU's per hour (loss)
Note that the glass amounts to only 11% of the gross wall surface area (21.5 ft.^{2} divided by 198 ft.^{2}) and yet, in this case, it accounts for 55% of the heat energy lost by conduction!

Problems — Occupational (Contextual) Math Concepts	Solutions
1. An exterior wall has a height of 8 feet and a length of 27 feet. Placed in that wall is a door (3' x 7') and a large sliding glass door (8' x 6'). What is the net wall area?	
2. An exterior wall has a height of 8 feet and a length of 12 feet. Placed in that wall is a large circular window with a diameter of 4 feet. What is the net wall area?	
3. An exterior basement wall is partially below ground. In fact the rectangular wall, which is 7 feet high and 12 feet long, is split diagonally between exposed and unexposed area, due to a sloping landscape. What is the net area of exposed wall?	
Problems — Related, Generic Math Concepts	**Solutions**
4. A health club has a circular jogging track with an outside diameter of 200 feet and the track is 15 feet wide. What is the area of the track?	
5. Your goal is to paint a mural that depicts a large yellow image of the Sun, risen half-way above the Eastern horizon. You buy a gallon of yellow paint and read that the manufacturer claims it will cover a 200 square foot wall. What is the diameter of the largest Sun you can paint?	
6. The installer plans to build a new patio with a 6 ft. (d) round hot tub in the center. What is the area of material needed around the hot tub pictured in the patio?	
Problems — PA Core Math Look	**Solutions**
7. Find the area of the figure pictured.	
8. Find the area of the figure pictured.	
9. Find the perimeter of the figure if c = 37 and b = 24.	

Problems **Occupational (Contextual) Math Concepts**	**Solutions**
1. An exterior wall has a height of 8 feet and a length of 27 feet. Placed in that wall is a door (3' x 7') and a large sliding glass door (8' x 6'). What is the net wall area?	Gross wall area (GWA) = (h x *l*) Net Wall area (NWA) = (Gross area) - (combined non-wall surface areas) $GWA = 8 \times 27 = 216$ ft.^{2} NWA = 216 - ((3 x 7) + (8 x 6)) NWA = 216 - (21 + 48) NWA = 147 ft.^{2}
2. An exterior wall has a height of 8 feet and a length of 12 feet. Placed in that wall is a large circular window with a diameter of 4 feet. What is the net wall area?	$GWA = 8 \times 12 = 96$ ft.^{2} $NWA = 96 - (\pi \times r^2)$ $NWA = 96 - (3.14 \times 2^2)$ NWA = 83.44 ft.^{2}
3. An exterior basement wall is partially below ground. In fact the rectangular wall, which is 7 feet high 12 feet long, is split diagonally between exposed and unexposed area, due to a sloping landscape. What is the net area of exposed wall?	Gross wall area (GWA) = $(h \times l)$ Net Wall area (NWA) = (Gross area) - (combined non-wall surface areas) $GWA = 7 \times 12 = 84$ ft.^{2} $NWA = 84 \div 2$ (2 equal triangles are formed) NWA = 42 ft.^{2}
Problems **Related, Generic Math Concepts**	**Solutions**
4. A health club has a circular jogging track with an outside diameter of 200 feet and the track is 15 feet wide. What is the area of the track?	The diameter of the smaller circle is (200 - (15 + 15)) feet Large circle area = $\pi\ (100 \times 100)$ Large circle area = $3.14 \times 10{,}000$, or 31,400 ft.^{2} Small circle area = 3.14×85 x 85, or 22,687 ft.^{2} Area of the track = Large Circle Area (31,400) - Small Circle Area (22,687), or 8,718 ft^2.
5. Your goal is to paint a mural that depicts a large yellow image of the Sun, risen half-way above the eastern horizon. You buy a gallon of yellow paint and read that the manufacturer claims it will cover a 200 square foot wall. What is the diameter of the largest Sun you can paint?	Base your estimations on a semi-circle whose area is 200 sq. ft. A full circle size would be 400 sq. ft. Area of a semi-circle: $\frac{1}{2}\pi r^2 = 200$ $2 \times \frac{1}{2}\pi r^2 = 2 \times 200$ Multiple both sides by 2. $\pi r^2 = 400$ $\frac{\pi r^2}{\pi} = \frac{400}{\pi}$ Divide both sides by π. $r^2 = 400 \div \pi$ $\sqrt{r^2} = \sqrt{\frac{400}{\pi}}$ Square root both sides. r = 11.28 Diameter = r $(11.28) \times 2$ Double the radius to find the diameter. Diameter = 22.5’
6. The installer plans to build a new patio with a 6 ft. (d) round hot tub in the center. What is the area of material needed around the hot tub pictured in the patio?	Area of patio = area of a trapezoid (patio shape) – area of the circle (hot tub shape) Area = $\frac{h(a+b)}{2} - \pi r^2$ $A = \frac{12(15+25)}{2} - \pi 3^2$ A= 240 – 28.26 = 211.74 ft.^{2}
Problems **PA Core Math Look**	**Solutions**
7. Find the area of the figure pictured.	Area = Area Rectangle + Area one full circle $= lw + \pi r^2$ (l = 45, w = 18, r = radius = $\frac{1}{2} \times 18$ = 9’) $= (45)(18) + \pi(9)^2$ = 810 + 254.3 = 1064.3 ft.^{2}
8. Find the area of the unshaded area if a = 5, b = 18, d = 3, and e = 1.	Area = Area triangle – Area circle 1 – Area circle 2 $= \frac{1}{2}bh - \pi r^2 - \pi r^2$ (radius circle 1 = $\frac{1}{2} \times 3$ = 1.5, radius circle 2 = $\frac{1}{2} \times 1$ = 0.5) $= \frac{1}{2}(18)(5) - \pi(1.5)^2 - \pi(0.5)^2$ = 45 – 7.1 - .8 = 37.1 units^2
9. Find the perimeter of the figure if c = 37 and b = 24.	Perimeter = c + b + semicircle with diameter a. $a^2 + b^2 = c^2$ $a^2 + 24^2 = 37^2$ $a^2 + 576 = 1369$ $a^2 + 576 - 576 = 1369 - 576$ $a^2 = 793$ $\sqrt{a^2} = \sqrt{793}$ a = 28.2 = diameter of semicircle circumference of semicircle =1/2 dπ = ½ (28.2)(3.14) = 44.3 Total perimeter = 37 + 24 + 44.3 = 105.3 units