HVAC (47.0201) T-Chart

Design and build equipment support systems	=	Understand and apply the Pythagorean Theorem to solve problems

Program Task: Design and build equipment support systems.

Program Associated Vocabulary:
BASE, ALTITUDE, HYPOTENUSE, DIAGONAL

Program Formulas and Procedures:
HVAC equipment is heavy and expensive, and by necessity, it is usually in close proximity to the end users. Weighty compressors, pumps, fans, and motors often hang just above the occupants they serve. Support systems are all that keep these items safely in place; the HVAC technician must provide strong and stable infrastructures in order to protect human life and the customer's investment.

The distinction between professional (safe) and amateur (unsafe) HVAC installations often lies in the quality of these support systems. Designing and fabricating strong shelves, brackets, pipe and duct hangers, suspensions systems, panel supports, guy wire assemblies, conduit systems, raceways, and other underlying components requires a fundamental knowledge of geometry, and almost always involves the Pythagorean Theorem.

Triangles are inherently strong (see picture above). Determining the dimensions of the triangle needed is an integral part of fabricating an appropriate support system.

Example:
In the picture shown above, a steel support bracket was fabricated and firmly attached to the building by bolting the vertical leg (a) of the triangle. The horizontal leg length (b) was based on the equipment depth. Finally, the diagonal leg, or the hypotenuse (c), was added, giving the structure impressive strength.

To determine the length of leg c, the technician used the Pythagorean Theorem $a^2 + b^2 = c^2$. If leg A is 3' and leg B is 3.5', what would leg C be?

$$a^2 + b^2 = c^2 \rightarrow 3^2 + 3.5^2 = c^2$$

$$9 + 12.25 = c^2 \rightarrow 21.25 = c^2$$

$$\sqrt{c^2} = \sqrt{21.25} \rightarrow c = 4.61' \text{ (rounded)}$$

PA Core Standard: CC.2.3.8.A.3

Description: Understand and apply the Pythagorean Theorem to solve problems.

Math Associated Vocabulary:
HYPOTENUSE, DIAGONAL, LEG, RIGHT ANGLE, RIGHT TRIANGLE, PYTHAGOREAN THEOREM, ROOT, SQUARE

Example 1: Solve for the hypotenuse, c, when given both legs.
A rectangle has side measurements of 8 inches and 12 inches. Find the length of the diagonal.

Step 1: Substitute known values into the Pythagorean theorem.

$$a^2 + b^2 = c^2 \quad 8^2 + 12^2 = c^2$$

Step 2: Square and add each number as directed by the theorem.

$$64 + 144 = c^2 \rightarrow 208 = c^2$$

Step 3: Take the square root of each side to solve for c.

$$\sqrt{208} = \sqrt{c^2} \rightarrow 14.4 = c$$

Example 2: Solve for a leg when given the hypotenuse and the other leg.
A right triangle has a hypotenuse that measures 10 inches and one of the legs measures 6 inches. Find the length of the other leg.

Step 1: Substitute known values into Pythagorean theorem.

$$a^2 + b^2 = c^2 \rightarrow 6^2 + b^2 = 10^2$$

Step 2: Square each number as directed by the theorem.

$$6^2 + b^2 = 10^2 \rightarrow 36 + b^2 = 100$$

Step 3: Subtract from both sides to isolate the variable.

$$36 - 36 + b^2 = 100 - 36 \rightarrow b^2 = 64$$

Step 4: Take the square root of each side to solve for the variable.

$$b^2 = 64 \rightarrow \sqrt{b^2} = \sqrt{64} \rightarrow b = 8$$

Instructor's Script - Comparing and Contrasting
In the example shown on the HVAC side of the T-Chart, the student must use the Pythagorean Theorem to solve for the diagonal, C. In many CTE applications, the diagonal is the missing dimension of the triangle. It is also important to show students how to solve for one of the legs of the right triangle. The computation is slightly different and more complex and this knowledge will provide the students with the ability to use the Pythagorean Theorem in other settings.

Example:
What is the maximum width of a room that an 80' emergency fuel line can be placed in, if the room is known to be 60' long?

$a^2 + b^2 = c^2$

$60^2 + b^2 = 80^2$

$3600 + b^2 = 6400$

$b^2 = 2800$

$b = \sqrt{2800}$

$b = 52.91'$

The room can be a maximum of $52.91'$ wide.

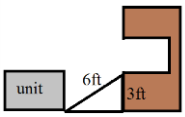
Common Mistakes Made By Students
Incorrectly identifying a, b, and c: Students often confuse the hypotenuse with one of the legs or incorrectly substitute values into the equation. To avoid this problem recognize that the diagonal often is used to describe a hypotenuse. Label your hypotenuse right away by quickly identifying the right angle and marking the side opposite the right angle as the hypotenuse.

Inability to manipulate the equation to solve for a or b: Solving for the hypotenuse is much simpler than solving for a leg of a right triangle. Students need to be given many opportunities to solve for all the variables in the Pythagorean Theorem.

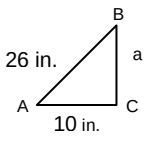
Inability to recognize the Pythagorean Theorem in multiple contexts: The Pythagorean Theorem appears in many contexts in standardized testing. Sometimes a test question will describe a right triangle and ask the student to solve for the missing side. Other times, the right triangle is drawn and the student must solve for the missing side. In many cases, a more complex picture is drawn and the student must use the Pythagorean Theorem to solve part of the problem. In these cases, it is not obvious that the Pythagorean Theorem is needed and the student must be able to select and use the theorem.

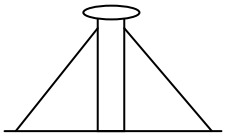
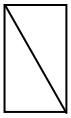
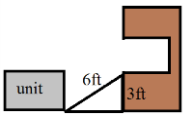
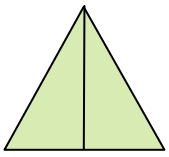
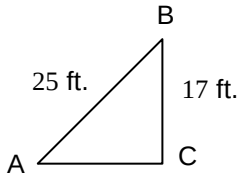
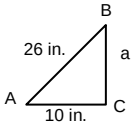
CTE Instructor's Extended Discussion
The Pythagorean Theorem is a handy formula for many installation applications. Avoid the temptation to call it the 3, 4, 5 method, as this confuses many students into thinking that the formula works only for triangles with those measurements or with measurements that are multiples of 3, 4, and 5, such as 6, 8, and 10, or 9, 12, and 15.

HVAC applications are numerous and include piping, ductwork, temporary ramps for rigging, and equipment installations. Use the Pythagorean Theorem to calculate the length anytime you have a right triangle and need any of the three sides

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A metal chimney is erected on the roof of a boiler room. It is 27' from the roofline to the top of the chimney where the guy wires attach. The base shackles are exactly 70' from the chimney base. What length of guy wires will be needed? Don't' cut it short; round up to the nearest whole number!		
2. An emergency fuel line must reach diagonally across a boiler room, corner to corner. The room is 40 ft. × 70 ft. How long must the fuel line be?		
3. You need a wooden board to slide a 300 lb. motor into a fan compartment that is 3' off the ground. If a wall is 5' away from the fan, would you be able to use a 6' board as a ramp?		

Problems	Related, Generic Math Concepts	Solutions
4. A tent has two slanted sides that are both 5 ft. long and the bottom is 6 ft. across. What is the height of the tent in feet at the tallest point?		
5. Three sides of a triangle measure 9 ft., 16 ft. and 20 ft. Determine if this triangle is a right triangle.		
6. On a baseball diamond, the bases are 90 ft. apart. What is the distance from home plate to second base using a straight line?		

Problems	PA Core Math Look	Solutions
7. The lengths of the legs of a right triangle measure 12 m. and 15 m. What is the length of the hypotenuse to the nearest whole meter?		
8. In a right triangle ABC, where angle C is the right angle, side AB is 25 ft. and side BC is 17 ft. Find the length of side AC to the nearest tenth of a foot.		
9. In the given triangle, find the length of a. 		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. A metal chimney is erected on the roof of a boiler room. It is 27' from the roofline to the top of the chimney where the guy wires attach. The base shackles are exactly 70' from the chimney base. What length of guy wires will be needed? Don't cut it short; round up to the nearest whole number!		$27^2 + 70^2 = c^2$ $729 + 4900 = c^2$ $5629 = c^2$ $\sqrt{5629} = c$ $76 \text{ ft.} = c$ 
2. An emergency fuel line must reach diagonally across a boiler room, corner to corner. The room is 40 ft. × 70 ft. How long must the fuel line be?		$a^2 + b^2 = c^2$ $\sqrt{40^2 + 70^2} = \sqrt{c^2}$ $\sqrt{1600 + 4900} = \sqrt{c^2}$ Fuel line must be $\sqrt{6500}$ or 80.6 ft. long 
3. You need a wooden board to slide a 300 lb. motor into a fan compartment that is 3' off the ground. If a wall is 5' away from the fan, would you be able to use a 6' board as a ramp?		$a^2 + b^2 = c^2$ $3^2 + b^2 = 6^2$ $9 + b^2 = 36$ $b^2 = 27$ $b = \sqrt{27}$ $b = 5.2$ NO, the 6 ft. board would create a triangle that is a little too wide for the hallway (5ft). 
Problems	**Related, Generic Math Concepts**	**Solutions**
4. A tent has two slanted sides that are both 5 ft. long and the bottom is 6 ft. across. What is the height of the tent in feet at the tallest point?		$a^2 + b^2 = c^2$ $a^2 + 3^2 = 5^2$ $a^2 + 9 = 25$ $a^2 = 16$ $a = 4$ ft. 
5. Three sides of a triangle measure 9 ft., 16 ft. and 20 ft. Determine if this triangle is a right triangle.		$a^2 + b^2 = c^2$ $16^2 + 9^2 = 20^2$ $256 + 81 \neq 400$ Therefore, it is not a right triangle.
6. On a baseball diamond, the bases are 90 ft. apart. What is the distance from home plate to second base using a straight line?		$90^2 + 90^2 = c^2$ $8100 + 8100 = c^2$ $16200 = c^2$ $\sqrt{16200} = c$ $127.28 \text{ ft.} = c$
Problems	**PA Core Math Look**	**Solutions**
7. The lengths of the legs of a right triangle measure 12 m. and 15 m. What is the length of the hypotenuse to the nearest whole meter?		$a^2 + b^2 = c^2$ $12^2 + 15^2 = c^2$ $144 + 225 = c^2$ $369 = c^2$ $\sqrt{369} = c$ $c = 19 \text{ m.}$
8. In a right triangle ABC, where angle C is the right angle, side AB is 25 ft. and side BC is 17 ft., find the length of side AC to the nearest tenth of a foot.		$a^2 + b^2 = c^2$ $17^2 + b^2 = 25^2$ $289 + b^2 = 625$ $b^2 = 336$ $\sqrt{b^2} = \sqrt{336}$ $b = 18.3 \text{ ft.}$ 
9. In the given triangle, find the length of a. 		$a^2 + b^2 = c^2$ $a^2 + 10^2 = 26^2$ $a^2 + 100 = 676$ $a^2 = 576$ $a = \sqrt{576}$ $a = 24 \text{ in.}$