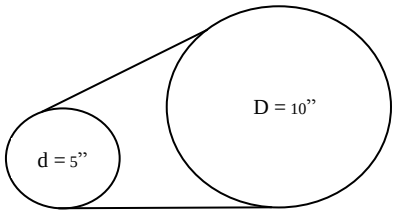
HVAC (47.0201) T-chart

Determine arc characteristics	=	Extend the concept of similarity to determine arc lengths and areas of sectors of circles

Program Task: Select, install, and adjust sheaves and belts for optimum performance.

Program Associated Vocabulary
ARC OF CONTACT (AOC) / MINIMUM BELT WRAP ANGLE, SHEAVE (PULLEY) CIRCUMFERENCE, DIAMETER, RATIO, CENTER TO CENTER DISTANCE



Program Formulas and Procedures:

Formula: d's $AOC° = 180 - \frac{60(D-d)}{C}$

D's $AOC° = 180 - \frac{60(d-D)}{C}$

Where:
D = larger sheave dia.
d = smaller sheave dia.
C = distance between sheaves, center to center
(18” center to center distance in this example)

Example:
Determine the arc of contact for both sheaves in figure 1.

$d \rightarrow AOC° = 180 - \frac{60(D-d)}{C}$

$AOC° = 180 - \frac{60(10-5)}{18}$

$AOC° = 163°$ (rounded to the nearest whole number)

$D \rightarrow AOC° = 180 - \frac{60(d-D)}{C}$

$AOC° = 180 - \frac{60(5-10)}{18}$

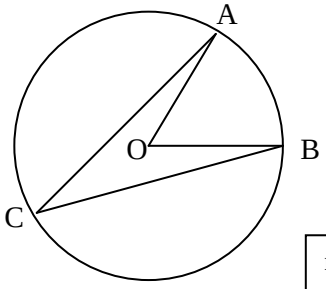
$AOC° = 197°$ (rounded to the nearest whole number)

Note that when sheave (pulley) sizes are equal, each shares half of the combined 360° AOC. As the sheave's size difference increases (fig 1), the larger sheave has increasingly more AOC than the smaller sheave. It becomes problematic when too little belt makes contact with either sheave. Increasing the distance between sheaves (C) helps, but its effects are minimal. If it is determined that the belt to sheave contact area must be increased, larger sheaves should be applied (same ratio and AOC but with larger circumferences). Another option would be to add multiple sheaves and use sets of matched belts.

PA Core Standard: CC.2.3.HS.A.9

Description: Extend the concept of similarity to determine arc lengths and areas of sectors of circles.

Math Associated Vocabulary:
ARC, MINOR ARC, MAJOR ARC, SEMICIRCLE, CENTRAL ANGLE, CHORD, INSCRIBED ANGLE



Vocabulary Examples:
Minor Arc: arc AB
Major Arc: arc ACB
Central Angle: ∠AOB
Chord: AC and BC
Inscribed Angle: <ACB

m<ACB = ½m∠AOB
arc AB = m∠AOB

Formulas and Procedures:

Arc: a part of a circle or a curve between two points

Minor Arc: an arc of a circle that is less than 180°

Major Arc: an arc of a circle that is more than 180°

Semicircle: an arc of a circle that is half of the circle (180°)

Central Angle: an angle whose vertex is the center of the circle

Chord: a segment whose endpoints lie on a circle.

Inscribed Angle: an angle whose vertex is on the circle and whose sides are chords of a circle

When an inscribed angle has the same endpoints as a central angle (subtended by the same arc), the measurement of the inscribed angle is half of the measurement of the central angle.

Example:
If m< AOB = 30°, then the m∠ ACB = 15°.
If m< ACB = 50°, then the m∠AOB = 100°.
Find the central angle measurement, given the arc length.

The length of arc AB (above) is 4 cm. and the circumference of the circle is 10 cm.

1. Find the fraction of the circle that the arc intercepts.

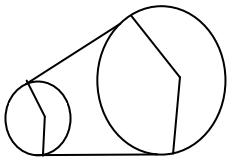
   $\frac{4}{10} = \frac{2}{5}$

2. Multiply this fraction by 360° to get the central angle measurement:

   $\frac{2}{5} \text{ x } 360° = 144°$

Instructor's Script - Comparing and Contrasting

When the pulley systems contain two equal sheaves, the arcs of contact for each of the sheaves form two semicircles of 180°. When the sheaves are of different size, the arc of contact of the larger sheave forms a major arc while the arc of contact of the smaller sheave forms a minor arc. In both situations, the arcs of contact form central angles with the sheaves.



To extend the problem, have the students determine the arc of contact of the drive belt in inches for the smaller gear. The procedure to do this for the problem presented in figure 2 is as follows:

1. Find the arc of contact of the smaller sheave (163°)
2. Find the circumference of the smaller circle.
   $c = 2\pi r$ or $c = \pi d$
   $c = 3.14(5) = 15.7\text{in.}$
3. Then find the fraction of the circle covered by the arc of contact.
   $\frac{163}{360}$
4. Use this fraction to set up a proportion
   $\frac{AOC}{360} = \frac{\text{arc(in.)}}{c} \rightarrow \frac{163}{360} = \frac{\text{arc(in.)}}{15.7\,\text{in.}}$
5. Cross-multiply and divide to find arc of contact length in inches:
   $\frac{163}{360} = \frac{\text{arc(in.)}}{15.7\text{in.}} \rightarrow \frac{163(15.7)}{360} = 7.1\text{in.}$

Common Mistakes Made By Students

- Lack of familiarity with and understanding of vocabulary associated with the concept.
- Setting up the proportion incorrectly to find arc lengths, given central angle measurements.

CTE Instructor's Extended Discussion

If the ratio of the driving sheave (pulley) to the driven sheave were the only important consideration when choosing sheave sizes, we'd save lots of shelf space by stocking sheaves that are only slightly larger in diameter than the shafts on which they are installed. Why use a motor sheave with a 5 inch diameter and a blower sheave with a 10 inch diameter when 1 inch and 2 inch diameter sheaves would provide the same ratio, hence the same blower speed? If you did that, expect to replace those belts frequently!

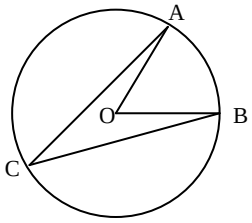
Belt drive systems are used in many HVAC applications. On low horsepower applications, one belt is usually sufficient. On larger systems, multiple (matched) belts are used to increase the belt to sheave contact area. In cases where it is determined that belts are wearing prematurely, the technician should consider the possibility that the sheave configuration has resulted in an unacceptably small Arc of Contact (AOC) on the smaller sheave.

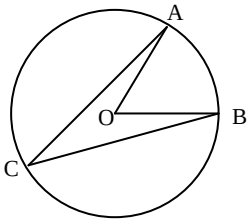
Of course, there are other, more likely problems. Poorly fitted belts (loose, tight, misaligned, wrong pitch or diameter) will significantly shorten the life of belts and sheaves. Belts should fit perfectly against the sheath walls, and without gaps. The perfect fit, alignment, and tension adjustment, in conjunction with adequate belt to sheave surface area contact (i.e., AOC) will determine the life of the equipment. Standard belts are often upgraded to Cog belts, which have irregularly shaped contact surfaces that offer the benefit of a higher coefficient of friction over standard belts (reduced slippage); they tend to run cooler, resulting in extended life.

Generally speaking, the area of belt to sheave contact is critical to performance, and the smaller sheave is of primary concern when considering contact area. The relationship between the sheave's diameters, and the center to center distance between the sheaves become critical factors in contact surface area calculations. Even though it is not done often, the proficient HVAC technician must understand this relationship and be able to calculate the AOC of sheave sets.

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Determine Arc of Contact for the smallest sheave. Sheave set A: motor sheave (d) = 5 and blower sheave (D) = 15; distance between sheave centers (C) = 18		
2. Determine Arc of Contact for the larger sheave. Sheave set B: motor sheave (d) = 5 and blower sheave (D) = 15; distance between sheave centers (C) = 24		
3. What is the length of the Arc of Contact for the larger sheave in problem number 2?		

Problems	Related, Generic Math Concepts	Solutions
4. To find the Earth's approximate Arc of Shade, use the formula: AOS = 180 - (((S-E) × 60) / C) Where: S = Sun's diameter (870,000 miles) E = Earth's diameter (7,926 miles) C = distance between Sun and Earth (93,000,000 miles)		
5. Imagine the Earth moved into Mercury's orbit, it would be only 28,600,000 miles from the Sun! Find its Arc of Shade, using the formula: AOS = 180 - (((S-E) × 60) / C).		
6. When a clock reads 9:00, the central angle between the two hands is 90°. If the arc length between the hands is 6 cm, how many cm of metal would be needed to frame the clock?		

Problems	PA Core Math Look	Solutions
7. Use the diagram shown to answer questions 7-9. If m∠ACB = 21.5°, find the measurement of ∠AOB. 		
8. If the length of arc AB = 3 cm. and the circumference of circle O = 15 cm. what is the measurement of ∠AOB?		
9. If the measurement of ∠AOB = 73° and segment OB = 8 cm. find the length of arc AB in centimeters.		

Problems	Occupational (Contextual) Math Concepts	Solutions
1. Determine Arc of Contact for the smallest sheave. Sheave set A: motor sheave (d) = 5 and blower sheave (D) = 15; distance between sheave centers (C) = 18	$d \rightarrow AOC° = 180 - \frac{60(D-d)}{C} \rightarrow AOC° = 180 - \frac{60(15-5)}{18}$ $AOC° = 147°$ (rounded to the nearest whole number)	
2. Determine Arc of Contact for the larger sheave. Sheave set B: motor sheave (d) = 5 and blower sheave (D) = 15; distance between sheave centers (C) = 24	$D \rightarrow AOC° = 180 - \frac{60(d-D)}{C} \rightarrow AOC° = 180 - \frac{60(5-15)}{24}$ $AOC° = 205°$ (rounded to the nearest whole number)	
3. What is the length of the Arc of Contact for the larger sheave in problem number2?	$C = 2\pi r = 2(3.14)(15) = 94.2 inches$ $\frac{205}{360} = \frac{x}{94.2} \rightarrow 360x = 19311$ $\frac{360x}{360} = \frac{19311}{360} \rightarrow x = 53.6 inches$	
Problems	**Related, Generic Math Concepts**	**Solutions**
4. To find the Earth's approximate Arc of Shade, use the formula: AOS = 180 - (((S-E) × 60) / C) Where: S = Sun's diameter (870,000 miles) E = Earth's diameter (7,926 miles) C = distance between Sun and Earth (93,000,000 miles)	AOS = 180 - (((S-E) × 60) / C) AOS = 180 - (((870,000 - 7,926) × 60) / 93,000,000) AOS = 180 - ((862,074 × 60) / 93,000,000)) AOS = 180 - .556 AOS = 179.4°	
5. Imagine the Earth moved into Mercury's orbit, it would be only 28,600,000 miles from the Sun! Find its Arc of Shade, using the formula: AOS = 180 - (((S-E) × 60) / C).	AOS = 180 - (((S-E) × 60) / C) AOS = 180 – 1.8 AOS = 178.2°	
6. When a clock reads 9:00, the central angle between the two hands is 90°. If the arc length between the hands is 6 cm, how many cm of metal would be needed to frame the clock?	$\frac{90}{360} = \frac{6}{x}$ $90x = 6(360)$ $x = \frac{6(360)}{90} = 24$ cm.	
Problems	**PA Core Math Look**	**Solutions**
7. Use the diagram shown to answer questions 7-9. If m∠ACB = 21.5°, find the measurement of ∠AOB. 	21.5 × 2 = 43°	
8. If the length of arc AB = 3 cm. and the circumference of circle O = 15 cm. what is the measurement of ∠AOB?	$\frac{3}{15} = \frac{x}{360} \rightarrow x = \frac{3(360)}{15} = 72°$	
9. If the measurement of ∠AOB = 73° and segment OB = 8 cm. find the length of arc AB in centimeters.	C = 16(3.14) = 50.26 cm. $\frac{73}{360} = \frac{x}{50.26} \rightarrow x = \frac{73(50.26)}{360} =$ 10.2 cm.	