

**Calculate drill depth = Construct and compare linear, quadratic, and exponential models to solve problems**

**Program Task:** Perform drilling on the drill press/ vertical milling machine.

**Program Associated Vocabulary:**  
PROPORTIONATE

**Program Formulas and Procedures:**  
When performing drilling operations it is important to know the length of the tip of a drill so that holes will be drilled to the proper depth.

The length of a 118° drill point is proportionate to its diameter and can be shown by the following formula. As the diameter of the drill increases, the length of the point will increase.

$$L = .3 \times D$$

Where L = length of the drill point, D = Diameter of the drill, and .3 is the constant

**Example:**  
If a group of 1/4" and 5/16" diameters, two holes are both to be drilled to a full diameter depth of .5", what would be the total drill depth required for the two hole sizes?

**Depth for 1/4" drill:**  
 $L = .3 \times .25 = .075"$   
Add .075" to the .5" to get a full depth of .575" for the 1/4" drill.

**Depth for the 5/16" drill:**  
 $L = .3 \times .3125 = .09375" \approx .094"$   
Add .094" to the .5" to get a full depth of .594" for the 5/16" drill.

**PA Core Standard:** CC.2.2.HS.C.5

**Description:** Construct and compare linear, quadratic, and exponential models to solve problems.

**Math Associated Vocabulary:**  
INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

**Formulas and Procedures:**

**Direct Variation:**  $y = kx$   
y is directly proportional to x with constant k not equal to zero.

**Inverse Variation:**  $y = k/x$   
y is directly proportional to x with constant k not equal to zero.

**Example: Inverse Variation**

$y = k/x$   
 $y = 20/x$   
 $k = 20$

x	1	2	4	10
y	20	10	5	2

As the x value increases the y value decreases.  
Notice if you solve the equation for k you get  $k = xy$ .  
For each value of x and y above  $xy = k$ . k is 20.

**Example: Direct Variation**

$y = kx$   
 $y = 3x$   
 $k = 3$

x	1	2	4	10
y	3	6	8	30

As the x value increases, the y value increases.  
Notice if you solve the equation for k you get  $k = y/x$ .  
For each value of x and y above  $y/x = k$ . k is 3.

## Machine Tool Technology (48.0501) T-Chart

### Instructor's Script – Comparing and Contrasting

The Machine Tool Technology example gives an example of direct variation. In this example as one variable increases, the other variable increases as well. This shows the importance of knowing how important it is to determine how one variable changes another variable. Keystone Exams and other standardized tests may contain a number of questions regarding different circumstances where one variable changes as another does. It may be direct or inverse variation. It may be exponential growth or decay.

It may be similar figures with the change in perimeter, area or volume. When you have similar figures, if the height is doubled the perimeter will also be doubled since the scale factor and the ratio of the perimeters is the same. The area of the figure would be multiplied by 4 since the ratio is  $1^2:2^2$ . The volume would be multiplied by 8 since the ratio is  $1^3:2^3$ .

### Common Mistakes Made By Students

Students often make the mistake of not knowing the correct formula to use. One of the best ways to figure out what the change in one variable would be if there is a change in another is to make a table of values to find the pattern. Also, some data may not have a pattern.

### CTE Instructor's Extended Discussion

Using basic formulas that show relationships between two factors can be used in many areas in machining.

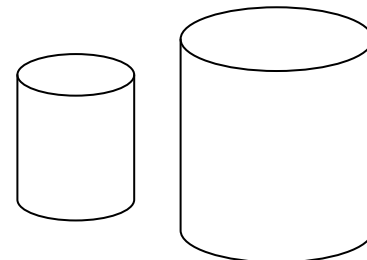
One case similar to drill point depth is countersinking depths. Increasing the cutting depth of a countersinking tool increases the diameter of the chamfer or countersink on a part. The factor depends on the angle of the tool.

When calculating RPM for machining operations, as tool diameter increases, RPM will decrease and vice versa. If a different cutting speed is applied to the same size tool for different materials, a lower cutting speed will result in a proportionately lower RPM and vice versa.

### Additional Example:

You are given two similar cylinders with a scale factor of 1:3.

- If the height of the smaller is tripled how does this change the height of the other?
- If the height of the smaller is tripled how does this change the surface area of the other?
- If the height of the smaller is tripled how does this change the volume of the other?



### Solution

- All of the linear measurements of the two cylinders will be in the same ratio. So if the height of the smaller is tripled then the height of the other will be tripled also.
- All of the area measurements of the cylinders will be in the ratio of  $1^2:3^2$ . So if the height of the smaller is tripled, then the surface area of the other is going to be multiplied by 9.
- The volume of the cylinders will be in the ratio  $1^3:3^3$ . So if the height of the smaller is tripled, then the volume of the other is going to be multiplied by 27.

Problems	Career and Technical Math Concepts	Solutions										
1. A countersink is creating a $\varnothing.225 \times 90^\circ$ chamfer on a VMC. If every .001" of depth increases the diameter by .002", what adjustment should be made to the programmed depth to make the diameter .210"? $D = 2d$ , where D = Diameter Change, d = depth change												
2. A print specifies a $\varnothing 5/8$ " hole to be drilled through a $3/4$ " thick plate. What is the minimum total depth that must be drilled? The point length is .3 times the diameter. L = Length of drill tip, D = Drill diameter												
3. Your company produces the same cylindrical part from both aluminum and stainless steel. The OD of the aluminum part is turned at 2600 RPM while the stainless steel is turned at 600 RPM. If the ID is bored at 1600 on the aluminum part, what should the RPM be for boring the stainless steel part? $S = PA$ , where A = RPM for aluminum, P = Factor of change from A to S												
Problems	Related, Generic Math Concepts	Solutions										
4. For the following problems determine if the values represent direct variation, indirect variation or neither.												
<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>2</td> <td>3</td> <td>4</td> <td>8</td> </tr> <tr> <td>y</td> <td>18</td> <td>12</td> <td>9</td> <td>4.5</td> </tr> </table>	x	2	3	4	8	y	18	12	9	4.5		
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Problems	PA Core Math Look	Solutions										
7. You are given two similar cones with a scale factor of 1:4. If the height of the smaller is multiplied by 4, how does this change the height of the other?												
8. If the height of the smaller is multiplied by 4, how does this change the surface area of the other?												
9. If the height of the smaller is multiplied by 4, how does this change the volume of the other?												

Problems	Career and Technical Math Concepts	Solutions										
1. A countersink is creating a $\varnothing.225 \times 90^\circ$ chamfer on a VMC. If every .001” of depth increases the diameter by .002”, what adjustment should be made to the programmed depth to make the diameter .210”? $D = 2d$ , where D = Diameter Change, d = depth change	.225-.210 = .015 The diameter needs to <i>decrease</i> by .015 $D = 2 \times d \rightarrow .015 = 2 \times d$ $\frac{.015}{2} = d \rightarrow d = .0075$	Decrease the programmed depth by .0075”										
2. A print specifies a $\varnothing 5/8$ ” hole to be drilled through a $3/4$ ” thick plate. What is the minimum total depth that must be drilled? The point length is .3 times the diameter. L = Length of drill tip, D = Drill diameter	$L = .3D$ $L = .3 \times .625 = .1875$	Add .1875 to .75 for a total depth of .9375”.										
3. Your company produces the same cylindrical part from both aluminum and stainless steel. The OD of the aluminum part is turned at 2600 RPM while the stainless steel is turned at 600 RPM. If the ID is bored at 1600 on the aluminum part, what should the RPM be for boring the stainless steel part? $S = PA$ , where A= RPM for aluminum, P = Factor of change from A to S	$S = P \times A$ $600 = P \times 2600$ $\frac{600}{2600} = P$ $P \approx .23$	$S = P \times A$ $S = P \times 1600$ $S = .23 \times 1600$ $S = 368 \text{ RPM}$										
Problems	Related, Generic Math Concepts	Solutions										
4. For the following problems determine if the values represent direct variation, indirect variation or neither. <table border="1" data-bbox="203 940 652 1039"> <tr><td>x</td><td>2</td><td>3</td><td>4</td><td>8</td></tr> <tr><td>y</td><td>18</td><td>12</td><td>9</td><td>4.5</td></tr> </table>	x	2	3	4	8	y	18	12	9	4.5	This is an example of inverse variation. As the x value increases, the y value decreases. Solving the equation $y = k/x$ for k gives $k = xy$ . For each pair of values k is equal to 36. (2 x 18, 3 x 12, 4 x 9, 8 x 4.5) The equation would be $y = 36/x$ . $k = 36$	
x	2	3	4	8								
y	18	12	9	4.5								
5. <table border="1" data-bbox="203 1102 652 1201"> <tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>y</td><td>14</td><td>21</td><td>28</td><td>30</td></tr> </table>	x	2	3	4	5	y	14	21	28	30	This is an example of neither. As the x value increases the y value increases, but the relationship is not consistent. $2/14 = 7$ , $3/21 = 7$ , $4/28 = 7$ , $5/30$ is not equal to 7. There is no value for k.	
x	2	3	4	5								
y	14	21	28	30								
6. <table border="1" data-bbox="191 1276 641 1375"> <tr><td>x</td><td>3</td><td>9</td><td>10</td><td>12</td></tr> <tr><td>y</td><td>9</td><td>27</td><td>30</td><td>36</td></tr> </table>	x	3	9	10	12	y	9	27	30	36	This is an example of direct variation. As the x value increases, the y value increases. Solving the equation $y = kx$ for k gives $k = y/x$ . For each pair of values k is equal to 3. ( $9/3$ , $27/9$ , $30/10$ , $12/36$ ) The equation would be $y = 3x$ .	
x	3	9	10	12								
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7. You are given two similar cones with a scale factor of 1:4. If the height of the smaller is multiplied by 4, how does this change the height of the other?	All of the linear measurements of the two cones will be in the same ratio. So if the height of the smaller is multiplied by 4 then the height of the other will be multiplied by 4 also.											
8. If the height of the smaller is multiplied by 4, how does this change the surface area of the other?	All of the area measurements of the cones will be in the ratio of $1^2:4^2$ . So if the height of the smaller is tripled, then the surface area of the other is going to be multiplied by 16.											
9. If the height of the smaller is multiplied by 4, how does this change the volume of the other?	The volume of the cones will be in the ratio $1^3:4^3$ . So if the height of the smaller is tripled, then the volume of the other is going to be multiplied by 64.											