

Approximate values to appropriate degree of precision

Apply the properties of rational and irrational numbers to solve real world or mathematical problems

**Program Task:** Weld blades for the vertical band saw.

**PA Core Standard:** CC.2.1.HS.F.2

**Program Associated Vocabulary:**  
APPROXIMATE, PI, ROUND

**Description:** Apply the properties of rational and irrational numbers to solve real world or mathematical problems.

**Math Associated Vocabulary:**  
IRRATIONAL NUMBER, SQUARE ROOT, PI

**Program Formulas and Procedures:**

Machinists work with irrational numbers nearly every day in industry. These are non-repeating & non-terminating decimal numbers. One of the most common values used is  $\pi$  (Pi). Normally the machinist simply uses the  $\pi$  key on the calculator. Using the  $\pi$  key maintain the level of accuracy that is required in machining operations.

**Formulas and Procedures:**

**Irrational Number:** a non-repeating & non-terminating decimal number that cannot be written as a fraction.

Further, when Pi is used in a calculation, the result of that calculation will also be an irrational number which can be rounded to the appropriate level of accuracy needed for the specific skill.

$\pi$ : The number  $\pi$  is a mathematical constant, commonly approximated as 3.14159.

In machining, irrational numbers are not being “located on a number line” but the same principle applies as those irrational numbers are approximated or rounded to an appropriate level of accuracy depending on the degree of precision required.

**Square Root:** The square root of a number is a number which, when multiplied by itself, yields that original number.

**Example:**

What length of saw blade is required for the band saw shown in the sketch with 24” diameter wheels and 48” center to center distance?

**Example 1:** Locate the following numbers on a number line.  $\sqrt{2}, \sqrt{5}, \pi$

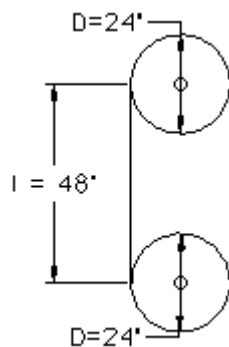
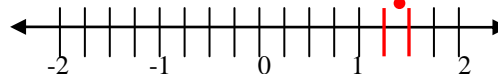
General steps:

1. Rewrite the number as a decimal to the nearest tenth or hundredth digit.
2. Use marks ( $1/4, 1/2, 3/4$ ) to approximate the location on the number line.

**Example 2:** Identify  $\sqrt{2}$  on the number line below:

$$\sqrt{2} = 1.41421356237... \approx 1.41$$

1.41 is between 1.25 and 1.5, but closer to 1.5.



$$L = \pi D + 2l$$

$$L = \pi \times 24 + 2 \times 48$$

$$L = 171.1398224... \text{ Inches}$$

What degree of accuracy is needed? In this application, rounding to the nearest inch is sufficient, so:

$$L = 171 \text{ Inches}$$

**Instructor's Script – Comparing and Contrasting**

The Common Core Standards and Machine Tool Technology concepts are very well connected. In this trade area it is very important to be accurate. In different trade areas the degree of accuracy may vary. Depending on the field and the application  $\pi$  can be estimated at 3.14, in other cases 3.1416, but the best estimate of Pi is usually given by using the  $\pi$  key on the calculator.

**Common Mistakes Made By Students**

**Taking the square root of a number:** This mostly occurs when the student is unfamiliar with a calculator. Some calculators require the student to press the number then the square root button; others require that the square root button is pressed before the number. It may be important to show students how to take the square root of 4, using both methods to evaluate which order gives the correct answer of 2.

**Using the appropriate rounding technique for the given situation:** In most cases, it is beneficial to round the number to the nearest hundredth. If the number line is broken into quarters, thirds, or tenths, then rounding the number to the closest hundredth would provide the information necessary to correctly identify the number's location.

**Being able to partition a number line and identify the location of the decimal number:** Sometimes the number line uses integer values only (...,-2, -1, 0, 1, 2, 3...). In this case, the student must be able to mentally divide the space between the integers into quarters or thirds to best approximate the location of the irrational number.

**CTE Instructor's Extended Discussion**

Another example of the use of irrational numbers is the utilization of the formulas for spindle RPM calculation:

$$\text{RPM} = \frac{12 \times \text{Cutting Speed}}{\pi \times \text{Diameter}} \quad \text{or} \quad \text{RPM} = \frac{3.82 \times \text{Cutting Speed}}{\text{Diameter}} \quad \text{where the 3.82 constant is the result of } \frac{12}{\pi}.$$

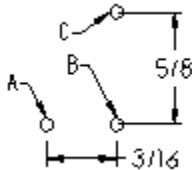
In the first formula, the irrational number Pi is used (as approximated by a calculator) and the resulting answer will be an irrational number. In the second formula, an approximation has already occurred but the calculation is commonly irrational as well. Both cases require the standard practice of rounding to the nearest whole number.

Other uses of irrational numbers are when using sine bars or when using the Pythagorean Theorem  $a^2 + b^2 = c^2$ . In these cases, frequently an irrational number answer must be approximated. In these situations, standard practice is to round to the nearest ten-thousandth or hundred-thousandth of an inch (the fourth or fifth decimal place value .xxxxx or .xxxxxx).

**Example:** What is the sine of 42 degrees? Round to the nearest hundred-thousandth.

On calculator, press  $\sin 42 = .669130606$  Rounded to the nearest hundred-thousandth, .66913

**Example:** What is the distance between holes "A" and "C"? Round to the nearest ten-thousandth of an inch.



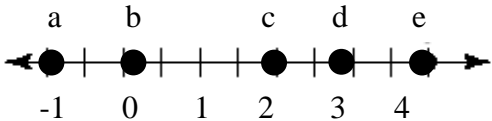
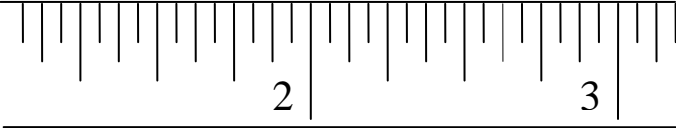
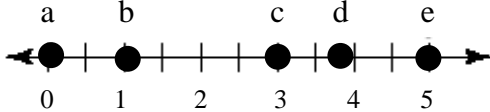
$$c^2 = a^2 + b^2 \rightarrow c^2 = \frac{3^2}{16} + \frac{5^2}{8}$$

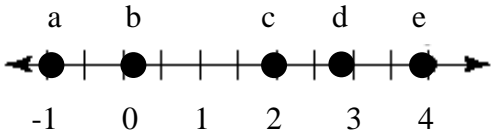
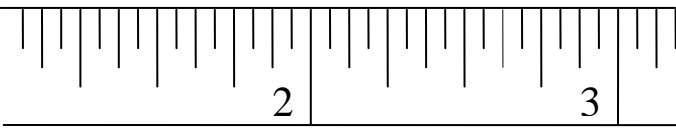
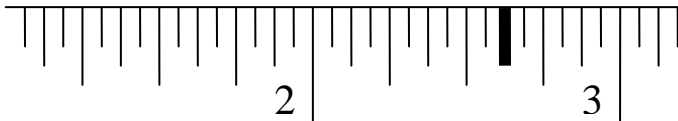
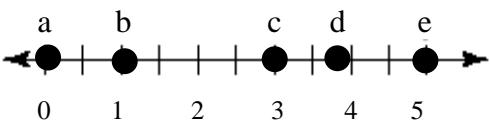
$$c^2 = .1875^2 + .625^2$$

$$c = \sqrt{.03515625 + .390625}$$

$$c = \sqrt{.42578125}$$

$$c = .6525$$

Problems	Career and Technical Math Concepts	Solutions
1. If a band saw has 14" diameter wheels spaced 36" between centers, what length saw blade is needed? (Round to the nearest inch.).		
2. Use the formula $RPM = \frac{12 \times \text{Cutting Speed}}{\pi \times \text{Diameter}}$ to calculate RPM to the nearest whole number for a .750 diameter endmill using a cutting speed of 140.		
3. A Pythagorean Theorem calculation gives the answer $\sqrt{3.90625}$ inches. What is the final answer to the nearest ten-thousandth of an inch?		
Problems	Related, Generic Math Concepts	Solutions
4. The location of $\sqrt{8}$ is closest to which point on the number line below?		
 <p>A number line is shown with arrows at both ends. It has tick marks and labels for integers from -1 to 4. Five points are marked with solid black circles and labeled above as a, b, c, d, and e. Point a is at -1, b is at 0, c is at 2, d is at 3, and e is at 4.</p>		
5. Why can't the square root of Pi be a rational number?		
6. Using the Pythagorean Theorem, a student finds that she needs $\sqrt{7}$ inches of material. Identify the location of this measurement on the measuring tape below.		
 <p>A measuring tape is shown with markings for inches. The numbers 2 and 3 are clearly visible. The tape is divided into millimeter increments.</p>		
Problems	PA Core Math Look	Solutions
7. Which of the following numbers would be located between 9 and 10 on the number line? a) $2\pi$ b) $3\pi$ c) $2\sqrt{5}$ d) $5\sqrt{2}$		
8. The location of $\sqrt{13}$ is closest to which point on the number line below?		
 <p>A number line is shown with arrows at both ends. It has tick marks and labels for integers from 0 to 5. Five points are marked with solid black circles and labeled above as a, b, c, d, and e. Point a is at 0, b is at 1, c is at 3, d is at 4, and e is at 5.</p>		
9. Which of the following would be closest to the value of $\sqrt{8}$ ? a) $2\frac{3}{4}$ b) $3\frac{1}{4}$ c) 4 d) $2\frac{1}{2}$		

Problems	Career and Technical Math Concepts	Solutions
1. If a band saw has 14" diameter wheels spaced 36" between centers, what length saw blade is needed? (Round to the nearest inch.).	$L = \pi D + 2l$ $L = \pi(14) + 2(36) = 115.9822972$ $L \approx 116"$	
2. Use the formula $RPM = \frac{12 \cdot \text{Cutting Speed}}{\pi \cdot \text{Diameter}}$ to calculate RPM to the nearest whole number for a .750 diameter endmill using a cutting speed of 140.	$RPM = \frac{12 \times \text{Cutting Speed}}{\pi \times \text{Diameter}}$ $RPM = \frac{12 \times 140}{\pi \times .750} = 713.0141451$ $RPM \approx 713$	
3. A Pythagorean Theorem calculation gives the answer $\sqrt{3.90625}$ inches. What is the final answer to the nearest ten-thousandth of an inch?	$\sqrt{3.90625} \approx 1.976423538$ inches rounded 1.9764 inches	
Problems	Related, Generic Math Concepts	Solutions
4. The location of $\sqrt{8}$ is closest to which point on the number line below?  	d) $\sqrt{8} = 2.828$	
5. Why can't the square root of Pi be a rational number?	Because Pi is an irrational number, and any rational number squared would produce a rational number.	
6. Using the Pythagorean Theorem, a student finds that she needs $\sqrt{7}$ inches of material. Identify the location of this measurement on the measuring tape below.  	Since $\sqrt{7} = 2.645751\dots$ , We round to 2.65 inches. $\frac{6}{10} = \frac{x}{16} \rightarrow 6(16) = 10x \rightarrow 96 = 10x \rightarrow 9.6 = x, 2^{10}/16$  	
Problems	PA Core Math Look	Solutions
7. Which of the following numbers would be located between 9 and 10 on the number line? a) $2\pi$ b) $3\pi$ c) $2\sqrt{5}$ d) $5\sqrt{2}$	b) $3\pi$	
8. The location of $\sqrt{13}$ is closest to which point on the number line below?  	d) $\sqrt{13} \approx 3.61$	
9. Which of the following would be closest to the value of $\sqrt{8}$ ? a) $2\frac{3}{4}$ b) $3\frac{1}{4}$ c) 4 d) $2\frac{1}{2}$	a) $2\frac{3}{4}$ $\sqrt{8} \approx 2.828$ If you convert $2\frac{3}{4}$ to its decimal format 2.75, then you find that answer a is the closest value to the $\sqrt{8}$ .	