

Calculate angles = Verify and apply geometric theorems as they relate to geometric figures

Program Task: Interpret engineering drawings.

PA Core Standard: CC.2.3.HS.A.3

Program Associated Vocabulary:
 ANGLE, COMPLEMENTARY ANGLE,
 QUADRILATERAL, SUPPLEMENTARY ANGLE

Description: Verify and apply geometric theorems as they relate to geometric figures.

Math Associated Vocabulary:
 CONGRUENT FIGURES, POLYGON, QUADRILATERAL,
 PARALLELOGRAM, PARALLEL, RECTANGLE, RHOMBUS,
 SQUARE, KITE, TRAPEZOID, ISOSCELES TRAPEZOID,
 SUPPLEMENTARY ANGLES

Program Formulas and Procedures:
 Engineering drawings, or prints, frequently show parts with angled sides. In order to set up and perform operations correctly, the machinist must often find angles that are not specifically dimensioned on the drawing. Non-dimensioned angles can often be found by first breaking profile, or shape, into four sided shapes (quadrilaterals). The fact that all quadrilaterals contain 360° aids in determining those non-dimensioned angles.

Formulas and Procedures:
 The following are the ways to prove a quadrilateral is a parallelogram:

Sometimes complementary ($90 - \text{given angle}$) and supplementary ($180 - \text{given angle}$) angles can also be used during the process to find the desired angle(s).

1. Both pairs of opposite sides are parallel.
2. Both pairs of opposite sides are congruent.
3. Both pairs of opposite angles are congruent.
4. One pair of opposite sides are both congruent and parallel.
5. The diagonals bisect each other.
6. An angle is supplementary to both of its consecutive angles.

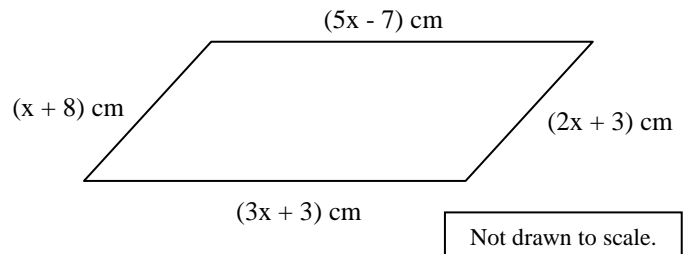
Example:
 Angle "A" is needed to set up for a milling machine operation.

In order to prove that a parallelogram has right angles, you can measure the diagonals. If the diagonals are equal in length then it has right angles.

If the diagonals of a parallelogram are perpendicular then it has equal side lengths.

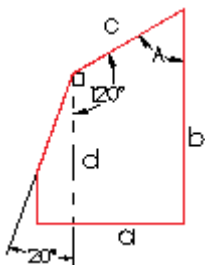


Example:



Adding the broken horizontal and vertical lines breaks the part profile into triangles and a quadrilateral with sides a, b, c, and d.

The figure above is a parallelogram. What are its side lengths?



$$\begin{aligned} (5x - 7) &= (3x + 3) && \text{Set opposite sides equal.} \\ 2x - 7 &= 3 && \text{Subtract } 2x. \\ 2x &= 10 && \text{Add } 7. \\ x &= 5 && \text{Divide by } 2. \\ 3x + 3 &= 3(5) + 3 = 18 && \text{Substitute.} \\ x + 8 &= 5 + 8 = 13 && \text{Substitute.} \end{aligned}$$

Then we can subtract 20° from the original 140° to get the 120° shown. Since all four sided figures contain 360° , angle A can be found by:

The side lengths are 18 cm., 18 cm., 13 cm. and 13 cm.

$$360 - (90 + 90 + 120) = 60$$

Angle A is 60°

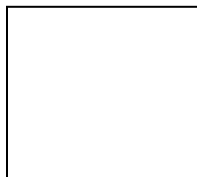
Instructor's Script – Comparing and Contrasting

The Machine Tool Technology examples given show a variety of different quadrilaterals and show how the properties are used through those real life examples.

Common Mistakes Made By Students

A common mistake made by students is to confuse the properties of different quadrilaterals. A rectangle and square have congruent diagonals, but not every parallelogram has that property. Another common mistake is looking at the picture that may not be drawn to scale and making assumptions about the figure.

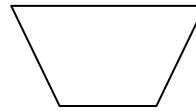
Examples of Quadrilaterals:



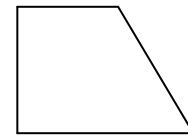
Square



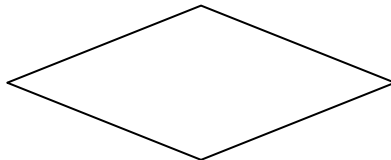
Rectangle



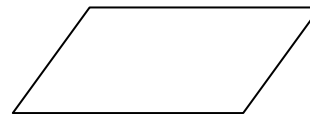
Trapezoid



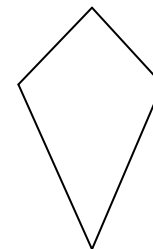
Right Trapezoid



Rhombus



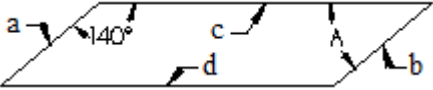

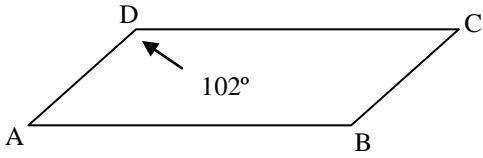
Parallelogram



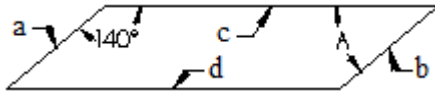

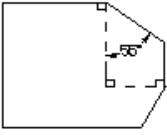
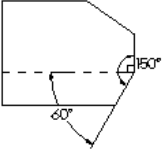
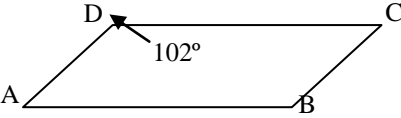
Kite

CTE Instructor's Extended Discussion

There is often more than one way to break a complex shape with angular sides into quadrilaterals and instructors should take time to show different ways during examples. Depending on how the shape is “broken”, complementary or supplementary angle use might be required to find the desired angle.

Problems	Career and Technical Math Concepts	Solutions
<p>1. You need to set the milling vise to angle A to machine the ends of this part. Sides a and b are parallel. Sides c and d are also parallel. What angle must the vise be set at to mill sides a and b?</p> 		
<p>2. Determine angle A. Then find its supplement as a setting for an angle vise to grind the part.</p> 		
<p>3. Determine angle B in the sketch above, and then find its supplement for the angle vise setting.</p>		
Problems	Related, Generic Math Concepts	Solutions
<p>4. If the angles of a quadrilateral are $(x + 7)^\circ$, $(5x - 7)^\circ$, $(2x + 2)^\circ$ and 10°, what is each angle?</p>		
<p>5. If one side of a parallelogram measures $(2x + 10)$ in. and the opposite side measures $(3x - 2)$ in., what is the length of these sides?</p>		
<p>6. If one diagonal of a rectangle is $(2x + 3)$ cm. and the other is $(x + 8)$ cm, what is length of the diagonal?</p>		
Problems	PA Core Math Look	Solutions
<p>7. You have drawn a rhombus in a coordinate plane. The slope of one of the diagonals is $\frac{1}{4}$. What is the slope of the other diagonal?</p>		
<p>8. What are the missing angles in the parallelogram?</p> 		
<p>9. Which type of quadrilateral has exactly one pair of opposite sides parallel?</p>		

Machine Tool Technology (48.0501) T-Chart

Problems	Career and Technical Math Concepts	Solutions
<p>1. You need to set the milling vise to angle A to machine the ends of this part. Sides A and B are parallel. Sides C and D are also parallel. What angle must the vise be set at to mill sides A and B?</p>		<p>Since a parallelogram has equal opposite angles and all four angles add up to 360, Angle A = $[360 - (2 \times 140)] \div 2 = 40^\circ$.</p>
<p>2. Determine angle A. Then find its supplement as a setting for an angle vise to grind the part.</p>		<p>Break the shape into a quadrilateral as shown. The 55° is found by subtracting 90° from the original 145°.</p> <p>Then find A by: Angle A = $360 - (90 + 90 + 55) = 125$ Supplement of A = $180 - 125 = 55^\circ$</p> 
<p>3. Determine angle B in the sketch above, and then find its supplement for the angle vise setting.</p>		<p>Again, break the profile into a quadrilateral. Subtract 90° from the original 150° to get the 60° shown. Then: Angle B = $360 - (90 + 90 + 60) = 120$ Supplement of B = $180 - 120 = 60^\circ$</p>
Problems	Related, Generic Math Concepts	Solutions
<p>4. If the angles of a quadrilateral are $(x + 7)^\circ$, $(5x - 7)^\circ$, $(2x + 2)^\circ$ and 10°, what is each angle?</p>		<p>$(x + 7) + (5x - 7) + (2x + 2) + 10 = 360$ $\rightarrow 8x + 12 = 360 \rightarrow 8x = 348 \rightarrow x = 43.5$ Substitute x to find the angles. $x + 7 = 43.5 + 7 = 50.5$, $5x - 7 = (5)(43.5) - 7 = 210.5$, $2x + 2 = (2)(43.5) + 2 = 89$ The angles of the quadrilateral are 50.5°, 210.5°, 89° and 10°.</p>
<p>5. If one side of a parallelogram measures $(2x + 10)$ in. and the opposite side measures $(3x - 2)$ in., what is the length of these sides?</p>		<p>Since opposite sides of a parallelogram are congruent, they are also equal in length. $3x - 2 = 2x + 10$ $1x - 2 = 10 \rightarrow 1x = 12 \rightarrow x = 12$ substitute to find sides $3x - 2 = (3 \times 12) - 2 = 34$ inches $2x + 10 = (2 \times 12) + 10 = 34$ inches</p>
<p>6. If one diagonal of a rectangle is $(2x + 3)$ cm. and the other is $(x + 8)$ cm. what is length of the diagonal?</p>		<p>Since the diagonals of a rectangle are congruent, $2x + 3 = x + 8 \rightarrow x + 3 = 8 \rightarrow x = 5$ Substitute to find the length of the diagonal: $2(5) + 3 = 13$ The diagonal has a length of 13 cm.</p>
Problems	PA Core Math Look	Solutions
<p>7. You have drawn a rhombus in a coordinate plane. The slope of one of the diagonals is $\frac{1}{4}$. What is the slope of the other diagonal?</p>		<p>Since the diagonals of a rhombus are perpendicular the other diagonal would have a slope that is the negative reciprocal of the given slope. The slope of the other diagonal is $-\frac{4}{1}$ or just -4.</p>
<p>8. What are the missing angles in the parallelogram?</p> 		<p>Since opposite angles are congruent, the measure of angle B is 102°. Since consecutive angles are supplementary, angles A and C are 78°. $(360 - 2(102))/2 = 78$</p>
<p>9. Which type of quadrilateral has exactly one pair of opposite sides parallel?</p>		<p>A trapezoid is a quadrilateral with exactly one pair of opposite sides parallel.</p>