Machine Tool Technology (48.0501) T-Chart

Perform turning operations	=	Verify and apply geometric theorems as they relate to geometric figures

Program Task: Perform turning operations.

Program Associated Vocabulary:
ANGLE, DIAGONAL, DIMENSION, INSCRIBED, SQUARE, BAR STOCK

Program Formulas and Procedures:
The machinist must often determine the diameter of round bar stock required to machine a particular size square.

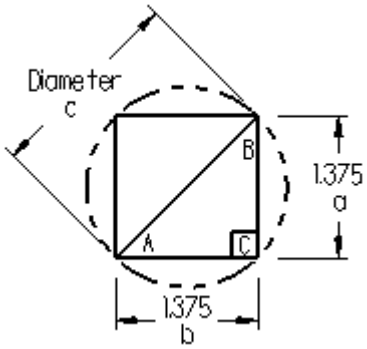
A sketch of a dimensioned square inscribed within a circle shows the relationship between the desired square and required bar stock diameter.

A diagonal drawn between opposing corners of the square represents the required diameter of the bar stock. This diagonal is side c in the Right Triangle Formula.

$$a^2 + b^2 = c^2$$

Sides a and b are equal in length because angles A and B are equal (45°). Substitute a and b in the formula and solve for c, which is equal to the required diameter.

Example:
What diameter of round bar stock is required to produce a 1.375" (inch) square?



$a^2 + b^2 = c^2$

$1.375^2 + 1.375^2 = c^2$

$1.890625 + 1.890625 = c^2$

$3.78125 = c^2$

$\sqrt{3.78125} = \sqrt{c^2}$

1.944544 = c, the required diameter

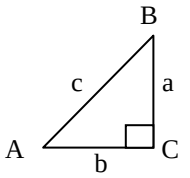
In practice, 1.944544" would be rounded up to 2" because bar stock is available in 1/16" (.0625") diameter increments.

PA Core Standard: CC.2.3.8.A.3

Description: Verify and apply geometric theorems as they relate to geometric figures.

Math Associated Vocabulary:
HYPOTENUSE, DIAGONAL, LEG, RIGHT ANGLE, RIGHT TRIANGLE, PYTHAGOREAN THEOREM, ROOT, SQUARE

Formulas and Procedures:



a = leg b = leg c = hypotenuse	**Pythagorean Theorem:** $a^2 + b^2 = c^2$

Example 1: Solve for the hypotenuse, c, when given both legs.
A rectangle has side measurements of 8 inches and 12 inches. Find the length of the diagonal.

Step 1: Substitute known values into the Pythagorean theorem.

$$a^2 + b^2 = c^2 \quad 8^2 + 12^2 = c^2$$

Step 2: Square and add each number as directed by the theorem.

$$64 + 144 = c^2 \rightarrow 208 = c^2$$

Step 3: Take the square root of each side to solve for c.

$$\sqrt{208} = \sqrt{c^2} \rightarrow 14.4 = c$$

Example 2: Solve for a leg when given the hypotenuse and the other leg.
A right triangle has a hypotenuse that measures 10 inches and one of the legs measures 6 inches. Find the length of the other leg.

Step 1: Substitute known values into Pythagorean theorem.

$$a^2 + b^2 = c^2 \rightarrow 6^2 + b^2 = 10^2$$

Step 2: Square each number as directed by the theorem.

$$6^2 + b^2 = 10^2 \rightarrow 36 + b^2 = 100$$

Step 3: Subtract from both sides to isolate the variable.

$$36 - 36 + b^2 = 100 - 36 \rightarrow b^2 = 64$$

Step 4: Take the square root of each side to solve for the variable.

$$b^2 = 64 \rightarrow \sqrt{b^2} = \sqrt{64} \rightarrow b = 8$$

Instructor's Script – Comparing and Contrasting

These are excellent examples of using the Pythagorean Theorem. There are many applications of this theorem. A ladder leaning against a wall can make a right triangle. Sight lines or kite strings can represent the hypotenuse of a right triangle. It is important to see real life examples, and it is also important to understand how this theorem can apply to numerous types of problems and a large variety of applications.

Common Mistakes Made By Students

Incorrectly identifying a, b, and c – Students will often confuse the hypotenuse with one of the legs or incorrectly substitute values into the equation. One way to avoid this is to recognize that diagonal often is used to describe a hypotenuse and label your hypotenuse right away by quickly identifying the right angle and marking the side opposite the right angle.

Inability to manipulate the equation to solve for a or b – Solving for the hypotenuse is much simpler than solving for a leg of a right triangle. Students need to be given many opportunities to solve for all the variables in the Pythagorean Theorem.

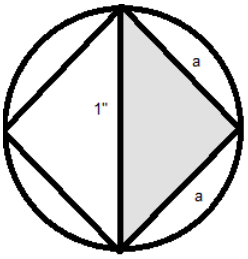
Inability to recognize the Pythagorean Theorem in multiple contexts – The Pythagorean Theorem appears in many contexts in standardized testing. Sometimes a test question will describe a right triangle and ask the student to solve for the missing side. Other times, the right triangle is drawn and the student must solve for the missing side. In many cases, a more complex picture is drawn and the student must use the Pythagorean Theorem to solve part of the problem. In these cases, it is not obvious that the Pythagorean Theorem is needed and the student must be able to select and use the theorem.

CTE Instructor's Extended Discussion

The Pythagorean Theorem, or Right Triangle Formula, $a^2 + b^2 = c^2$ can also be used to determine the largest square which can be machined from a given round stock diameter. In these cases, side c of the triangle is known, and sides a and b must be calculated. Since a square has equal length sides, sides a and b of the right triangle are also equal. Since a = b, the formula can be rewritten as $a^2 + a^2 = c^2$.

The Pythagorean Theorem can also be used to determine distance between holes.

Example: What is the largest square that can be machined from 1.5"diameter round stock?



$a^2 + b^2 = c^2$

$a^2 + a^2 = c^2$

$2a^2 = c^2$

$2a^2 = 1.5^2$

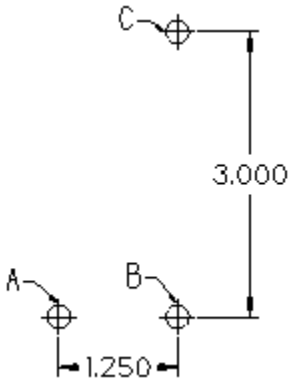
$2a^2 = 2.25$

$a^2 = 2.25 / 2$

$a = \sqrt{2.25/2}$

a = 1.06 inches on a side

Example: What is the distance between holes "A" & "C"?



$c^2 = a^2 + b^2$

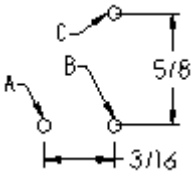
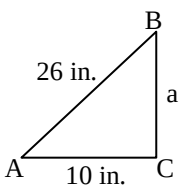
$c^2 = 1.250^2 + 3.000^2$

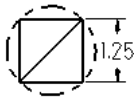
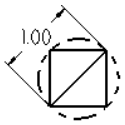
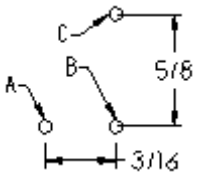
$c^2 = 1.5625 + 9$

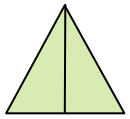
$c = \sqrt{1.5625 + 9}$

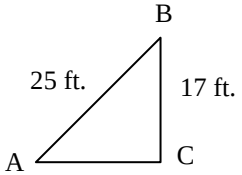
$c = \sqrt{10.5625}$

c = 3.250

Problems	Career and Technical Math Concepts	Solutions
1. What diameter round stock is required to machine a 1.25" square?		
2. What is the largest square that can be machined from 1" diameter round stock?		
3. What is the distance between holes "A" and "C"? 		
Problems	**Related, Generic Math Concepts**	**Solutions**
4. A tent has two slanted sides that are both 5 feet long and the bottom is 6 feet across. What is the height of the tent in feet at the tallest point?		
5. Three sides of a triangle measure 9 feet, 16 feet and 20 feet. Determine if this triangle is a right triangle.		
6. On a baseball diamond, the bases are 90 feet apart. What is the distance from home plate to second base using a straight line?		
Problems	**PA Core Math Look**	**Solutions**
7. The lengths of the legs of a right triangle measure 12 meters and 15 meters. What is the length of the hypotenuse to the nearest whole meter?		
8. In a right triangle ABC, where angle C is the right angle, side AB is 25 feet and side BC is 17 feet. Find the length of side AC to the nearest tenth of a foot.		
9. In the given triangle, find the length of a. 		

Problems	Career and Technical Math Concepts	Solutions
1. What diameter round stock is required to machine a 1.25" square?	$a^2 + b^2 = c^2 \rightarrow 1.25^2 + 1.25^2 = c^2$ $1.5625 + 1.5625 = c^2$ $3.125 = c^2$ $1.7678 = c$ Would round up to 1.8125 (1 13/16) because stock is only available in 1/16" diameter increments.	
2. What is the largest square that can be machined from 1" diameter round stock?	$a^2 + a^2 = c^2 \rightarrow a^2 + a^2 = 1.00^2$ $2a^2 = 1.00^2 \rightarrow 2a^2 = 1$ $a^2 = ½$ a = .7071inches per side	
3. What is the distance between holes "A" and "C"?	$c^2 = a^2 + b^2 \rightarrow c^2 = (3/16)^2 + (5/8)^2$ $c^2 = .1875^2 + .625^2$ $c = \sqrt{.03515625 + .390625}$ $c = \sqrt{.42578125}$ c = 0.652619 inches	

Problems	Related, Generic Math Concepts	Solutions
4. A tent has two slanted sides that are both 5 feet long and the bottom is 6 feet across. What is the height of the tent in feet at the tallest point?	$a^2 + b^2 = c^2$ $a^2 + 3^2 = 5^2$ $a^2 + 9 = 25$ $a^2 = 16$ a = 4 feet	
5. Three sides of a triangle measure 9 feet, 16 feet and 20 feet. Determine if this triangle is a right triangle.	$a^2 + b^2 = c^2$ $16^2 + 9^2 = 20^2$ $256 + 81 \neq 400$ Therefore, it is not a right triangle.	
6. On a baseball diamond, the bases are 90 feet apart. What is the distance from home plate to second base using a straight line?	$90^2 + 90^2 = c^2$ $8100 + 8100 = c^2$ $16200 = c^2$ $\sqrt{16200} = c$ 127.28 ft. = c	

Problems	PA Core Math Look	Solutions
7. The lengths of the legs of a right triangle measure 12 meters and 15 meters. What is the length of the hypotenuse to the nearest whole meter?	$a^2 + b^2 = c^2 \quad 12^2 + 15^2 = c^2$ $144 + 225 = c^2 \quad 369 = c^2$ $\sqrt{369} = c \quad 19 = c$ c = 19 meters	
8. In a right triangle ABC, where angle C is the right angle, side AB is 25 feet and side BC is 17feet. Find the length of side AC to the nearest tenth of a foot.	$a^2 + b^2 = c^2$ $17^2 + b^2 = 25^2$ $289 + b^2 = 625$ $b^2 = 336$ $\sqrt{b^2} = \sqrt{336}$ b = 18.3 feet	
9. In the given triangle, find the length of a.	$a^2 + b^2 = c^2 \quad a^2 + 10^2 = 26^2$ $a^2 + 100 = 676 \quad a^2 = 576$ $a = \sqrt{576}$ a = 24 inches