

Calculate countersink depth =

Verify and apply geometric theorems as they relate to geometric figures

Program Task: Perform countersinking on the drill press and milling machine.

Pa Core Standard: CC.2.3.HS.A.3

Program Associated Vocabulary:
COUNTERSINK, INCLUDED ANGLE, BISECT

Description: Verify and apply geometric theorems as they relate to geometric figures.

Program Associated Vocabulary:
CONGRUENT, SIMILAR, EQUILATERAL, ISOSCELES, SCALENE, RIGHT, OBTUSE, ACUTE, MEDIAN, ALTITUDE, ANGLE BISECTOR, PERPENDICULAR BISECTOR, CENTROID, ORTHOCENTER, INCENTER, CIRCUMFERENCE

Program Formulas and Procedures:
When countersinking a hole, the diameter of the countersink is normally given on a print. The machinist needs to determine the depth to feed the tool to reach the correct diameter. For a 90° countersink, the calculation uses some basic properties of triangles.

Formulas and Procedures:
Procedures for proving triangles congruent: SSS (side-side-side), SAS (side-angle-side), ASA (angle-side-angle), AAS (angle-angle-side), HL (hypotenuse-leg).

Procedures for proving triangles similar: AA (angle-angle similarity), SAS (side-angle-side similarity), SSS (side-side-side similarity).

Example:
Countersink an existing .312" diameter hole to .80" diameter using a 90° countersink.

The Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Since the countersink has a 90° included angle, bisect that angle with the centerline of the hole to create two equal 45° angles.

The sum of the angles of a triangle is 180°.

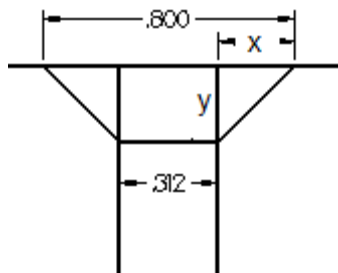
Subtract the existing hole diameter from the countersink diameter. Then divide the answer by two because ½ the difference is on each side of the hole.

The sides of a 45-45-90 degree triangle are always in the ratio

$$.800 - .312 = .488$$

$$.488 \div 2 = .244$$

$$x = .244$$



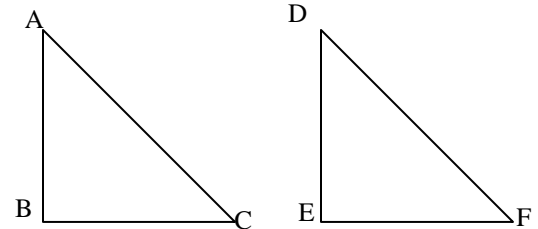
The legs of a 45-45-90 are equal so the depth that the tool needs to be fed (y) is also .244".

The sides of a 30-60-90 degree triangle are always in the ratio

$$1:1:\sqrt{2}$$

$$1:\sqrt{3}:2$$

Example:



$$\triangle ABC \cong \triangle DEF$$

What are the congruent parts?

$$\overline{AB} \cong \overline{DE}$$

$$\overline{BC} \cong \overline{EF}$$

$$\overline{AC} \cong \overline{DF}$$

Congruent Sides

$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

Congruent Angles

Instructor’s Script – Comparing and Contrasting

The examples here from machine tool technology are good real life examples. There are so many properties of triangles that it is hard to give an extensive list. Special triangles are the basis for higher levels of math and additional real life examples. Trigonometric functions are based on relationships and patterns that we begin learning about in geometry.

Common Mistakes Made By Students

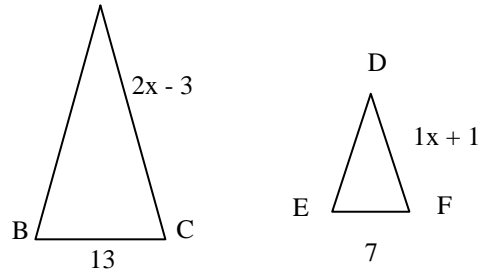
- Lack of familiarity with the concept and understanding of the vocabulary associated with the concept.
- Setting up the proportion incorrectly.
- Confusing the properties of congruent triangles with the properties of similar triangles.
- Confusing congruent and equal. Congruent figures are the same in both shape and size. Two triangles can have the same areas without being congruent.

Example:

$\Delta ABC \sim \Delta DEF$

Find the length of AC.

Since the triangles are similar the ratio of the corresponding sides is proportional.



$$\frac{2x - 3}{1x + 1} = \frac{13}{7} \rightarrow 7(2x - 3) = 13(x + 1) \rightarrow 14x - 21 = 13x + 13 \rightarrow x = 34$$

Using $x = 34$, you can substitute to find the length of AC.

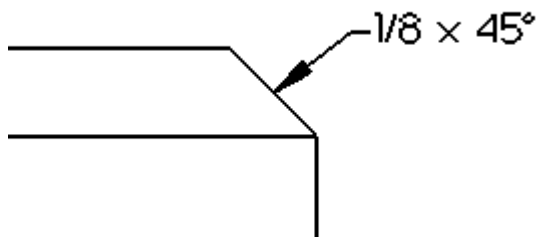
$AC = 2x - 3$

$2(34) - 3 = 65$

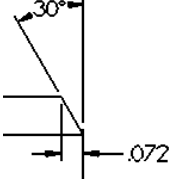
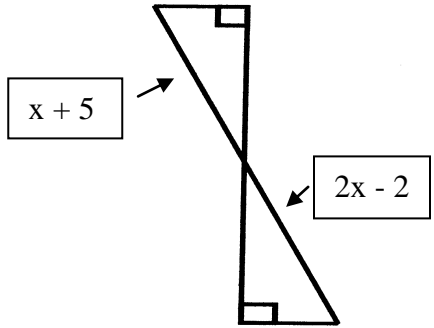
AC would be 65 units.

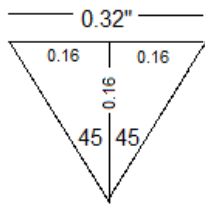
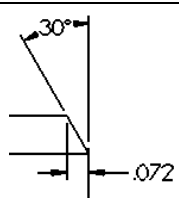
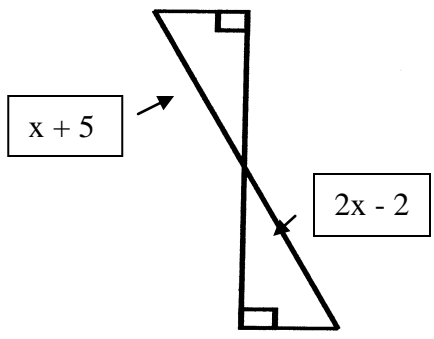
CTE Instructor’s Extended Discussion

Bisecting the included angle of any angular tool or dimension gives what is referred to as the centerline angle. This concept can also be used when milling bevels because many chamfering milling tools are identified by their included angles. A drawing may specify a 45° bevel, but a 90° chamfering tool would be used to machine the bevel.



This concept is also used when cutting threads on the lathe. The compound rest is set to $1/2^\circ$ less than $1/2$ of the thread’s included angle. Bisect the thread’s included angle and subtract $1/2^\circ$. For example, determine the compound rest setting for a unified thread with a 60° included angle. First, $60 \div 2 = 30$. Then, $30 - 1/2 = 29 1/2$ (29.5).

Problems	Career and Technical Math Concepts	Solutions
1. A series of holes in a component to be machined on a VMC need $\text{Ø}.32'' \times 90^\circ$ chamfers. What countersink depth must be programmed?		
2. What included angle tool is needed to mill the 30° bevel shown on the drawing? 		
3. When threading on the lathe, the compound rest is set to advance the tool at $1/2^\circ$ less than the thread's centerline angle. At what angle should the rest be set to cut an Acme thread with a 29° included angle?		
Problems	Related, Generic Math Concepts	Solutions
4. $\triangle ABC \cong \triangle XYZ$ Name all congruent parts.		
5. Which of the following side lengths can NOT make a triangle? a) 3, 4, 4 b) 2, 4, 5 c) 3, 7, 9 d) 10, 14, 2		
6. If an isosceles triangle has a vertex angle that is 3 times the measure of the base angles, what are the angles of the triangle?		
Problems	PA Core Math Look	Solutions
7. What value of x would make the triangles congruent? 		
8. What congruence theorem could you use to prove the triangles congruent?		
9. What would be the length of the congruent sides?		

Problems	Career and Technical Math Concepts	Solutions
1. A series of holes in a component to be machined on a VMC need $\varnothing.32''$ x 90° chamfers. What countersink depth must be programmed?		One half of the $\varnothing.32''$ is on each side of the hole's centerline creating two 45-45-90 triangles with leg lengths equal to $\frac{1}{2}$ of the $.32''$. The programmed depth should be $.16''$. 
2. What included angle tool is needed to mill the 30° bevel shown on the drawing?		Since the bevel represents the bisected angle of the tool, double the 30° and use a 60° tool.
3. When threading on the lathe, the compound rest is set to advance the tool at $1/2^\circ$ less than the thread's centerline angle. At what angle should the rest be set to cut an Acme thread with a 29° included angle?		Bisecting the 29° thread angle $29 \div 2 = 14.5^\circ$. $1/2^\circ$ less than 14.5° is 14° .
Problems	Related, Generic Math Concepts	Solutions
4. $\triangle ABC \cong \triangle XYZ$ Name all congruent parts.		$\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\angle C \cong \angle Z$ $\overline{AB} \cong \overline{XY}$ $\overline{BC} \cong \overline{YZ}$ $\overline{AC} \cong \overline{XZ}$
5. Which of the following side lengths can NOT make a triangle? a) 3, 4, 4 b) 2, 4, 5 c) 3, 7, 9 d) 10, 14, 2		d) $10 + 2$ is not greater than 14, so this can NOT make a triangle.
6. If an isosceles triangle has a vertex angle that is 3 times the measure of the base angles, what are the angles of the triangle?		$3x + x + x = 180^\circ \rightarrow 5x = 180^\circ \rightarrow x = 36^\circ$ $3x = 3(36) \rightarrow 3x = 108^\circ$ The vertex angle is 108° , and the base angles are 36° each.
Problems	PA Core Math Look	Solutions
7. What value of x would make the triangles congruent? 	$2x - 2 = x + 5$ $x - 2 = 5$ $x = 7$	The sides would need to be equal in length for the triangles to be congruent. Subtract x from each side of the equation. Add 2 to each side of the equation.
8. What congruence theorem could you use to prove the triangles congruent?		You would use the ASA (angle-side-angle) congruence postulate.
9. What would be the length of the congruent sides?		$x + 5 = 7 + 5 \rightarrow x = 12$ $2x - 2 = 2(7) - 2 \rightarrow x = 12$