Sales, Distribution, and Marketing Operations (52.1801) T-Chart

Forecast and estimate worker productivity = Use reasoning to solve equations and justify the solution method

Program Task: Students will forecast and estimate productivity for employees.

Program Associated Vocabulary:
PRODUCTIVITY, LABOR COSTS, FORECASTING

Program Formulas and Procedures:
Businesses experience seasonal changes in demand for products. When this happens, managers need to be able to evaluate the productivity information for their employees and determine the number of employees to hire to meet the demands of the business.

Example: (This example represents an inverse proportion. Direct proportion examples used in the marketing field are described on page 2.)

At Keystone Khakis it takes 15 employees four hours to make 75 pairs of khakis. The fall season is rapidly approaching and they are expecting an increase in the demand for khakis. If they increase the staff to 20 employees, how long will it take them to produce 75 pairs of khakis?

Step 1: Set up the proportion.

$$\frac{15 \text{ employees}}{20 \text{ employees}} = \frac{4 \text{ hours}}{x \text{ hours}}$$

Step 2: Invert one ratio, since it is an inverse proportion.

$$\frac{15 \text{ employees}}{20 \text{ employees}} = \frac{x \text{ hours}}{4 \text{ hours}}$$

Step 3: Cross multiply.

$20x = 15(4)$

Step 4**:** Divide

$$\frac{20x}{20} = \frac{60}{20}$$

Step 5: Write out your solution.

$x = 3$ hours

It would take 20 employees three hours to make 75 pairs of khakis. This information will help the manager evaluate the cost of increasing production by hiring additional employees or possibly by offering overtime to the employees who currently work for the company.

PA Core Standard: CC.2.2.HS.D.9

Description: Use reasoning to solve equations and justify the solution method.

Math Associated Vocabulary:
INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT

Formulas and Procedures:
Direct Proportions:
Two quantities, A and B, are directly proportional if by whatever factor A changes, B changes by the same factor.

Example 1: Take the formula, distance = rate x time. If the rate remains constant, at 30 miles per hour, then the time and distance are directly proportional.

$d = 30t$
when $t = 2$, $d = 60$
when $t = 4$, $d = 120$

*Note that when the time doubles, so does the distance.

Example 2: If speed is directly proportional to distance, and a car can travel 100 miles at 50 miles per hour, how far can that car travel during the same time if it travels at 70 mph?

Step 1: Set up proportion.

$$\frac{50 \text{ mph}}{70 \text{ mph}} = \frac{100 \text{ mi.}}{x}$$

Step 2: Cross multiply and divide to solve.

$50x = 70(100) \rightarrow 50x = 7000 \rightarrow x = 140$ miles

Inverse Proportions:
Two quantities, A and B, are inversely proportional if by whatever factor A changes, B changes by the multiplicative inverse, or reciprocal of that factor.

Example 1: Take the formula, distance = rate x time. If the distance, 100 miles is constant, then as the rate increases, the time decreases.

$100 = rt$
When $r = 100$, $t = 1$
When $r = 50$, $t = 2$

*Note that when the rate doubles, the time is halved.

Example 2: The time needed to complete a job is inversely proportional to the number of people working. If it takes one person 8 hours to pain the room alone, how long would it take 4 people to paint a room?

Step 1: Set up the proportion. Step 2: Invert (flip) one ratio.

$$\frac{1 \text{ person}}{4 \text{ people}} = \frac{8 \text{ hours}}{x \text{ hours}} \qquad \frac{1 \text{ person}}{4 \text{ people}} = \frac{x \text{ hours}}{8 \text{ hours}}$$

Step 3: Cross-multiply and divide to solve.
$4x=8$, $x = 2$
4 people can paint the room in 2 hours.

Instructor's Script - Comparing and Contrasting
The example shown on page 1 on the Marketing side of the T-Chart represents an inverse proportion. Increasing the number of workers will decrease the time it takes to complete the job. Inverse proportions are often difficult for students to understand. It should be noted that this sample problem can be adapted to provide trade related applications of direct proportions. For instance, if the number of workers increase and the time remains the same, then the number of khakis produced will also increase. Similarly, if the number of workers remains constant and the time allocated increases, then the number of khakis produced will also increase. When teaching proportional relationships, it is very important to teach students how to identify whether inverse or direct proportions exist for the situation given.

Common Mistakes Made By Students
When students compare direct and inverse proportional relationships, they may become confused and have difficulty differentiating one from the other. One way to keep them straight is to:

1. Set up one pair of values on the same line, e.g., 12 = 100 lbs.
2. Beneath that line, place the other pair of values, 24" x lbs.
3. Students need to be aware that direct proportions mean that as one variable increases so does the other variable. An inverse proportion means that one variable increases when the other one decreases. Students struggle with this concept.
4. If the problem is a direct proportion, students should cross multiply (24 times 100) and (12 times x) and then divide to solve the problem.
5. If an inverse relationship exists, then students should first invert one ratio before cross multiplying and dividing to solve the problem.
6. If need be, have the student set up the problem and do it both ways to see which answer makes sense! We know in problem #9, for example, that it won't take 5 rabbits more time than it took 1 rabbit to eat 20 carrots, so it must be an inverse proportion.

CTE Instructor's Extended Discussion
Using the direct and inverse forms of proportions provides managers with tools to evaluate productivity for employees. Managers who use these tools have a variety of different methods they can use to mathematically evaluate data. This information can be used to evaluate delivery methods for products and it can be used to forecast employee needs of a company based on worker production data. It provides managers with a tool to compare data in a variety of different methods.

There are several different types of inventory management used by companies: perpetual, just in time, and physical. Each system handles inventory data a little differently; using direct and inverse proportions can help a manager forecast supply needs for production.

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Problems	Career and Technical Math Concepts Solutions
1. A delivery truck driver covers 60 miles per hour and travels a distance of 120 miles in two hours. How many miles will the truck travel in 4 hours at the same rate of speed?	
2. It takes four students 3.5 hours to process 100 cases of shipment for the store. How long will it take six students to process 100 cases of shipment?	
3. The Widget Company production supervisor told management that it takes six employees two hours to produce 90 widgets. How long would it take 10 employees to produce 90 widgets?	
Problems	**Related, Generic Math Concepts Solutions**
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?	
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m3 is 0.5 atm, what would the pressure be of 0.060 m3 of the same gas at the same temperature?	
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?	
Problems	**PA Core Math Look Solutions**
7. Given that y and x are **directly** proportional and y = 2 when x = 5, find the value of y when x = 15.	
8. Given that y and x are **inversely** proportional and y = 2 when x = 5, find the value of y when x = 15.	
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?	

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Problems	Career and Technical Math Concepts Solutions
1. A delivery truck driver covers 60 miles per hour and travels a distance of 120 miles in two hours. How many miles will the truck travel in 4 hours at the same rate of speed?	**(Direct)** $\frac{2}{120} = \frac{4}{x}$ $2x = 120(4) \rightarrow 2x = 480 \rightarrow x = 240$ miles
2. It takes four students 3.5 hours to process 100 cases of shipment for the store. How long will it take six students to process 100 cases of shipment?	**(Inverse)** $\frac{4}{6} = \frac{3.5}{x}$ $\frac{4}{6} = \frac{x}{3.5}$ Invert one ratio $6x = 4(3.5) \rightarrow 6x = 14 \rightarrow x = 2.33$ hours
3. The Widget Company production supervisor told management that it takes six employees two hours to produce 90 widgets. How long would it take 10 employees to produce 90 widgets?	**(Inverse)** $\frac{6}{10} = \frac{2}{x}$ $\frac{6}{10} = \frac{x}{2}$ Invert one ratio $10x = 6(2) \rightarrow 10x = 12 \rightarrow x = 1.2$ hours

Problems	Related, Generic Math Concepts Solutions
4. If you need 5 pounds of chicken to serve 20 people, how many pounds will you need to serve 50 people?	**(Direct)** $\frac{5 \text{ pounds}}{20 \text{ people}} = \frac{x \text{ pounds}}{50 \text{ people}} \rightarrow 20x = 5(50) \rightarrow 20x = 250$ $x = 12.5$ pounds
5. The pressure of a gas and its corresponding volume are inversely proportional. If the pressure of 0.24 m3 is 0.5 atm, what would the pressure be of 0.060 m3 of the same gas at the same temperature?	**(Inverse)** $\frac{0.24 \text{ m}^3}{0.060 \text{ m}^3} = \frac{0.5 \text{ atm}}{x \text{ atm}}$ (Invert one ratio since it is an inverse proportion.) $\frac{0.24 \text{ m}^3}{0.060 \text{ m}^3} = \frac{x \text{ atm}}{0.5 \text{ atm}}$ $.24 \times 0.5 = .060x \rightarrow x = 2$ atm
6. If it takes 26 lbs. of metal to make 10 castings, how many pounds of metal will be needed to make 14 castings?	**(Direct)** $\frac{10 \text{ castings}}{14 \text{ castings}} = \frac{26 \text{ lbs.}}{x \text{ lbs.}}$ $10x = 26(14) \rightarrow x = 36.4$lbs.

Problems	PA Core Math Look Solutions
7. Given that y and x are **directly** proportional and y = 2 when x = 5, find the value of y when x = 15.	**(Direct)** $\frac{5}{15} = \frac{2}{y}$ $\uparrow 5y = 2(15)$ $\uparrow y = 6$
8. Given that y and x are **inversely** proportional and y = 2 when x = 5, find the value of y when x = 15.	**(Inverse)** $\frac{5}{15} = \frac{y}{2} \rightarrow 15y = 2(5) \rightarrow y = 0.667$
9. If one rabbit can chew 20 carrots in 15 hours, how long will it take 5 rabbits to chew the same number of carrots?	**(Inverse)** $\frac{1}{5} = \frac{x}{15} \rightarrow 5x = 1(15) \rightarrow x = 3$ hours