Math Lesson Plan
Eligible Content M11.A.1.1.1
Find the square root of an integer to the nearest tenth using either a calculator or estimation.

Objective: Use a calculator or estimation to find square roots to the nearest tenth.

Introduction

Tell students that the day's lesson will be about learning how to use a calculator or estimation to find the square root to the nearest tenth. This method is used when an exact answer is not needed.

Skill Review

M11.A.1.3.2 Compare and/or order any real numbers (rational and irrational may be mixed).

Definitions

<u>Square Root</u>
The number n is a square root of b if $n^2 = b$.

Lesson

The square root of a positive integer can be found on a calculator and rounded to a certain decimal place or it can be estimated to a certain decimal place.

If you are using a calculator that has a square root button, use the square root function on your calculator and round the answer to one decimal place.

If your calculator does not have a square root button, then you are going to have to estimate the square root. This procedure will take more time and effort.
Procedure:

1 – Find two perfect squares that the number is between.
2 – Divide the number by one of those square roots.
3 – Take the average of the result and the number you divided by.
4 – Use the result from step 3 to repeat steps 2 and 3 until you have reached an accurate approximation.

Examples

Find $\sqrt{28}$.

<u>Work</u> Using a calculator: enter $\sqrt{28}$ using the square root function on your calculator.
You will see 5.291502622 in the window.
Round this to 5.3

If your calculator does not have a square root button, use the procedure listed above.
Step 1: $\sqrt{28}$ is between $\sqrt{25} = 5$ and $\sqrt{36} = 6$.
Step 2: $28 \div 5 = 5.6$
Step 3: The average of 5.6 and 5 is $(5.6 + 5) \div 2 = 5.3$
Step 4: $28 \div 5.3 = 5.283018868$
$(5.283018868 + 5.3) \div 2 = 5.291509434 \approx 5.3$

Find the length of the side of a square, to the nearest tenth, if its area is 68 square inches.

<u>Work</u> Area of square = Side squared

$A = s^2$

$68 = s^2$

$s = \sqrt{68}$

$s = 8.2$ (using a calculator)

The length of the side of the square is 8.2 inches.

Practice

1. Find $\sqrt{54}$

   (A) 7.3

   (B) 7.4

   (C) 27

   (D) 2916

2. The formula for the area of a circle is A = πr^2. You want to put a circular carpet that has an area of 30 square feet on your dining room floor. What is the radius of the carpet?

Practice Answers

1. A

Using the calculator: The $\sqrt{54}$ is 7.348469228 which rounds to 7.3

2. To find the radius of the carpet, use the formula for the area of a circle. Substitute the area of the carpet for A and solve the equation for r.

A = πr^2
30 = πr^2
Divide both sides by π.
r^2 = 9.549296586
Take the square root of both sides.
r = 3.1

The length of the radius of the carpet is 3.1 feet.

Math Lesson Plan
Eligible Content M11.A.1.1.2
Express numbers and/or simplify expressions using scientific notation (including numbers less than 1)

Objective: Read and write numbers in scientific notation

Introduction

Numbers such as 1200, 528, 17.3 and 0.0045 are written in decimal form. Scientific notation uses powers of 10 to express decimal numbers. For example, 2.83×10^4 is a number written in scientific notation.

Skill Review

None

Definitions

<u>Scientific Notation</u>
A number written in the form $c \times {}^{10n}$, where $1 \leq c \leq 10$ and n is an integer.

Lesson

When given a number in scientific notation, it can be rewritten in decimal form using the following steps.

		2.83×10^4
1.	Rewrite the number at the beginning of the problem	2.83
2.	Move decimal the right (when n is positive) or left (when n is negative) n spaces. Add zeros when necessary.	28,300 (add two zeros)

When given a number in decimal form, it can be rewritten in scientific notation using the following steps.

		0.0000045
1.	Move the decimal place until the number is greater than 1,	4.5
2.	Count the number of spaces the decimal moved	6
3.	Determine if the number is greater or less than 1 (if greater use positive exponent, less use negative)	less than, negative exponent
4.	Rewrite in scientific notation	4.5×10^{-6}

To multiply and divide numbers written in scientific notation.

1. Enter first number in the calculator using the EE key.

2. Enter appropriate operation

3. Enter second number in the calculator using the EE key

Use a calculator to multiply 7.48×10^{-7} by 2.4×10^{9}

KEYSTROKES

7.48
EE
−7
×
2.4
EE
9
ENTER

Answer 1.7952×10^{3} or 1795.2

Examples

Write the following number in decimal form 1.23×10^{-3}

Work 0.00123

Write the following number in scientific notation 2,300,000,000

Work 2.3×10^{9}

Perform the indicated operation. $\dfrac{8.9 \times 10^{4}}{6.4 \times 10^{-3}}$

Use calculator

13906250 or approximately 1.3906×10^{7}

Practice

1. The diameter of a red blood cell, in inches, is 3×10^4. This expression is the same as which of the following numbers

   A. 0.00003
   B. 0.0003
   C. 3,000
   D. 30,000

2. In 1867, the United States purchased Alaska from Russia for $7.2 million. The total area of Alaska is about 5.9 × 105 square miles. What was the price per square mile?

Practice Answers

1. Negative exponent, move decimal 4 places to the left.

   Answer: B

2. Price per square mile = $\dfrac{\text{Total price}}{\text{Number of Square Miles}}$

   $= \dfrac{7.2 \times 10^6}{5.9 \times 10^5}$ 7,200,000 for the total price

   $= 1.22 \times 101$ (convert this number to decimal form)

   = \$12.20 per square mile (must list correct monetary unit, not \$12.2)

Math Lesson Plan
Eligible Content M11.A.1.1.3
Simplify square roots.

Objective: Use the rules to simplify square roots.

Introduction

Tell students that the day's lesson will be about learning how to simplify square roots to find the exact value in simplest form. This method is used when we want to work with the exact value and not an estimated value, which is what we get when we use a calculator.

Skill Review

M11.A.1.1.1 Find the square root of an integer to the nearest tenth using a calculator.

Definitions

<u>Radical Sign</u>
A radical sign ($\sqrt{}$) is used to indicate a root.

<u>Radicand</u>
The number under the radical sign is the radicand. In $\sqrt{a}$, a is the radicand.

Lesson

The square root of a positive integer is in simplest form when no integral factor of the radicand is a perfect square, other than 1. The best way to do this is to find the prime factorization of the radicand first.

Procedure:

1 – Find the prime factors of the radicand.
2 – If there are any pairs (2 of a kind) of prime factors, cross of each pair under the radical sign and write one of those factors in front of the radical sign. You are actually taking the square root of that product by doing this.
3 – Multiply all factors that are in front of the radical sign and keep that number in the front. Multiply all numbers under the radical sign and keep that number under the sign.

Examples

Simplify $\sqrt{72}$

Work

$\sqrt{72}$

Step 1: $\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$

Step 2: $2 \cdot 3\sqrt{2}$

Step 3: $6\sqrt{2}$

Find the length of the side of a square whose area is equal to the area of a 3 – foot by 15 – foot rectangle.

Work

Area of square = Area of rectangle

$s^2 = lw$

$s^2 = 15 \cdot 3$

$s^2 = 45$

$s = \sqrt{45}$

$s = \sqrt{3 \cdot 3 \cdot 5}$

$s = 3\sqrt{5}$ inches

The length of the side of the square is $3\sqrt{5}$ inches.

Practice

1. Simplify $\sqrt{147}$

   (A) 12.12

   (B) $7\sqrt{3}$

   (C) $3\sqrt{7}$

   (D) $49\sqrt{3}$

2. A rectangle has sides that measure 16 cm and 12 cm. What is the length of the side of a square that has the same area as the rectangle?

Practice Answers

1. B

The prime factors of 147 are 3, 7 and 7. Cross off the two sevens and write one seven on the outside. The factor of 3 remains on the inside. Thus $\sqrt{147} = \sqrt{3\cdot 7\cdot 7} = 7\sqrt{3}$.

2. To find the length of the side of the square set the area of the square equal to the area of the rectangle and solve this equation for s.

$$s^2 = lw$$
$$s^2 = 16\cdot 12$$
$$s^2 = 192$$
$$s = \sqrt{192}$$
$$s = \sqrt{2\cdot 2\cdot 3\cdot 4\cdot 4}$$
$$s = 2\cdot 4\sqrt{3}$$
$$s = 8\sqrt{3}$$

The length of the side of the square is $8\sqrt{3}$ cm.

Math Lesson Plan
Eligible Content M11.A.1.2.1
Find the Greatest Common Factor (GCF) and/or the Least Common Multiple (LCM) for sets of monomials

Objective: Use factors to find the GCF and LCM for monomial terms.

Introduction

Tell students that the day's lesson will be about learning how to find a Greatest Common Factor (GCF) and/or a Least Common Multiple (LCM). A GCF is used to factor algebraic expressions while an LCM is needed when you are adding or subtracting algebraic fractions.

Skill Review

You need to know how to find the prime factorization of numbers and how to write an exponent in factored form.

Definitions

Monomial
A monomial is a real number, a variable, or the product or quotient of a real number and variables. Examples of monomials are 5x, - 4, and y^2.

Greatest Common Factor
The greatest common factor (GCF) of two or more monomials is the largest number or expression that divides evenly into each monomial. The GCF of 4, 8, and 16 is 4.

Least Common Multiple
The least common multiple (LCM) of two or more monomials is the smallest number or expression that each of the given monomials divides into evenly. The LCM of 6 and 15 is 30.

Lesson

The GCF is used when you need to find the largest number or expression that will divide evenly into a group of monomials. When you find the GCF, it can be used to factor an expression.

Procedure:
1 – Write the prime factorization of the real number portion of each monomial.
2 – Write each variable in factored form.
3 – Multiply all common factors to get the GCF.

The LCM is used when you need to find the smallest number or expression that each monomial will divide into evenly. This will be needed anytime you are adding or subtraction fractions that do not have the same denominator.

Procedure:

1 – Write the prime factorization of the real number portion of each monomial.
2 – Write each variable in factored form.
3 – To find the LCM you must write the largest amount of each factor for every different number or variable and multiply these values together.

Examples

Find the GCF of $24x^2$ and 6xy

Work

Factors of $24x^2$ are: $2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$
Factors of 6xy are : $2 \cdot 3 \cdot x \cdot y$
Common factors are: $2 \cdot 3 \cdot x$
GCF is 6x

Find the LCM of $24x^2$ and 6xy

Work

Factors of $24x^2$ are: $2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$
Factors of 6xy are : $2 \cdot 3 \cdot x \cdot y$
Largest number of each factor: $2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y$
LCM is $24x^2y$

If you are asked to simplify the following subtraction problem $\frac{x}{8b} - \frac{y}{6c}$, what would your lowest common denominator be?

Work

Factors of 8b are: $2 \cdot 2 \cdot 2 \cdot b$
Factors of 6c are: $2 \cdot 3 \cdot c$
Largest number of each factor: $2 \cdot 2 \cdot 2 \cdot 3 \cdot b \cdot c$
LCM is 24bc

The lowest common denominator would be 24bc.

Practice

1. Find the GCF of $35x^2y$ and $42x^2yz$

   (A) $42x^2yz$

   (B) $210x^2yz$

   (C) $7x^2y$

   (D) $35x^2y$

2. In order to add $\frac{2}{12bc} + \frac{3}{8ac}$ you have to find a lowest common denominator. What would the lowest common denominator be?

Practice Answers

1. C

   Factors of $35x^2y$ are: $5 \cdot 7 \cdot x \cdot x \cdot y$
   Factors of $42x^2yz$ are: $2 \cdot 3 \cdot 7 \cdot x \cdot x \cdot y \cdot z$
   Common factors are: $7 \cdot x \cdot x \cdot y$
   GCF is $7x^2y$

2. To find the lowest common denominator you will have to find the LCM.

   Factors of 12bc are: $2 \cdot 2 \cdot 3 \cdot b \cdot c$
   Factors of 8ac are: $2 \cdot 2 \cdot 2 \cdot a \cdot c$
   Largest number of each factor: $2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot c$
   LCM is 24abc

   The lowest common denominator of 12bc and 8ac is 24abc.

Math Lesson Plan
Eligible Content M11.A.1.3.1
Locate/identify irrational numbers at the approximate location on a number line.

Objective: Graph irrational numbers on a number line.

Introduction

A *rational number* is any number that can be written as a ratio of two integers. In other words, a number is rational if we can write it as a fraction where the numerator and denominator are both integers.
The term "rational" comes from the word "ratio," because the rational numbers are the ones that can be written in the ratio form p/q where p and q are integers. Irrational, then, means all the numbers that are not rational.

Skill Review

M11.A.1.1.1 Calculate square roots

Definitions

Integer
Integers are the whole numbers, negative whole numbers, and zero.

Rational Number
A number that can be written as a ratio of two integers (fraction). These numbers include the integers, fractions, terminating decimals, and repeating decimals.

Irrational Number
Numbers that can be written as decimals, but not as fractions.

Lesson

An irrational number is any real number that is not rational. By "real" number I mean, loosely, a number that we can conceive of in this world, one with no square roots of negative numbers.

An example of an irrational numbers is π, which does not have an exact decimal equivalent, although 3.1415926 is good enough for many applications. Other common examples are square root values of numbers that are not perfect squares. The square root of 2 is another irrational number that cannot be written as a fraction. Not all square roots are irrational numbers. If the square root of a numbers is an integer, a terminating decimal, or a repeating decimal, then the number is defined as rational.

To determine the approximate value of the square root of 2, use a calculator.

$\sqrt{2} \approx 1.414$

These numbers can be placed on a number line by using their approximate decimal value. The value $\sqrt{2}$ would be placed at almost half the distance between +1 and +2.

Examples

Determine if the following numbers are rational or irrational.

1. $\sqrt{27}$

2. $-\sqrt{19}$

3. π

4. $-\sqrt{2.25}$

5. 0.8

Work:

1. irrational because $\sqrt{27} \approx 5.19615$ (does not terminate or repeat)

2. irrational because $-\sqrt{19} \approx -4.35889$

3. irrational, π is defined as an irrational number

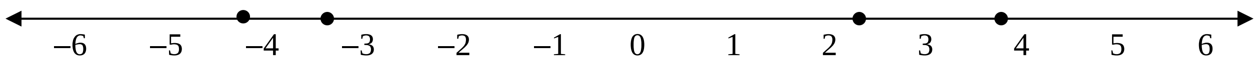
4. rational, because $-\sqrt{2.25} = -1.5$ (the answer is rational)

5. rational, because 0.8 is a terminating decimal

Place the following numbers on a number line.

π, $\sqrt{5}$, $-\sqrt{11}$, $-\sqrt{18}$

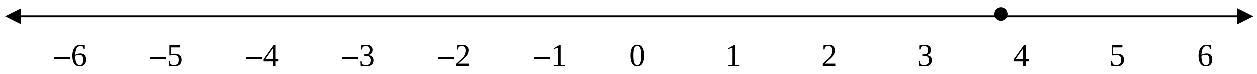
Work:



$\sqrt{15} \approx 3.87$
$\sqrt{5} \approx 2.24$
$-\sqrt{11} \approx -3.32$
$-\sqrt{18} \approx -4.24$

Practice

1. Which irrational number would be located at the position indicated on the number line?



A. $-\sqrt{11}$
B. $-\sqrt{2}$
C. π
D. $\sqrt{15}$

2. Graph the following numbers on a number line.

$\sqrt{27}$, π, $\sqrt{8}$, $-\sqrt{21}$, $-\sqrt{30}$, $\sqrt{14}$

Practice Answers

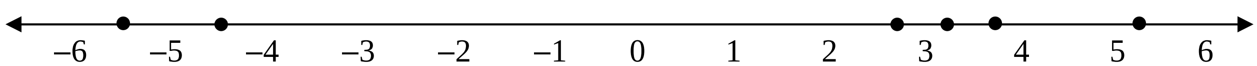
1. A and B are not potential answers because the point is on the positive side of the number line.

$\pi \approx 3.14$
$\sqrt{15} \approx 3.87$

The answer is D because the point is located closer to the 4 on the number line.

2. $-\sqrt{30} \approx -5.48$ Points graphed below

$-\sqrt{21} \approx 4.58$
$\sqrt{8} \approx 2.83$
$\pi \approx 3.14$
$\sqrt{14} \approx 3.74$
$\sqrt{27} \approx 5.20$



Math Lesson Plan
Eligible Content M11.A.1.3.2
Compare and/or order any real numbers.

Objective: Compare real numbers

Introduction

For all real numbers a and b, exactly one of the following must be true.

(1) $a < b$
(2) $a = b$
(3) $a > b$

This lesson will be used to learn how to compare real numbers to determine which is greater or to place numbers in order from least to greatest.

Skill Review

M11.A.1.3.2 Irrational numbers

Definitions

Real Number

The set of numbers that includes all rational and irrational numbers.

Lesson

Real numbers can be represented in many forms. Real numbers can be whole numbers, integers, fractions, decimals, or even the root of a number. To be able to compare numbers in different forms, it is easiest to convert all numbers to a standard form, decimal being the form of choice. Once all numbers have been placed in decimal form, numbers can be placed in order from least to greatest

Examples

Use <, =, or > to compare each pair of real numbers.

1. $0.451451451 \ldots\ldots\ldots \frac{451}{999}$

Work $\frac{451}{999} \approx .4514514515$ the numbers are =

2. $\sqrt{108}$.............$10.\overline{35}$

Work $\sqrt{108} \approx 10.3923$

$10.3923 > 10.\overline{35}$

Practice

1. Place the following numbers in order from least to greatest.

$$-2.6,\ \sqrt{5},\ 2.24,\ -\sqrt{10},\ -2\frac{2}{3},\ 2.\overline{23},\ -\frac{215}{98},\ -2.216215214,\ -\sqrt{5}$$

A. $2.24,\ \sqrt{5},\ 2.\overline{23},\ -\frac{215}{98},\ -2.216215214,\ -\sqrt{5},\ -2.6,\ -2\frac{2}{3},\ -\sqrt{10}$

B. $-\sqrt{10},\ -2\frac{2}{3},\ -2.6,\ -\sqrt{5},\ -2.216215214,\ -\frac{215}{98},\ \sqrt{5},\ 2.\overline{23},\ 2.24$

C. $-\sqrt{10},\ -2\frac{2}{3},\ -2.6,\ -\sqrt{5},\ -2.216215214,\ -\frac{215}{98},\ 2.\overline{23},\ \sqrt{5},\ 2.24$

D. $-\sqrt{10},\ -2\frac{2}{3},\ -\sqrt{5},\ -2.6,\ -2.216215214,\ -\frac{215}{98},\ \sqrt{5},\ 2.\overline{23},\ 2.24$

2. Explain why $\sqrt{\frac{25}{36}}$ is not between $\frac{7}{8}$ and 0.9.

Practice Answers

1. Convert all numbers to decimals and order from least to greatest.

   Answer: C

2. The value of $\sqrt{\frac{25}{36}}$ is 0.83333333

   The value of $\frac{7}{8}$ is 0.875

   The value of the third value is 0.9

   0.83333333 is not between 0.875 and 0.9. 0.83333333 < 0.875

Math Lesson Plan
Eligible Content M11.A.2.1.1.
Solve problems using operations with rational numbers including rates and percents

Objective: Use operations with rational numbers including rates and percents to solve problems.

Introduction:
Tell students the lesson will be about percent and ratios. Ask students where percents are used in the workplace as well as in everyday life. There are many examples where percents are used such as discounts and markups in a business they may own or manage. In everyday life we are constantly bombarded with percentages as they apply to paycheck deductions and financial transactions such as car loans, mortgages and savings accounts.

Skill Review:

M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation.)
M11.A.3.2.1	Use estimation to solve problems.

Definitions:

<u>Percentage</u>
The rate or number of parts per hundred parts

<u>Proportion</u>
An equation or statement that two ratios are equal

<u>Ratio</u>
The comparison of two quantities by division

Lesson:

Ask students how to take the percent of a number.
Make sure they know that to take a percent of a number they need to <u>multiply</u>.
Also make sure that they know they need to move the decimal point two places to the left in the percent before they multiply. (An optional way to do percent problems is to use the proportion: %/100 = is/of.)
When working with proportions (two equal fractions) with one unknown number the students will need to multiply and then divide. (See examples)

Examples:

1. Find the sale price of an item that is advertised at 25% off if the original price is $29.95.

Work $29.95 x .25 = $7.49

$29.95 –$ 7.49 = $22.46

The sale price is $22.46

Optional method to find percent: 25/100 =x/$29.95

100x = 25 x $29.95

100x = $749

x =$ 749 ÷ 100

X = $7.49

Note: When doing math problems students should think about whether their answer is reasonable.

2. When John exercises, he walks at an average rate of 1.75 miles every half hour. At this rate, how many miles can John walk in 3 hours?

Work $\frac{1.75 \text{ miles}}{\frac{1}{2} \text{ hour}} = \frac{y \text{ miles}}{3 \text{ hours}}$

½ y = (1.75) x 3

½ y = 5.25

Y = 5.25 ÷ ½

Note: A fraction can be changed to a decimal by dividing (1÷2 = .5). A decimal might be easier to work with than a fraction.

Y = 10.5

John will walk 10.5 miles in 3 hours.

Note: Make sure you stress to students that they label their answers with units such as miles.

Practice:

1. If you want to markup something you are selling by 30% and the item cost you $14.49, what will you charge the customer?
   (A) $19.32

   (B) $14.79

   (C) $18.84

   (D) $ 19.49

2. If you are able to do a certain job in 2 ½ hours, how many of those jobs can you complete in a day?

Practice Answers:

1. C

Thirty percent of \$14.49 can be found by multiplying .30 times \$14.49.

\$14.49 x .30 = \$4.35 Note: It is important to emphasize that it is necessary to:

a. Move the decimal two places from 30.% to .30

b. 4.347 will need to be rounded to the nearest cent.

\$14.49 +\$4.35 = \$18.84

You will charge the customer \$18.84.

2. Set up a proportion (two equal fractions.)

$$\frac{1 \text{ job}}{2\tfrac{1}{2} \text{ hours}} = \frac{y \text{ jobs}}{8 \text{ hours}}$$

Note: We are using the fact that a "normal" work day is 8 hours. The letter y represents the number of jobs.

(2 ½ hours) y = (1 job) times 8 hours

(2 ½) y = 8

Y = 8 ÷ 2 ½ or y = 8 ÷ 2.5

Y = 3.2 jobs

Note: Since this is not a whole number this can lead to a discussion about whether a fourth job should be started. For example, in cosmetology a customer cannot get "part" of a perm but in cabinetmaking perhaps a fourth project can be started and finished the next day.

Math Lesson Plan
Eligible Content M11.A.2.1.2
Solve problems using direct and inverse proportions

Objective: Use either direct or inverse proportions to solve problems.

Introduction

Tell students that the day's lesson will be about learning how to determine the difference between a direct and inverse proportion and then to be able to solve the proportion.

Skill Review

M11.A.2.1.1
M11.A.3.1.1

Definitions

<u>Direct Proportion</u>
When a relationship can be expressed by an equation of the form y = kx, where k $\neq$ 0, then y is said to be directly proportional to x. The number k is called the constant of variation.

<u>Inverse Proportion</u>
When a relationship can be expressed by an equation of the form $y = \frac{k}{x}$, where k $\neq$ 0, then y is said to be inversely proportional to x. The number k is called the constant of variation.

Lesson

A direct proportion occurs when you are comparing two things that change in the same direction. For example, when you purchase hamburger by the pound; the more hamburger (lbs) you buy, the more money it costs.

Procedure:

1 – Write the proportion $\frac{x_1}{y_1} = \frac{x_2}{y_2}$
2 – Substitute all known values.
3 – Solve for the unknown value by cross multiplying.

An inverse proportion occurs when you are comparing two things that change in opposite directions. For example, the bigger a gear gets, the slower it turns.

Procedure:

1 – Write the proportion $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

2 – Substitute all known values.

3 – Solve for the unknown value by cross multiplying.

Examples

Y is directly proportional to x, if y = 4 when x = -2, find x when y = 6.

<u>Work</u>

Let y_1 = 4, x_1 = -2, y_2 = 6 and x_2 is your unknown.

Substitute into the proportion from the procedure list: $\frac{-2}{4} = \frac{x_2}{6}$

Cross multiply: $4x_2$ = (-2)(6)

Solve: $4x_2$ = - 12

x_2 = - 3

Gear A drives Gear B. Gear A has 60 teeth and speed 5400 rpm. Gear B has 45 teeth. Find the speed of Gear B.

<u>Work</u>

The speed of gears is inversely proportional.
Let x_1 = 60, y_1 = 5400, x_2 = 45, and y_2 is the unknown.

Substitute into the proportion from the procedure list: $\frac{60}{45} = \frac{y_2}{5400}$

Cross multiply: $45y_2$ = (60)(5400)

Solve: $45y_2$ = 324000

y_2 = 7200

Gear B turns at a speed of 7200 rpm's.

Practice

1. When y = 4, x = 16; and y varies inversely as x. Find y when x = 8.

   (A) 2

   (B) -2

   (C) -8

   (D) 8

2. If y varies directly with x and x is doubled, what happens to y?

Practice Answers

1. D

Let $y_1 = 4$, $x_1 = 16$, $x_2 = 8$ and y_2 is your unknown.

Substitute into the proportion from the procedure list: $\frac{16}{8} = \frac{y_2}{4}$

Cross multiply: $8y_2 = (16)(4)$

Solve: $8y_2 = 64$

$y_2 = 8$

2. Y varies directly with x means that x and y are directly proportional. Therefore both values will change at the same rate. Thus, if x is doubled, y will also double.

Math Lesson Plan
Eligible Content M11.A.2.1.3
Identify and/or use proportional relationships in problem solving settings

Objective: Use proportional relationships in problem solving settings.

Introduction

Murals are often created by enlarging an original drawing. Different methods are used to make sure that all parts of the enlargement are in proportion to the original drawing. One common method used in mural making is to enlarge each piece of art by the same percentage. If a drawing is enlarged 300% of its original size, then the length and width of the enlargement will each be three times the size of the original. Similar methods are used in construction when using blueprints, models, or scale diagrams and then multiplying by the appropriate scale factor when building the actual object. The most common examples of scale diagrams are scale measurements on maps and common kitchen recipes.

Skill Review

M11.A.2.1.1, M11.A.2.1.2 and M11.A.2.1.3. Solve problems using operations with rational numbers, direct and inverse proportions.

Definitions

A **proportion** is an equation that states that two ratios are equal. In the proportion; $\frac{a}{b} = \frac{c}{d}$, the numbers a and d are the **extremes** of the proportion and the numbers b and c are the **means** of the proportion.

Reciprocal Property

If two ratios are equal, their reciprocals are also equal.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

Cross Product Property

The product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$, then ad = bc.

Solving for the variable in a proportion is called **solving the proportion**.

Lesson

A proportion is a mathematical sentence that states that two ratios are equivalent.

Express the proportion by using two fractions such as $\frac{4}{6} = \frac{10}{15}$; the proportion is read "4 is to 6 as 10 is to 15"

Use the 'cross product property' to show the product of the extremes is equal to the product of the means. $4 \bullet 15 = 6 \bullet 10 \longrightarrow$ both products equal 60.

If any three of the four terms of a proportion are known quantities, $\frac{3}{7} = \frac{x}{56}$ determine the fourth (unknown) term by:

(1) Considering the proportion as a fractional equation and solving it.

$$\frac{3}{7} = \frac{x}{56}$$

$(56)\left(\frac{3}{7} = x\right)$ Multiply both sides by 56 to solve for x.

24 = x Simplify.

(2) Or transforming the proportion to a simple equivalent equation by writing the product of the extremes equal to the product of the extremes (Cross Product Property) and then solving it.

$$\frac{3}{7} = \frac{x}{56}$$

$(56)(3) = 7x$ Cross Product Property

$\frac{(56)(3)}{7} = x$ Divide both sides by 7 to solve for x.

24 = x Simplify.

Perform the following examples and give the assignment.

Examples

1. Solve the following proportion: $\frac{4}{x} = \frac{5}{7}$

$\frac{4}{x} = \frac{5}{7}$ *Write the original proportion.*

$\frac{x}{4} = \frac{7}{5}$ *Use the reciprocal property to rewrite the proportion.*

$x = 4\left(\frac{7}{5}\right)$ *Multiply each side by 4 in order to isolate the variable.*

$x = \frac{28}{5} = 5.6$ *Simplify.*

2.

The above map shows the approximate location of some of the U.S. state capitals. On the map, 1 inch represents 656 miles.

A) The distance between Albany, New York, and Nashville, Tennessee, is approximately 852.8 miles. Write a proportion to find the map distance between these two capitals. Then solve your proportion to find the map distance if you were to measure with a ruler.

B) On this map, the distance between Denver, Colorado, and Carson City, Nevada is $1\frac{1}{8}$ inches. Write and solve a proportion to find the actual distance between these two capitals.

Solution.

A) $\frac{mapdis\tan ce}{actualdis\tan ce} = \frac{1in}{656mi} = \frac{x}{852.8mi}$ *Write the original proportion.*

$x = 852.8mi\left(\frac{1in}{656mi}\right)$ *Multiply each side by 852.8 mi. in order to isolate the variable.*

x = 1.3 in *Cancel appropriate units and Simplify.*

* *Since most rulers do not come marked in 'tenths' of inches any measurement that you would make would be very close to* $1\frac{1}{4}in$.

B) $\frac{1in}{656mi} = \frac{1\frac{1}{8}in}{x}$ *Write the original proportion.*

$\frac{656mi}{1in} = \frac{x}{1\frac{1}{8}in}$ *Use the reciprocal property to rewrite the proportion.*

$x = 1.125in\left(\frac{656mi}{1in}\right)$ *Multiply each side by 1.125 in(1* $\frac{1}{8}$ *in = 1.125 in) in order to isolate the variable.*

x = 738 mi *Cancel appropriate units and Simplify.*

Practice

1. Mr. Morris is making a dollhouse with toy furniture. He uses 0.5 inches to represent 1 foot. What would be the dimensions of a toy table representing a table 6 feet long, 3 feet wide, and 30 inches high?

   A) 3 inches long, 1.25 inches wide and 1.5 inches high.
   B) 3 inches long, 1.5 inches wide and 1.25 inches high.
   C) 12 inches long, 6 inches wide and 5 inches high.
   D) 3 inches long, 1.5 inches wide and 15 inches high.

**** Special Note – The above problem is taken directly from the PDE website.***

2. A lemonade recipe calls for ¾ cup of lemon juice for 2 quarts of lemonade. How much lemon juice should you use for an 8 quart jug of lemonade?

Practice Answers

1. Write the ratio described in the problem

$$\frac{dollhouse\ \dim ension}{actualobject\ \dim ension} = \frac{0.5in}{1ft}$$

Since the dollhouse objects are proportional to the actual house objects all corresponding dimensions will also be proportional. Therefore

Length	Width	Height
$\frac{0.5in}{1ft} = \frac{l}{6ft}$	and $\frac{0.5in}{1ft} = \frac{w}{3ft}$	and $\frac{0.5in}{1ft} = \frac{h}{30in}$

Solve each proportion separately.

$l = 6ft\left(\frac{0.5in}{1ft}\right) = 3in$, $w = 3ft\left(\frac{0.5in}{1ft}\right) = 1.5in$, substitute 30in = 2.5ft

$$h = 2.5ft\left(\frac{0.5in}{1ft}\right) = 1.25in$$

l = length = 3 in w = width = 1.5 in h = height = 1.25 in

B) is the answer.

2. Write the ratio described in the problem

$$\frac{\frac{3}{4} cuplemonjuice}{2qtslemonade} = \frac{x}{8qts}$$

Solve the proportion for the variable; x = the necessary amount of lemon juice.

$$x = 8qts\left(\frac{\frac{3}{4}cups}{2qts}\right)$$

Cancel the appropriate units and Simplify

x = 3 cups of lemon juice needed to make 8 qts of lemonade

Math Lesson Plan
Eligible Content M11.A.2.2.1
Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers – exponents should not exceed power of 10).

Objective: To simplify and/or evaluate expressions involving exponents, roots, and absolute values using the rules for exponents.

Introduction

This lesson will involve many aspects of the real numbers and operations. Students will need to know how and when to use the rules of exponents, roots, and absolute values. This process will be used anytime you are simplifying an expression or solving an equation that involves exponents.

Skill Review

M11.A.1.1.1
M11.A.1.13
M11.A.3.1.1

Definitions

Exponent
An exponent tells how many times a number or letter, known as the base, is used as a factor.

Absolute Value
The distance the number is away from zero on the number line is called the absolute value. Since absolute value is a distance, it is always a positive value.

Lesson

Review the Rules for Exponents

1. Multiplying when the base is the same:

   $a^{x} \cdot a^{y} = a^{x+y}$

2. Raising an exponent to a power:

   $(a^{x})^{y} = a^{xy}$

3. Power of a product:

   $(ab)^{x} = a^{x} b^{x}$

4. Negative exponent:

$$a^{-x} = \frac{1}{a^x} \qquad \text{or} \qquad \frac{1}{b^{-x}} = b^x$$

To simplify an expression:

1. Numbers with exponents must be multiplied out
2. Each variable is written only once
3. All exponents must be positive

To evaluate an expression:

1. Substitute the given values into the expression where indicated.
2. Simplify according to the rules of exponents

Examples

Simplify: $2\left(\frac{1}{4}\right)^2$

Work $2\left(\frac{1}{4}\right)^2 = 2\left(\frac{1^2}{4^2}\right) = 2\left(\frac{1}{16}\right) = \frac{2 \bullet 1}{16} = \frac{2}{16} = \frac{1}{8}$

Simplify: $\sqrt{\frac{5^5}{5^3}}$

Work $\sqrt{\frac{5^5}{5^3}} = \sqrt{5^{5-3}} = \sqrt{5^2} = 5$

Simplify: $\left| -x^2 \bullet y^{-2} \right|$ *where* $x = 6$ *and* $y = -3$

<u>Work</u>

$\lvert -6^2 \bullet (-3)^{-2} \rvert$	substitute into the original equation
$\left| -6^2 \bullet \dfrac{1}{(-3)^2} \right|$	simplify negative exponent
$\left| -36 \bullet \dfrac{1}{9} \right|$	simplify exponents
$\lvert -4 \rvert$	multiply numbers
4	take absolute value of number

Practice

1. Evaluate the expression $\left(x\sqrt{x}\right)^2$ where x = 3

(A) $\left(3\sqrt{3}\right)^2$

(B) 81

(C) 9

(D) 27

2. Simplify $9\left(\frac{2}{5}\right)^4$

Practice Answers

1. D

$\left(3\sqrt{3}\right)^2$	substitution
$3^2 \bullet \left(\sqrt{3}\right)^2$	rules of exponents – power of a product
$9 \bullet 3$	rules of exponents
27	multiplication

2. $$9\left(\frac{2}{5}\right)^4 = 9\left(\frac{2^4}{5^4}\right) = 9\left(\frac{16}{625}\right) = \frac{9 \bullet 16}{625} = \frac{144}{625}$$

Math Lesson Plan
Eligible Content M11.A.2.2.2
Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 \cdot x^7 = x^{13}$), power of powers (e.g. $(x^6)^7 = x^{42}$) and powers of products (e.g. $(2x^2)^3 = 8x^6$), (positive exponents only).

Objective: To be able to simplify and/or evaluate expressions involving exponents using the rules for exponents.

Introduction
Tell students that the day's lesson will be about learning how to simplify expressions that involve exponents. They will need to use the rules of exponents to do this. This process will be used anytime you are simplifying an expression or solving an equation that involves exponents.

Skill Review

M11.A.2.2.1
M11.A.3.1.1

Definitions

<u>Exponent</u>
An exponent tells how many times a number or letter, known as the base, is used as a factor.

Lesson

RULES FOR EXPONENTS

1 – Multiplying when the base is the same:

$$a^x \cdot a^y = a^{x+y}$$

2 – Raising an exponent to a power:

$$(a^x)^y = a^{xy}$$

3 – Power of a product:

$$(ab)^x = a^x b^x$$

Each of these rules can be shown by expanding on the definition of an exponent. If you forget the rule, rely on the definition to help simplify.

1 - $x^2 \cdot x^3 = x \cdot x \bullet x \cdot x \cdot x = x^5 = x^{2+3}$

2 - $(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x \cdot x \bullet x \cdot x \bullet x \cdot x = x^6 = x^{2 \cdot 3}$

3 - $(xy)^3 = (xy)(xy)(xy) = (x \cdot x \cdot x)(y \cdot y \cdot y) = x^3 y^3$

Simplifying an expression means:

1. Numbers with exponents must be multiplied out
2. Each variable is written only once
3. All exponents must be positive

Examples

Simplify: $x \cdot x^2 \cdot x^4$

Work $x \cdot x^2 \cdot x^4 = x^{1+2+4} = x^7$

Simplify: $(3^2)^3$

Work $(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$

Simplify: $(ab)^4$

Work $(ab)^4 = (ab)(ab)(ab)(ab) = (a\,a\,a\,a)(b\,b\,b\,b) = a^4 b^4$

Practice

1. Which of the following is NOT correct?

   (A) $3^{8} = (3^{4})^{2}$

   (B) $3^{4} \cdot 3^{4} = 3^{16}$

   (C) $3^{4} \cdot 3^{4} = 3^{8}$

   (D) $(3 \cdot 3)^{4} = 3^{8}$

2. Explain the difference between $a^{3} \cdot a^{5}$ and $(a^{3})^{5}$.

Practice Answers

1. B

$$3^{4} \cdot 3^{4} = 3^{4+4} = 3^{8}$$

2. According to the rules for exponents $a^{3} \cdot a^{5} = a^{3+5} = a^{8}$ and $(a^{3})^{5} = a^{3 \cdot 5} = a^{15}$

Math Lesson Plan
Eligible Content M11.A.3.1.1
Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).

Objective: Simplify or evaluate expressions where order of operations must be taken into consideration.

Introduction

Tell students that the day's lesson will be about learning how to simplify and/or evaluate expressions. In these problems you will have to remember to work with order of operations, otherwise it would be possible to get more than one correct answer.

Skill Review

None

Definitions

Algebraic expression
An algebraic expression is made up of numbers, variables and symbols for the operations.
Example: 2x + 5

Numerical expression
A numerical expression consists of only numbers and symbols for the operations.
Example: 6 – 8 + 2(6)

Lesson

To simplify an algebraic or numeric expression you must follow the correct order of operations.

Procedure:

1 – Simplify within grouping symbols. These may include, but are not limited to, parentheses, brackets, fraction bars, and absolute value signs.
2 – Simplify all terms containing exponents.
3 – Multiply or divide in order from left to right.
4 – Add or subtract in order from left to right.

You can use the mnemonic PEMDAS to help remember this order.

P = Parentheses or other grouping symbols
E = Exponents
M or D = Multiply or Divide in order from left to right
A or S = Add or Subtract in order from left to right

Examples

Simplify $440 \div (2 + 18)$

Work

1 – Perform the operations inside the parentheses first.

$440 \div 20$

2 – Perform the division.

22

Evaluate the algebraic expression $3a^2 + 4ab - b^2$ for $a = 2$ and $b = -1$

Work

1 –Substitute the values for a and b into the expression.

$3(2)^2 + 4(2)(-1) - (-1)^2$

2 – Simplify using PEMDAS.

There is nothing to simplify inside parentheses.

Exponents: $3(4) + 4(2)(-1) - 1$

Multiply or Divide in order: $12 + (-8) - 1$

Add or Subtract in order: 3

Write a numerical expression for the following phrase and then simplify: twenty-one minus the sum of fifteen and five.

Work

1 – Write the numerical expression.

$21 - (15 + 5)$

2 – Simplify the expression using PEMDAS.

Parentheses: $21 - (20)$

Subtract: 1

Pat thinks that if $y = 5$, the expression $-y^2$ and the expression $(-y)^2$ will result in the same value. Write an explanation to agree or disagree.

Work

Disagree. $-y^2 = -5^2 = -25$ since the exponent must be done first. When we have parentheses in the problem, they have to be done first which changes the problem. $(-y)^2 = (-5)^2 = 25$.

Practice

1. Which expression has a value of 18?

   (A) $3 \cdot 2 + 4$

   (B) $(18 – 10) \div 4 + 15$

   (C) $4 \cdot 2 + 3 - 2$

   (D) $27 – 13 \cdot 2 + 17(6 – 5)$

2. Write a word problem for the numerical expression $3(4 + 3) + 2$. Then simplify the expression.

Practice Answers

1. D

$A = 10$, $B = 17$, $C = 9$ and $D = 18$

$27 - 13 \cdot 2 + 17(6 - 5) = 27 - 13 \cdot 2 + 17(1) = 27 - 26 + 17 = 18$

2. Answers will vary. Sample: Matt bought 4 pairs of blue socks and 3 pairs of black socks at $3.00 a pair. He also bought a hat for $2.00. What was the total cost of his purchase?

$3(4 + 3) + 2 = 3(7) + 2 = 21 + 2 = 23$

The total cost of his purchase was $23.00.

Math Lesson Plan
Eligible Content M11.A.3.2.1
Compute accurately and fluently and make reasonable estimates.

Objective:
Use estimation to solve problems.

Introduction
Tell students that today's lesson will be about learning how to estimate when solving problems. Estimation is used to determine sums, differences, products and quotients when exact numbers are not necessary. Estimation is also used to check whether a solution is reasonable.

Skill Review

None

Definitions

Round – to replace by the nearest multiple of 10, with 5 being increased to the next highest multiple:
12,627 is rounded to 12,630 or 12,600, or 13,000.

Estimate – to calculate approximately the value, amount, magnitude, size, or position of something.

Lesson

Procedure:
1 – Round each number in the problem to the nearest place value desired.

2 – Compute, using the indicated operation

Examples

1. A student is pricing new car stereo systems. One system sells for $1895 and another system sells for $1524. Round each price to the nearest hundred dollars to estimate the difference in price of these systems.

Work

Step 1: Round each number to the nearest hundred dollars as indicated in the problem.

$1895 rounds to $1900
$1524 rounds to $1500

Step 2: Subtract the rounded values to get the estimated difference of $400. (The exact difference is $371).

2. Suppose you scored 89, 97, 100, 75, and 82 on your biology tests. Round each score to the nearest ten and (a) estimate your total score, (b) estimate your average test grade.

Work

Step 1: Round each number to the nearest ten as indicated in the problem.

89 rounds to 90
97 rounds to 100
100 rounds to 100
75 rounds to 80
82 rounds to 90

Step 2: Add the rounded values to get a total score of 450. (Exact total score is 443) Answer to part (a).

Step 3: Divide the estimated total score by 5 (five test grades) to get the estimated average test grade of 90. (Exact average is 88.6). Answer to part (b).

450/5 = 90

Practice

1. At the start of the month, the counter on the copy machine read 6,583. At the end of the month, it read 82,110. The copies cost $1\frac{1}{3}$ cents a piece. What was the approximate total cost of the copies for this month?

   A. $10,000.00

   B. $2,200.00

   C. $1,000.00

   D. $200.00

(Pennsylvania Department of Education)

2. Mrs. Ditters and her daughter went to lunch. Their bill came to $27.29. If a fair tip is between 15 and 20 percent, what would be a fair tip to leave their waiter?

(Pennsylvania Department of Education)

Practice Answers

1. C

2. Round the bill of $27.29 to $27.00.
   Multiply $27 by 2 for a twenty percent tip. The result is 54.
   Move the decimal one place because 20% = .2. The result is 5.4.
   Round the 5.4 to $5.00.
   A fair tip is $5.00

Lesson Plan
Assessment Anchor M11.B.2.1.1
Measure and/or compare angles in degrees (up to 360°) (protractor must be provided or drawn)

Objective:
To use a protractor to measure angles.

Prerequisite Skills:

Materials:

protractor

Definitions:

A protractor is an instrument used to measure and/or draw angles.

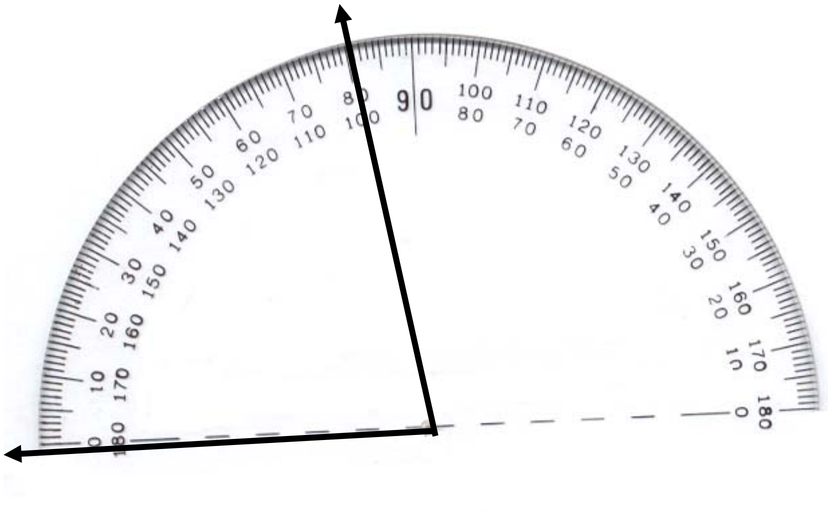
Lesson

Angles are measured in degrees. The symbol is °. There are 360° in a circle. A line is 180°.

To draw an angle:
1. Draw a ray.
2. Place the center of the protractor on the endpoint of the ray.
3. Use the degree markings on the protractor to mark off the desired angle measurement.
4. Connect the mark to the endpoint of the first ray.
5. Label the angle.

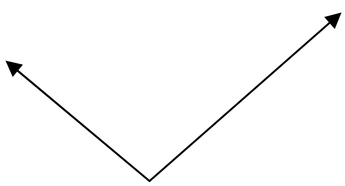
To measure an angle less than 180°:
1. Extend the length of the sides of the angle using a straight edge, if necessary.
2. Place the center of the protractor on the vertex of the angle.
3. Line up the protractor's 0° line with one ray of the angle.
4. Determine the angle measurement by reading the degree where the other side of the angle lines up on the protractor.
5. Write the angle measurement using the appropriate notation.

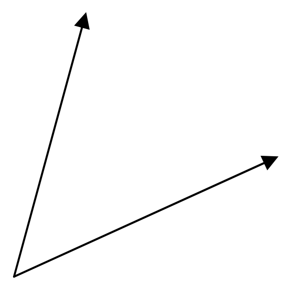
Example: What is the measure of the given angle?



Solution: 81°

Example: Which angle is larger?

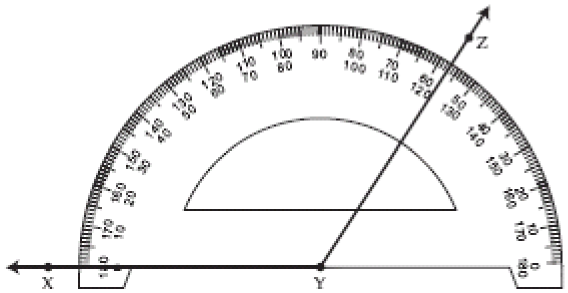




Solution: The first angle is larger since the measurement of the angle is greater.

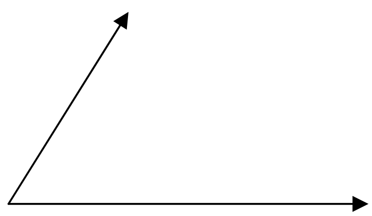
Practice

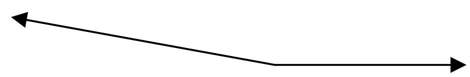
1. What is the measure of ∠XYZ?



A. 57°
B. 63°
C. 123°
D. 137°

2. Which angle is larger? Explain why.





Practice Answers

1. C

    A is the measure of its supplementary angle

    B measured using the wrong scale – counted up from 60

    D measured using the wrong scale – counted up from 130

2. The second angle is larger since the measurement of the angle is greater than the measurement of the first angle.

Math Lesson Plan
Assessment Anchor M11.B.2.2.1
Calculate the surface area of prisms, cylinders, cones, pyramids, and/or spheres

Berks Career & Technology Center

Objective:

To calculate the surface area of solids using formulas provided on the PSSA reference sheet.

Introduction:
To introduce the lesson, ask students if they think a standard piece of paper that is 8.5 by 11 inches will completely cover a cube that is 4 inches in length on every side. In order to do this, students will need to calculate the area of the piece of paper. A piece of paper is rectangular, and the area of a rectangle can be calculated by multiplying length by width.

$$8.5 \cdot 11 = 93.5\, in^2$$

This will then need to be compared to the combined area of each of the faces of a cube. Any face of a cube is a square, and to calculate the area of a square, students again need to multiply length by width. In this case, the length and width of each side is 4, so the area of one face is $4 \cdot 4 = 16$. However, this is only a small part of the entire area that must be covered. To completely cover the cube with a piece of paper, all six faces must be accounted for. So, if one face of the cube is 16 in.^{2}, and there are 6 faces, the total area to be covered is

$$16 \cdot 6 = 96 \text{ in.}^2$$

So, in this case, the paper would not be enough to completely cover the cube. Explain to students that in this lesson, they will examine surface area, which is the sum of all the areas of the faces a solid. They have just found the surface area of a cube by calculating the area of every face of the cube. However, some solids are more complex than a cube, and formulas have been developed to find the surface area of common solids. These formulas can be found on the formula sheet distributed with the PSSA examination.

Prerequisite Skills:

M11.C.1.1.1 Identify/use the properties of radius, diameter, and/or tangent of a circle.
M11.C.1.2.1 Identify/use properties of triangles.
M11.C.1.2.2 Identify/use properties of quadrilaterals.

Materials:

PSSA Formula Sheet

Definitions:

Solid: A three dimensional geometric figure

Surface Area: The sum of the areas of all the faces of a solid

SSA examination.

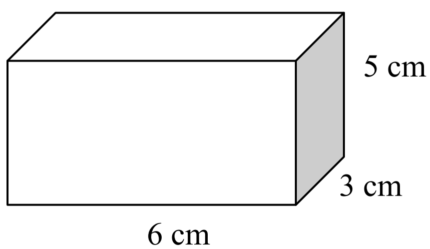
Lesson

Inform students that the best method of calculating the surface area of any solid is to follow the following steps:

1. Identify the solid
2. Write the formula for calculating the surface area of that solid using the formula sheet
3. Identify the properties of the solid used in the formula
4. Plug the actual properties of the solid into the formula
5. Perform the necessary mathematical operations to obtain your answer
6. Write the appropriate unit after your answer.

Example 1: Calculate the surface area of a prism

Calculate the surface area of the following figure.



Have students find a prism on the PSSA formula sheet. Then, identify the formula for finding the surface area (abbreviated SA) of a prism.

SA $= 2lw + 2lh + 2wh$

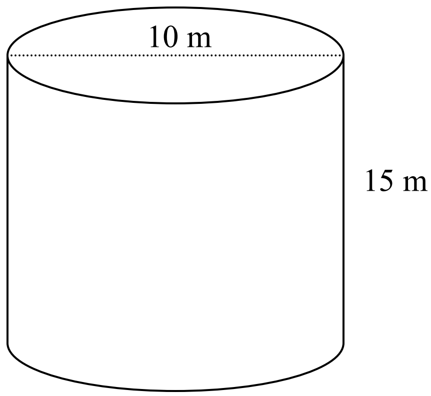
In this example, $l = 6,\ w = 3,\ h = 5$. Substitute these values into the formula.

$$\begin{aligned} SA &= 2 \cdot 6 \cdot 3 + 2 \cdot 6 \cdot 5 + 2 \cdot 3 \cdot 5 \\ &= 36 + 60 + 30 \\ &= 126 \text{ cm}^2 \end{aligned}$$

Inform students that even though they are examining three dimensional objects, they are still calculating two dimensional areas, so the appropriate unit is squared, not cubed.

<u>Example 2:</u> Calculate the surface area of a cylinder

Calculate the surface area of the following figure.



According to the formula sheet, the formula for calculating the surface area of a cylinder is,

SA = $2\pi r^2 + 2\pi rh$

For π, the number 3.14 can be substituted.

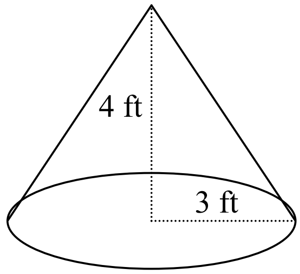
Note that this formula requires the radius of the figure. However, diameter is given.

Radius = $\frac{1}{2}$ Diameter = $\frac{1}{2} \cdot 10 = 5$. Using this information,

SA = $2 \cdot 3.14 \cdot 5^2 + 2 \cdot 3.14 \cdot 5 \cdot 15$
SA = 157 + 471
SA = 628 m^2

<u>Example 3</u>: Calculate the surface area of a cone

Calculate the surface area of the following figure.

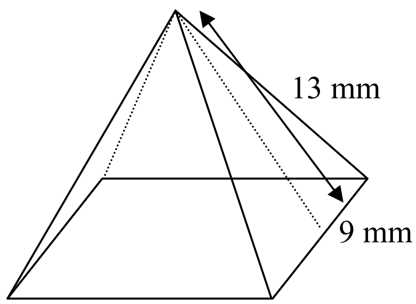


$$SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$

In this example, $r = 3$, $h = 4$, and again, 3.14 can be used to approximate π. Substituting,

$$SA = 3.14 \cdot 3^2 + 3.14 \cdot 3\sqrt{3^2 + 4^2}$$
$$SA = 28.26 + 9.42\sqrt{9 + 16}$$
$$SA = 28.26 + 9.42 \cdot 5$$
$$SA = 28.26 + 47.1$$
$$SA = 75.36 \text{ ft}^2$$

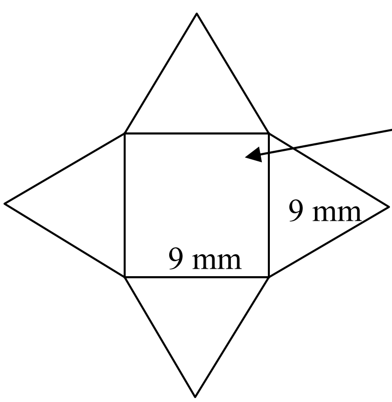
Example 4: Calculate the surface area of a square pyramid



The formula to calculate the surface area of a pyramid is

$$SA = (\text{Area of the base}) + \frac{1}{2}l(\text{number of base sides})b$$

This formula requires a bit of explanation. In order to differentiate the base and the base sides, it may be useful to "unfold" the figure and view it as a two dimensional figure.



This is the *base* of the pyramid.
It is a square with a side length of 9.
Area = $l^2 = 9 \cdot 9 = 81\text{mm}^2$

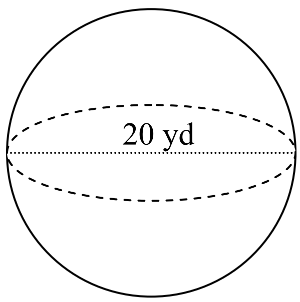
The other four sides are called the *base sides.*

So, the area of the base = 81, l = 13, the number of base sides = 4, and b = 9

$$SA = 81 + \frac{1}{2} \cdot 13 \cdot 4 \cdot 9$$
$$SA = 81 + 234$$
$$SA = 315 \text{ mm}^2$$

<u>Example 5:</u> Calculate the surface area of a sphere

Calculate the surface area of a sphere with a diameter of 20 yards.



$$SA = 4\pi r^2$$

This formula requires the radius of the sphere. However, diameter is given. To get the radius,

Radius = $\frac{1}{2}$ Diameter = $\frac{1}{2}(20) = 10$ yd

So, using the values $\pi = 3.14$ and $r = 10$,

$$SA = 4 \cdot 3.14 \cdot 10^2$$
$$SA = 1256 \text{ yd}^2$$

Practice

1. Find the surface area of a cone that has a height, *h,* of 22 cm and a radius, *r,* of 4 cm. Use 3.14 for π. Round your answer to the nearest hundredth.

   A.) 331.09 cm^3 B) 110.36 cm^2

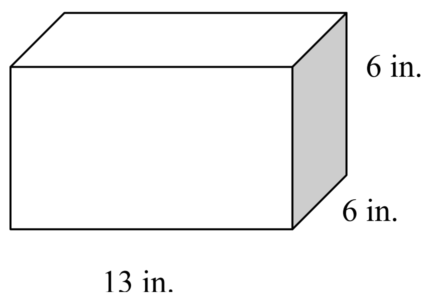
   C) 331.09 cm^2 D) 110.36 cm^3

2. Find the exact surface area of a cylinder with a height, *h*, of 5 m and a diameter of 15.2 m. Use $\pi = 3.14$. Round your answer to the nearest tenth.

   A) 219.8 m^2 B) 601.4 m^2

   C) 1,928.2 m^2 D) 358.7 m^2

3. The figure below is a pattern for a simple storage box. If a carpenter builds a larger box, using a scale of 1:5, what is he surface area of the new box? How does the surface area of the new box compare to that of the box shown in the pattern?



Practice Answers

1. C

   SA = $\pi r^2 + \pi r\sqrt{r^2 + h^2}$

   SA = $3.14(4^2) + 3.14(4)\sqrt{4^2 + 22^2}$

   SA = 331.09 cm^2

2. B

   $r = \frac{1}{2}$ diameter $= \frac{1}{2} \cdot 15.2 = 7.6$ m

   SA = $2\pi r^2 + 2\pi rh$

   SA = $2(3.14)(7.6^2) + 2(3.14)(7.6)(5)$

   SA = 601.4 m^2

3. Multiplying the length width and height of the figure by 5, one obtains new dimensions of l = 65 in., w = 30 in., h = 30 in.

   Substituting these dimensions:

   SA = $2lw + 2lh + 2wh$

   SA = $2(65)(30) + 2(65)(30) + 2(30)(30)$

   SA = 9600 in.^2

   The original surface area is:

   SA = $2lw + 2lh + 2wh$

   SA = $2(13)(6) + 2(13)(6) + 2(6)(6)$

   SA = 384 in.^2

   Using a ratio to compare the surface areas, $\frac{9600}{384} = \frac{25}{1}$. So, the larger box has 25 times as much surface area.

Math Lesson Plan
Assessment Anchor M11.B.2.2.2
Calculate the volume of prisms, cylinders, cones, pyramids, and/or spheres

Berks Career & Technology Center

Objective: To calculate the volume of solids using formulas provided on the PSSA reference sheet.

Introduction:
To introduce volume, inform students that is often useful to know how much space an object either takes up or can hold. The amount of space occupied by an object is known as volume. Industries often calculate the volume of freight shipments and try to determine the most cost efficient way to transport those shipments. For example, solid objects are often better transported in containers shaped like rectangular prisms, while liquids are most efficiently shipped in cylindrical containers.

It is often necessary to calculate volume for residential purposes as well. When building a home or renovating one, it is necessary to know the volume of each room or floor in order to purchase the appropriate heating and cooling units that regulate temperatures. Air conditioners are actually classified and sold by the volume of the space they are capable of cooling.

In this lesson, students will explore how the volumes of common solid shapes are calculated. Formulas have been developed to quickly determine the total space an object occupies using easily measured properties of solids, such as length, height, radius, and so forth.

Prerequisite Skills:

M11.C.1.1.1 Identify/use the properties of radius, diameter, and/or tangent of a circle.
M11.C.1.2.1 Identify/use properties of triangles.
M11.C.1.2.2 Identify/use properties of quadrilaterals.

Materials:

PSSA Formula Sheet

Definitions:

Solid: A three dimensional geometric figure

Volume: A quantification of the three dimensional space occupied of a solid

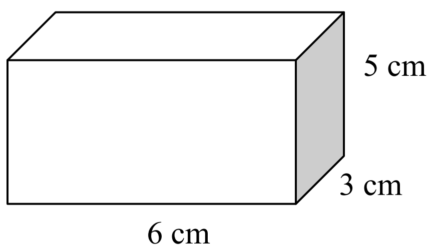
Lesson

Inform students that the best method of calculating the volume of any solid is to follow the following steps:

1. Identify the solid
2. Write the formula for calculating the volume of that solid using the formula sheet
3. Identify the properties of the solid used in the formula
4. Substitute the actual properties of the solid into the formula
5. Perform the necessary mathematical operations to obtain your answer
6. Write the appropriate unit after your answer.

<u>Example 1:</u> Calculate the volume of a prism

Calculate the volume of the following figure.



Have students find a prism on the PSSA formula sheet. Then, identify the formula for finding the volume (abbreviated V) of a prism.

$V = lwh$

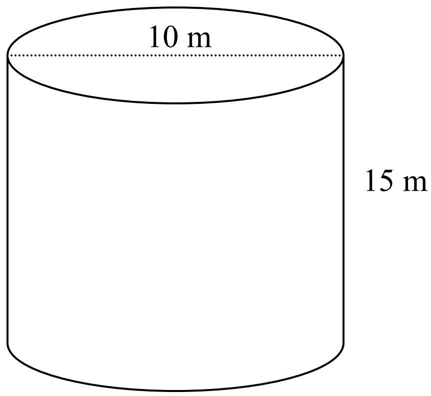
In this example, $l = 6,\ w = 3,\ h = 5$. Substitute these values into the formula.

$$\begin{aligned} V &= 6 \cdot 3 \cdot 5 \\ &= 90 \text{ cm}^3 \end{aligned}$$

Inform students that solids are three dimensional units, and the appropriate unit for the volume of a solid is always cubed to reflect that. Tell your students to remember that they are working in three dimensions, so their unit should be to the third power.

<u>Example 2:</u> Calculate the volume of a cylinder

Calculate the volume of the following figure.



According to the formula sheet, the formula for calculating the volume of a cylinder is,

$$V = \pi r^2 h,$$

where r is the radius of the circular part of the cylinder and h is the height of the cylinder.

For π, the number 3.14 can be substituted.

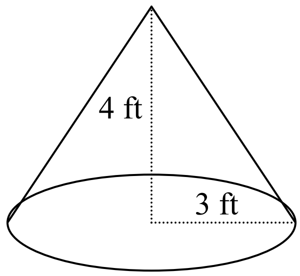
Note that this formula requires the radius of the figure. However, diameter is given.

Radius = $\frac{1}{2}$ Diameter = $\frac{1}{2} \cdot 10 = 5$. Using this information,

$V = (3.14)(5^2)(15)$
$V = 1177.5 \text{ m}^3$

<u>Example 3</u>: Calculate the volume of a cone

Calculate the volume of the following figure.



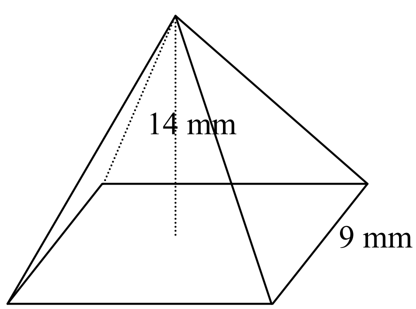
V = $\frac{1}{3}\pi r^2 h$

In this example, r = 3, h = 4, and again, 3.14 can be used to approximate π. Substituting,

V = $\frac{1}{3}(3.14)(3^2)(4)$

V = 37.68 ft^3

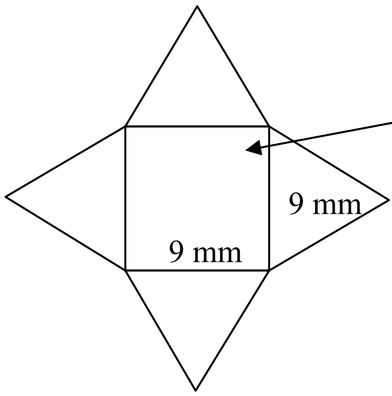
<u>Example 4:</u> Calculate the volume of a square pyramid



The formula to calculate the volume of a pyramid is

V = $\frac{1}{3}$(Area of the base)h

This formula requires a bit of explanation. In order to differentiate the base and the base sides, it may be useful to "unfold" the figure and view it as a two dimensional figure.



This is the *base* of the pyramid.
It is a square with a side length of 9.
Area = $l^2 = 9 \cdot 9 = 81\,\text{mm}^2$

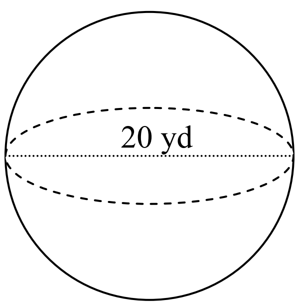
The other four sides are called the *base sides.*

So, the area of the base = 81, and the height, h = 14.

$$V = \frac{1}{3}(81)(14)$$
$$V = 378\ \text{mm}^3$$

<u>Example 5:</u> Calculate the volume of a sphere

Calculate the volume of a sphere with a diameter of 20 yards.



$$V = \frac{4}{3}\pi r^3$$

This formula requires the radius of the sphere. However, diameter is given. To get the radius,

$$\text{Radius} = \frac{1}{2}\,\text{Diameter} = \frac{1}{2}(20) = 10\ \text{yd}$$

So, using the values $\pi = 3.14$ and $r = 10$,

$$V = \frac{4}{3}(3.14)(10^3)$$
$$V = 4{,}186.\overline{6}\ \text{yd}^3$$

Practice

1. Calculate the volume of a rectangular prism fish tank with a length of 20 inches, a width of 18 inches and a height of 15 inches.

A) 5,400 in.^{2} B) 53 in.^{3} C) 360 in.^{3} D) 5,400 in.^{3}

2. A cylindrical storage container has a diameter of 150 feet, and is 80 feet high. The container is currently filled to 50% of its maximum capacity. What is the volume of the product currently inside the container? Use 3.14 to approximate π in your calculations.

A) 5,652,000 ft.^{3} B) 1,413,000 ft.^{3} C) 706,500 ft.^{3} D) 2,826,000 ft.^{3}

Practice Answers

1. **D**

V = *lwh*
V = (20 in.)(18 in.)(15 in.)
V = 5,400 in.3

2. **C**

The formula for volume requires the radius of the cylinder, which is $150 \div 2 = 75$ feet.

V = $\pi r^2 h$
V = (3.14)(75 ft)^2(80 ft)
V = 1,413,000 ft.^{3}

However, note that the container is only 50% full. To find the volume of the product inside the container, multiply 50% times the maximum volume.

$50\% \bullet 1{,}413{,}000 = 0.50 \bullet 1{,}143{,}000$ =706,500 ft.^{3}

Math Lesson Plan
Eligible Content M11.B.2.2.3
Estimate area, perimeter or circumference of an irregular figure.

Objective: Use known perimeter, circumference and area formulas to solve for the area and distance around irregular figures.

Introduction

The area of many shapes cannot be defined as a simple geometric figure. Often you are given a figure that is a combination of geometric figures such as circles, rectangles and triangles. A combination of formulas can be used to calculate the distance around an irregular figure or the area of the figure.

Skill Review

None

Formulas

Circumference $C = 2\pi r$

Area of a Rectangle $A = lw$

Area of a Triangle $A = \frac{1}{2}bh$

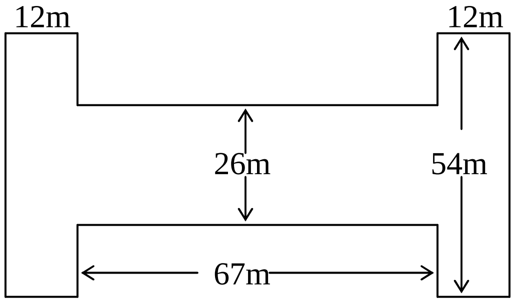
Area of a Circle $A = \pi r^2$

Lesson

An irregular figure is a figure that cannot be classified into specific shapes that you have studied. To find the distance around irregular figures, add the lengths of the sides. If the sides of the figures include circles, use the circumference formula to calculate the length of that portion of the figure and add it to the total of the other sides. To find the area of an irregular figure, separate the figure into shapes of which you can calculate the area. The sum of the areas of each smaller figure is the area of the irregular figure.

Examples

Find the perimeter of the region below.

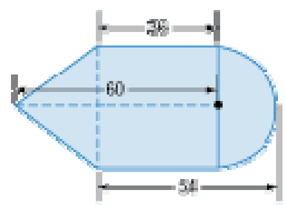


The 4 pieces on the inside of the figure (without dimensions), would each be 14 cm. This number can be achieved by subtracting 26 from 54 and dividing by 2.

Add the lengths of the sides $2 \times 54 + 4 \times 12 + 2 \times 67 + 4 \times 14 = 346$

Find the area of the irregular figure at the right.

The figure can be separated into a rectangle, a triangle, and a semicircle.



Find the dimensions of the three figures.

The radius of circle is 54 - 36 or 18.

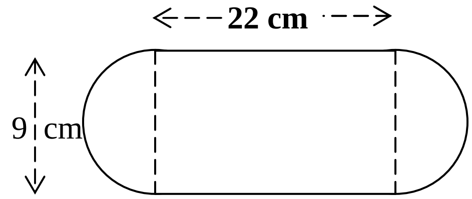
The length of the rectangle is marked as 36 and the height of the rectangle is the diameter of the circle, 2(18) or 36. The rectangle is a square.

The length of the base of the triangle is the length of the side of the square 18. The height of the triangle is 60 – 36 or 24.

Area of the triangle	½ (18)(24) = 216 sq units
Area of the square	(36)(36) = 1296 sq units
Area of the semi-circle	½ (π)$(18)^2 \approx$ 509 sq units
Total Area	216 + 1296 + 509 = 2021 sq units

Practice

1. Find the perimeter of the figure at right.



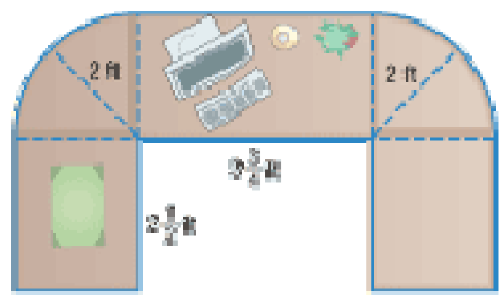
(A) 105.4 cm

(B) 72.3 cm

(C) 78.5 cm

(D) 88 cm

2. Jodi’s desk in her office is shaped as shown. Find the surface area of the top of the desk.



Practice Answers

1. A

The ends of the figure form one circle connected by two segments. To find the perimeter, add the circumference of the circle with the length of the two segments.

$$2 \times \pi \times 9 + 22 + 22 = 105.4$$

2. To find the area of the figure, separate the diagram into 2 quarter circles and 3 rectangles.

The quarter circles form one semicircle. To find their combined area, take ½ the area of a circle with a radius of 2.

Area of the Quarter Circles - $\frac{1}{2}(\pi \times r^2)$

$$\frac{1}{2}(\pi \times 2^2) = 6.28$$

Area of middle rectangle $3\frac{3}{4} \times 2 = 7.5$

Area of end rectangles (2) $2(2 \times 2\frac{1}{2}) = 10$

Total Area 6.28 + 7.5 + 10 = 23.78 square feet

Math Lesson Plan
Eligible Content M11.B.2.2.4
Find the measurement of a missing length given the perimeter, circumference, area or volume.

Objective: Use formulas and rules of algebra to find the measurement of a missing length given the perimeter, circumference, area or volume.

Introduction

Tell students that the day's lesson will be about using formulas to find missing lengths. Many trades use formulas when giving estimates to customers as well as using formulas to do the actual job. When students graduate and have places of their own they will need to use formulas in order to do renovations such as replacing carpet, etc.

Skill Review:

M11.A.2.2.1 Simplify/evaluate expressions involving positive and negative exponents, roots and /or absolute value (may contain all types of real numbers – exponents should not exceed power of 10).
M11.B.2.2.1 Calculate the surface area of prisms, cylinders, cones, pyramids and /or spheres. Formulas are provided on a reference sheet.
M11.B.2.2.2 Calculate the volume of prisms, cylinders, cones, pyramids and /or spheres. Formulas are provided on a reference sheet.
M11.D.2.1.3 Write, solve and/or apply a linear equation (including problem situations).

Definitions

Perimeter
The distance around a figure.

Circumference
The perimeter of a circle.

Area
The number of nonoverlapping square units contained in the interior of a figure such as square inches, square feet, square yards, square centimeters, etc.

Volume
The number of nonoverlapping cubic units contained in the interior of a three-dimensional figure such as cubic feet, cubic yards, cubic meters, etc.

Lesson

The following are some formulas that you can give students.

These formulas and other formulas are included on the PSSA formula sheet.

	Area:	Perimeter:	Volume:
Rectangle:	$A = lw$	$P = 2l + 2w$	where l is the length and w is the width.
Triangle:	$A = \frac{1}{2}bh$	$P = a + b + c$	where b is the base and h is the height and a, b, and c are the lengths of the sides.
Circle:	$A = \pi r^2$	$C = 2\pi r$	where $\pi = 3.14$ and r is the radius (distance from the center to a point on the circle).
Cylinder:	$SA = 2\pi rh + 2\pi r^2$		$V = \pi r^2 h$

Explain to students that the following operations are opposites of each other:
1. Addition and subtraction
2. Multiplication and division
3. Powers and roots

These operations can be used to solve for unknown lengths in a formula.

Examples

1. Find the width of a rectangle given the length is 15 feet and the area is 180 square feet.

Work:

Substitute the given numbers into the formula for the area of a rectangle:

$A = lw$

$180 = 15w$ Divide both sides of the equation by 15 since division is the opposite of multiplication.

$12 = w$

The width of the rectangle is 12 feet.

2. If the area of a circle is 78.5 square yards, what is the diameter of the circle?

Work:

Substitute the given numbers into the formula for the area of a circle:

$A = \pi r^2$	
$78.5 = 3.14r^2$	Divide both sides of the equation by 3.14 since division is the opposite of multiplication.
$25 = r^2$	Take the square root of both sides since the opposite of a power is a root.
$5 = r$	The radius of the circle is 5 yards. Since the diameter is twice the radius, the diameter of the circle is 10 yards.

3. If the perimeter of a rectangle is 100 feet and the length is 30 feet, what is the width?

$P = 2l + 2w$	Substitute the given numbers into the formula for perimeter.
$100 = 2(30) + 2w$	Multiply 2 by 30.
$100 = 60 + 2w$	Subtract 60 from both sides of the equation since subtraction is the opposite of addition.
$40 = 2w$	Divide both sides of the equation by 2 since division is the opposite of multiplication.
$20 = w$	The width of the rectangle is 20 feet.

4. If a cylinder holds 88 cubic centimeters and its height is 7 centimeters, what is its radius?

$V = \pi r2h$	Substitute the given numbers into the volume formula.
$88 =3.14r2(7)$	Multiply 3.14 by 7.
$88 =22r^2$	Divide both sides of the equation by 22.
$4 = r^2$	Take the square root of both sides of the equation.
$2 = r$	The radius of the cylinder is 2 centimeters.

Practice

1. Find the width of a rectangular room if a customer wants you to construct a room whose length is 22 feet and whose area is 396 square feet.

   (A) 18 feet
   (B) 176 feet
   (C) 99 feet
   (D) 20 feet

2. If a customer wants a circular patio that is 200 square feet, what would the diameter of the patio have to be?

   (A) 8 feet
   (B) 64 feet
   (C) 32 feet
   (D) 16 feet

3. If you have 166 feet of custom fencing and you are fencing in a rectangular area that is 34 feet on one side, what will be the length of the other side of the rectangle?

4. If a silo needs to hold 2500 cubic feet of feed for animals and the height of the silo is 32 feet, what is the radius of the silo?

Practice Answers

1. Find the width of a rectangular room if a customer wants you to construct a room whose length is 22 feet and whose area is 396 square feet. (A)

   Work: $A = l \times w$
   Substitute the numbers into the area formula for a rectangle:
   $396 = 22 \times w$ (The opposite of multiplication is division. Divide both sides of the equation by 22)
   The width of the room is 18 feet.

2. If a customer wants a circular patio that is 200 square feet, what would the diameter of the patio have to be? (D)

   Work:
   Area = $\pi r2$ Substitute the numbers into the area formula for a circle.
   $200 = 3.14r2$ Divide both sides of the equation by 3.14 to isolate r2.

   $64 = r2$ The opposite of a power is a root. Take the square root of both sides.
   $\sqrt{64} = r$
   8 feet = the radius of the circle to the nearest whole number.
   The diameter would be twice the radius or 16 feet.

3. If you have 166 feet of custom fencing and you are fencing in a rectangular area that is 34 feet on one side, what will be the length of the other side of the rectangle?

   Work:
   $P = 2l + 2w$ Substitute the given numbers into the formula for perimeter.
   $166 = 2l + 2\times34$ Multiply 2 by 34.
   $166 = 2l + 64$ Subtract 64 from both sides of the equation.

   $102 = 2l$ Divide both sides of the equation by 2.
   $51 = l$

   The length of the rectangle is 51 feet.

4. A silo needs to hold 2500 cubic feet of feed. The height is 32 feet.

   Work: A silo is a cylinder. The formula for the volume of a cylinder is:
   $V = \pi r2h$ Substitute the given numbers into the formula.
   $2500 = 3.14r2(32)$ Multiply 3.14 by 32.
   $2500 = 100r2$ Divide each side of the equation by 100.
   $25 = r2$ Take the square root of both sides of the equation
   $5 = r$
   The radius of the silo to nearest foot would be 5 feet.

Math Lesson Plan
Eligible Content M11.B.2.3.1
Describe how a change in the linear dimension of a figure affects its perimeter, circumference, area or volume.

Objective: Use formulas and rules of algebra to show how a change in a dimension of a figure affects its perimeter, circumference, area or volume.

Introduction: Tell students that the day's lesson is about what happens to perimeter, circumference, area or volume of a figure when one of the dimensions such as its length, width, or height is doubled or tripled. Discuss the need to work with formulas for perimeter, circumference, area or volume in such shops as the construction cluster, horticulture and everyday life.

Skill Review:

M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers – exponents should not exceed power of 10).
M11.A.3.1.1	Simplify/evaluate expressions using the order of operations.
M11.B.2.2.1	Calculate the surface area of prisms, cylinders, cones, pyramids and/or spheres.
M11.B.2.2.2	Calculate the volumes of prisms, cylinders, cones pyramids and/or spheres.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).

Definitions:

Perimeter:
The distance around a figure.

Circumference:
The perimeter of a circle.

Area:
The number of non-overlapping square units contained in the interior of a figure such as square inches, square feet, square yards, square centimeters, etc.

Volume:
The number of non-overlapping cubic units contained in the interior of a three-dimensional figure such as cubic feet, cubic yards, cubic meters, etc.

Lesson:

The following are some formulas that you can give to your students.
These formulas and other formulas are included on the PSSA formula sheet in the PSSA test.

	Area:	Perimeter:	Volume:
Circle:	$A = \pi r^2$	$C = 2\pi r$ or $C = \pi d$	Where π =3.14 and r is the radius (distance from the center to a point on the circle and d is the diameter or twice the radius).
Cube:	$SA = 2lw + 2lh + 2wh$		$V = lwh$ where l is the length, w is the width and h is the height.
Triangle:	$A = ½ bh$	$P = a + b + c$	Where b is the base and h is the height and a, b, and c are the lengths of the sides.

Ask students what they think will happen to the circumference of a circle if the length of the radius is doubled or tripled. Then do the first example to discover what happens.

Ask students what they think will happen to the volume of a cube ("box" with all sides equal) when the edge (side) is doubled or tripled. Then do the second example to discover what happens.

Ask students what they think will happen to the area of a triangle when the length of the base is doubled or tripled. Then do the third example to discover what happens.

Examples:

1. a. The circumference of a circle whose radius is 3 feet is: $C = 2\pi r$ or $C = 2(3.14)(3) = 18.84$ feet.

b. The circumference of a circle whose radius is doubled to 6 feet is: $C = 2(3.14)(6) = 37.68$ feet.

Since 2 times 18.84 is 37.68, doubling the radius doubled the circumference.

c. The circumference of a circle whose radius is tripled to 9 feet is: $C = 2(3.14)(9) = 56.52$ feet.

Since 3 times 18.84 is 56.52, tripling the radius tripled the circumference.

This would lead us to believe that <u>changing the length of the radius of a circle by a certain multiplier</u> <u>would change the circumference by the same multiplier</u>. This is a true hypothesis.

2. a. The volume of a cube whose edges are 9 inches is: $V = lwh = (9)(9)(9) = 9^3 = 729$ cubic inches.

b. The volume of a cube whose edges are doubled to 18 inches is: $V = (18)(18)(18) = 18^3 = 5832\ in^3$.

Most students might be inclined to say that doubling the edges would double the volume.

However, since $5832 \div 729 = 8$, doubling each edge did more than double the volume.

Since each edge was doubled, the volume $= (2\times9)(2\times9)(2\times9) = 2\times2\times2\times9\times9\times9 = 2^3\times 9^3 = 8\times729=5382$.

Hence, doubling each edge changed the volume by $2^3 = 8$ times.

c. The volume of a cube whose edges are tripled to 27 inches is: $V = (27)(27)(27) = 19683$ cubic inches.

Students might be inclined to think that tripling the edges would triple the volume.

However, since $19683 \div 729 = 27$, tripling each edge did more than triple the volume.

Hopefully, some of the students are starting to see a pattern.

Since each edge was tripled, the volume $= (3\times9)(3\times9)(3\times9) = 3\times3\times3\times9\times9\times9=3^3\times9^3= 27\times729=19683\ in^3$

Hence, tripling each edge changed the volume by $3^3 = 27$ times.

Hopefully, the students will recognize the pattern that if each edge is multiplied by a number n, the volume will be multiplied by n^3.

3. a. The area of a triangle whose base is 3 meters and whose height is 8 meters is: $A = \frac{1}{2} bh = \frac{1}{2}(3)(8) = 12\ m^2$.

b. The area of triangle whose base is doubled to 6 meters is: $A = \frac{1}{2}(6)(8) = 24\ m^2$.
Since $24 \div 12 = 2$, doubling the base of the triangle doubled the area of the triangle.

c. The area of a triangle whose base is tripled to 9 meters is: $A = \frac{1}{2}(9)(8) = 36\ m^2$.
Since $36 \div 12 = 3$, tripling the base of the triangle tripled the area of the triangle.

This would lead us to believe that <u>changing the length of the base of a triangle by a multiplier would</u> <u>change the area of the triangle by the same multiplier</u>. This is a true hypothesis.

Practice:

1. If the length of the radius of a circle is multiplied by 4, how many times larger is the circumference of the circle?

   (A) ¼ as large

   (B) 4^2 or 16 times larger

   (C) 4^3 or 64 times larger

   (D) Four times larger

2. If the length of the edge of an original cube is 5 inches, how much larger is the volume of a second cube whose edges are 6 times longer or 30 inches each ?

3. The length of the base of one triangle is 7 centimeters and the height is 11 centimeters. The length of the base of a second triangle is 5 times as long as the base of the first triangle or 35 centimeters. The height of the second triangle is the same as the height of the first triangle. How does the area of the second triangle compare to the area of the first triangle?

Practice Answers:

1. (D)

Since the radius of the second circle is 4 times larger, the circumference would be 4 times larger.

The circumference of the first circle with a radius r is $2\pi r$.
The circumference of the second circle with radius 4r is $2\pi(4r) = 8\pi r$.
$8\pi r = 4$ times $2\pi r$.

2. The volume of the second cube would be 6^3 or 216 times larger.

The volume of the first cube is $lwh = (5)(5)(5) = 125\ in^3$.
The volume of the second cube is $(30)(30)(30) = 27000\ in^3$.
Since $125 \times 216 = 27000$, the answer is justified,

3. The area of the second triangle would be 5 times larger than the area of the first triangle.

The area of the first triangle is $\frac{1}{2} bh = \frac{1}{2} (7)(11) = 38.5\ cm^2$.
The area of the second triangle is $\frac{1}{2} (35)(11) = 192.5\ cm^2$.
Since $38.5 \times 5 = 192.5$, the answer is justified.

Math Lesson Plan
Eligible Content M11.C.1.1.1
Identify and/or use the properties of a radius, diameter, and/or tangent of a circle (given numbers should be whole.)

Objective: Use the properties of radius, diameter, and tangent.

Introduction

Emphasize the importance of gaining a basic understanding of the relationship between radius and diameter. Students may be given a formula that includes one of these terms and not the other. To be able to use the formula you need to know the relationship between the two basic geometric terms. Tangent is another basic mathematical term that the students will benefit from knowing.

Skill Review

M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems

Definitions

Radius

Of a circle: A segment connecting the center of a circle to any point on the circle. It is also the length of a segment connecting the center of a circle to any point on the circle.

Of a Sphere: A segment connecting the center of a sphere to any point on the sphere. It is also the length of a segment connecting the center of a sphere to any point on the sphere.

Diameter

Of a circle: A segment connecting two points on the circle and passing through the center. The term is also used to denote the length of such a segment.

Of a Sphere: A segment connecting two points on a sphere and passing through the center.

Tangent

To a curve: A line that touches a curve at one point. The point of contact is called the point of tangency.

Circles: Circles that touch each other at one point.

Lesson

When you are working with circles it is important to know some basic facts about the relationship between the parts of the circle. In this lesson we will learn and apply these basic facts.

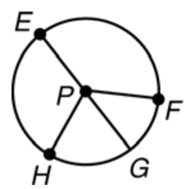
The diameter is always twice the radius.

A tangent to a circle forms a right angle with the radius of that circle at the point of tangency.

Two tangents to the same circle from the same external point will be congruent and therefore equal in measure.

Examples and Work

Use the picture below for examples 1 and 2.



Example 1. Given that diameter EG is 30 meters, what is the length of radius EP?
Since the diameter is always twice the radius, you can use the equation D= 2r.

D = 2r	Write the equation
30 meters = 2r	Substitute known values into the equation.
15 meters = r	Divide each side of the equation by 2.

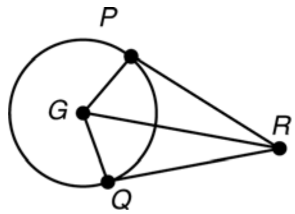
The length radius EP is 15 meters.

Example 2. Given that radius GP is 7 yards, what is the length of diameter EG?
Since the diameter is always twice the radius, you can use the equation D= 2r.

D = 2r	Write the equation
D = (2)(7 yards)	Substitute known values into the equation.
D = 14 yards	Simplify

The length of the diameter is 14 yards.

Use the picture below for examples 3 and 4.



Example 3. Given that tangent RP is 5 cm, what is the length of tangent RQ?

Since two tangents to the same circle from the same external point will be congruent and therefore equal in measure, the measure of tangent RQ is 5 cm.

Example 4. Given that line RP is tangent to circle G, what is the measure of angle RPG?

Since a tangent to a circle forms a right angle with the radius of that circle at the point of tangency, the measure of angle RPG will be 90 degrees.

Practice

1. If a circle has a diameter of 12 units, then it has _____.

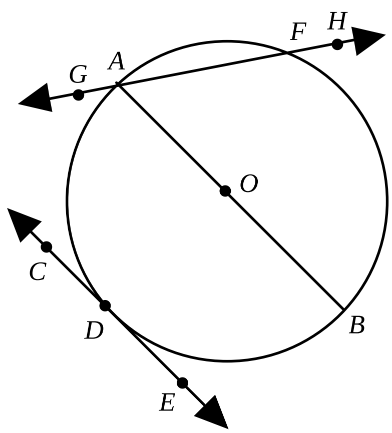
[A] a radius of 24 units

[B] a radius of 6 units

[C] a radius of 4 units

[D] a diameter of 6 units

For practice problem 2 use the figure below.



2. Name the tangent.

Practice Answers

1. Given that diameter is 12 units, what is the length of the radius?
   Since **the diameter is always twice the radius,** you can use the equation **D= 2r.**

D = 2r	Write the equation
12 units = 2r	Substitute known values into the equation.
6 meters = r	Divide each side of the equation by 2.B.

The answer is B.

2. Since a tangent is a line that touches a curve at one point. The answer would be line CD. This line could also be called line DC, line DE, line ED, line CE, or line EC.

Math Lesson Plan
Eligible Content M11.C.1.1.2
Identify and/or use the properties of arcs, semicircles, inscribed angles and/or central angles.

Objective: Use the properties of arcs, semicircles, inscribed angles and central angles.

Introduction

In real life we often use angles and arcs without realizing that we are using them. In this lesson we will learn how arcs are related to the angles they intercept.

Skill Review

M11.C.1.1.1 Identify and/or use the properties of radius, diameter, and/or tangent of a circle.

Definitions

Arc - A part of a circle or a curve between two points.

Minor Arc – An arc of a circle that is less than half the circle.

Major Arc - An arc of a circle that is more than half the circle.

Semicircle - An arc of a circle that is half the circle.

Central Angle – An angle whose vertex is the center of the circle.

Chord – A segment whose endpoints are on a circle.

Inscribed Angle – An angle whose vertex is on the circle and whose sides are chords of the circle.

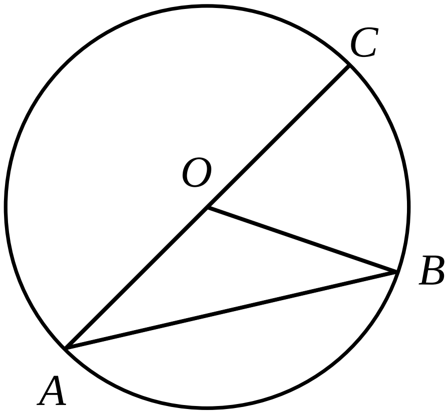
Lesson

Any arc that is **less than 180 degrees** is a **minor arc**.
Any arc that is **greater than 180 degrees** is a **major arc**.
Any arc **equal to 180 degrees** is a **semicircle**.

Inscribed angles are equal to half the arc the intercept.
Central angles are equal to the arc they intercept.

Examples and Work

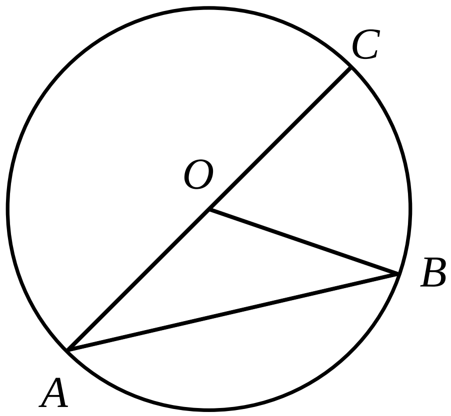
Examples 1-4 will use the figure below.



1. **In Circle O, arc AC is an example of a semicircle.**
2. **In Circle O, arc AB and arc BC are examples of minor arcs.**
3. **In Circle O, arc ACB is a major arc.**
4. **In Circle O, angle COB is a central angle.**
   If the measure of arc CB is 60 degrees than the measure of angle COB is also 60 degrees, since measure of the central angle is the same as the measure of the arc that is intercepts.
5. **In Circle O, angle CAB is an inscribed angle.**
   If the measure of arc CB if 60 degrees than the measure of angle CAB is 30 degrees, since the measure of the inscribed angle is half the measure of its intercepted arc.

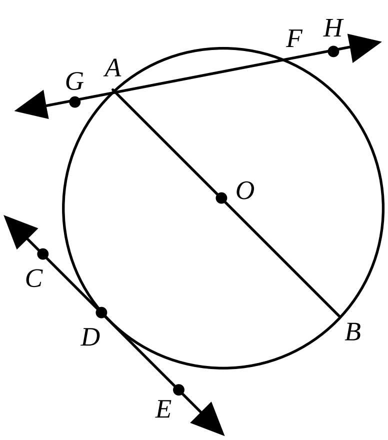
Practice

#1 Use the figure below.



If the measure of central angle COB is 65°, what is the measure of inscribed angle CAB?

#2 Use the figure below.



If the measure of arc FB is 106°, what is the measure of inscribed angle FAB?

A. 106°
B. 53°
C. 35.3°
D. 46°
E. Not Enough Information

Practice Answers

1. If central angle COB is 65°, then arc CB is also 65°, since the central angle is equal to the arc it intercepts. Since the arc is 65°, **the inscribed angle CAB is 32.5°**. This is because any inscribed angle is half the measure of the arc it intercepts.

2. **B.** The measure of inscribed angle FAB is 53°, since it is half the measure of the intercepted arc FB.

Math Lesson Plan
Eligible Content M11.C.1.2.1
Identify and/or use properties of triangles.

Objective: Identify and use the properties of triangles.

Introduction

This lesson will cover some basic vocabulary and basic properties of triangles. There are two ways that we typically classify triangles. One is to classify them by sides (equilateral, isosceles and scalene). The other is to classify them by angle (right, acute and obtuse). We will also define the terms median, altitude, angle bisector, and discuss side/angle relationships and the triangle inequality theorem. If you would like more information on isosceles and equilateral triangles see the M11.C.1.2.3 lesson.

Skill Review

M11.A.3.1.1 Simplify/Evaluate expressions using the order of operations to solve problems
M11.A.2.1.1 Simplify/Evaluate expressions involving exponents.
M11.A.2.1.3 Identify and/or use proportional relationships in problem solving settings.

Definitions

Congruent figures have the same shape and size.

An Equilateral Triangle is a triangle with all three sides congruent.

An Isosceles Triangle is a triangle with at least two sides congruent. By definition an equilateral triangle is also isosceles.

A **Scalene Triangle** is a triangle with no sides congruent.

A **Right Triangle** is a triangle is a triangle with a right (90°) angle.

An **Obtuse Triangle** is a triangle with an obtuse angle (an angle greater than 90°).

An **Acute Triangle** is a triangle with all acute angles (angles that are less then 90°).

The **Median of a Triangle** is the line segment connecting a vertex to the midpoint of the opposite side. The point of intersection of the medians of a triangle is called the **centroid**. This is the balancing point of the triangle.

The **Altitude of a Triangle** is the perpendicular distance from the vertex of the triangle to the opposite side. The altitude can be inside, on or outside the triangle. The point of intersection of the altitudes of a triangle is the **orthocenter.** This point is an equal distance to each side of the triangle.

The **Angle Bisector of a Triangle** is segment ray of line that divides an angle into two congruent parts. In a triangle the **inceter** is the point where the three angle bisectors of a triangle meet. This point is an equal distance to each vertex of the triangle.

Lesson

In a triangle, the sides and angles are related to each other so that if certain patterns occur then the triangles are congruent. These patterns include:

SSS (side-side-side) If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

SAS (side-angle-side) If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

ASA (angle-side-angle) If two angles and an included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

AAS (angle-angle-side) If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of a second triangle, then the two triangles are congruent.

HL (hypotenuse-leg) If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

In a triangle, the sides and angles are related to each other so that if certain patterns occur then the triangles are similar. These patterns include:

Side-Side-Side Similarity (sss) If the lengths of the three sides of a triangle are proportional to the corresponding lengths of second triangle then the two triangles are similar.

Side-Angle-Side Similarity (sas) If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional then the two triangles are similar.

Angle-Angle Similarity (aa) If two angles of one triangle are congruent to two angles of a second triangle, then the two triangles are similar.

<u>The Triangle Inequality Theorem</u>

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

<u>Given the sides of a triangle you can determine if it is acute, obtuse or right.</u>

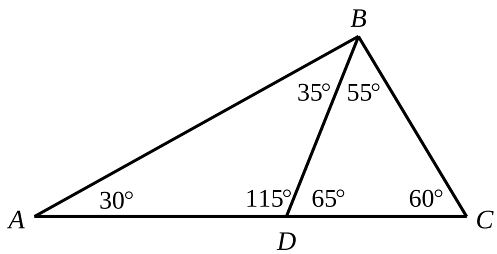
In any triangle with sides a, b, and c with c being the longest of the three sides if $c^2 = a^2 + b^2$, then it is a right triangle. If $c^2 < a^2 + b^2$, then it is an acute triangle. If $c^2 > a^2 + b^2$, then it is an obtuse triangle.

June 2008

Examples and Work

Example 1

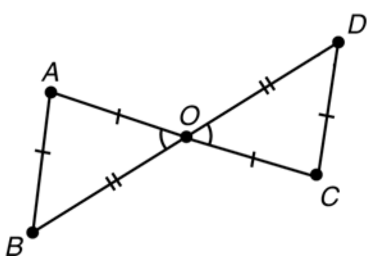
Name an acute triangle.



ΔBCD is an acute triangle since all three of the angles are less than 90°.

Example 2

State two postulates or theorems that can be used to conclude that $\Delta AOB \cong \Delta COD$.



In the picture side AO, side AB, side CO and side CD are all marked congruent, side OB and side OD are also marked as congruent. This makes ΔAOB congruent to ΔCOD by SSS.

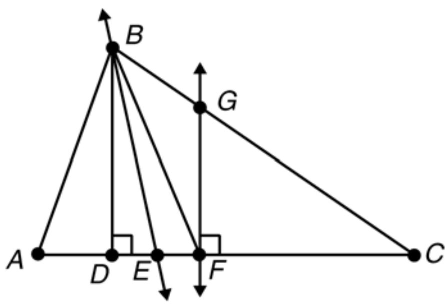
Another way to prove the triangles congruent would be by using side AO congruent to side CO, side OB congruent to side OD, and $\angle$AOB congruent to $\angle$COD. This would be using SAS.

Example 3

Refer to the figure below

Given: $\overline{AF} \cong \overline{FC}$, $\angle ABE \cong \angle EBC$

Which line is a perpendicular bisector in ΔABC?



In the picture
line segment GF is a perpendicular bisector,
line segment BE is an angle bisector,
line segment BD is an altitude and
line segment BF is a median.

Example 4

Which side lengths allow you to construct a triangle?

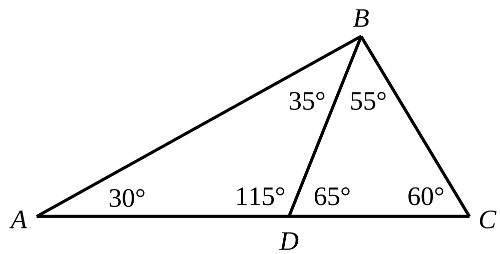
A.) 2, 3, and 8 B.) 6, 8, and 10
C.) 4, 1, and 9 D.) 7, 2, and 2

(2 + 3) < 8 therefore this cannot be a triangle.
(4 + 1) < 9 therefore this cannot be a triangle.
(2 + 2) < 7 therefore this cannot be a triangle.

(6 + 8) > 10 therefore this can be a triangle, and the answer would be B.

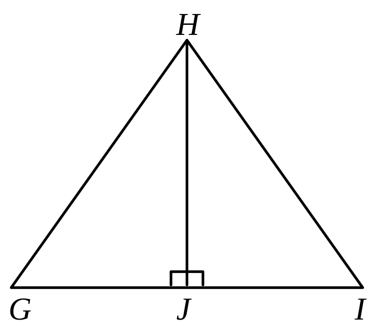
Practice

1. Name a right triangle.



A.) ΔABC B.) ΔADB C.) ΔBDC D.) none of these

2. Refer to the figure below. Which of the following statements is true?



$\overline{GJ} \cong \overline{JI}$

A.) There are no congruent triangles.

B.) $\Delta GIJ \cong \Delta JHG$ by SSS

C.) $\Delta GHJ \cong \Delta IHJ$ by SAS

D.) $\Delta GJH \cong \Delta IJH$ by SSS

Practice Answers

1. Δ ABC is a right triangle since the sum of 35° and 55° is 90°, and by definition a right triangle has an angle of 90°.

2. The answer is C. The given information to shows that one pair of sides is congruent and the picture shows that one pair of angles are right angles and therefore congruent. Since the triangles share a side, that side will be congruent to itself and thus the triangles are congruent by SAS (side-angle-side).

Math Lesson Plan
Eligible Content M11.C.1.2.2
Identify and/or use properties of quadrilaterals.

Objective Identify and use the properties of quadrilaterals.

Introduction
This lesson will cover some basic vocabulary and basic properties of quadrilaterals. Some of these properties include parallel sides, diagonals that bisect each other or are congruent, congruent sides, congruent angles, and supplementary angles.

Skill Review

M11.A.3.1.1 Simplify/Evaluate expressions using the order of operations to solve problems

Definitions

Congruent figures have the same shape and size.

A **polygon** is a plane geometric figure with three or more straight sides.

A **quadrilateral** is a polygon with four sides.

A **parallelogram** is a quadrilateral with opposite sides parallel.

Parallel sides have the same slope.

A **rectangle** is a parallelogram with four congruent angles.

A **rhombus** is a parallelogram with four congruent sides.

A **square** is a parallelogram with both with four congruent sides and four congruent angles.

A **kite** is a quadrilateral with two pairs of consecutive sides congruent and exactly one pair of opposite angles congruent.

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

An **isosceles trapezoid** is a quadrilateral with exactly one pair of parallel sides, congruent legs and congruent base angles.

Supplementary angles are angles whose sum is 180 degrees.

Lesson

Properties of Quadrilaterals

The sum of interior angles of any polygon is (N – 2) 180º.
N is the number of sides of the polygon.
The sum of the interior angles of a quadrilateral is 360º = (4 – 2) 180º.

A **kite** is a quadrilateral with two pairs of consecutive congruent sides; exactly one pair of opposite angles congruent and the diagonals are perpendicular.

A **trapezoid** has exactly one pair of parallel sides.

An **isosceles trapezoid** has congruent legs; congruent base angles and the diagonals are perpendicular.

Properties of Parallelograms

Opposite sides are parallel.
Opposite angles are congruent.
Opposite sides are congruent.
Consecutive angles are supplementary.
Diagonals bisect each other.

You can prove that a quadrilateral is a parallelogram if you can prove any of the above properties are true.

Squares, rectangles and rhombuses are special parallelograms and have all of the properties of parallelograms as well as some other special properties.

Rectangles have four congruent angles, and the diagonals are congruent.

Rhombuses have four congruent sides, perpendicular diagonals, and the diagonals also bisect the opposite angles.

Squares are both rectangles and rhombuses and have the properties of both.

Examples and Work

Example 1

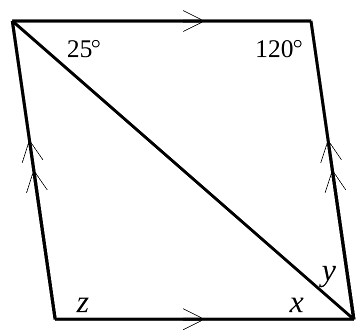
Which type of quadrilateral has no parallel sides?

[A] rhombus
[B] trapezoid
[C] kite
[D] rectangle

C. A kite is a quadrilateral with no parallel sides.

Example 2

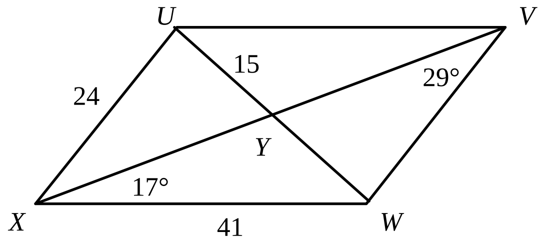
Find the value of the variables in the parallelogram.



[A] $x = 35°$, $y = 25°$, $z = 120°$
[B] $x = 60°$, $y = 12.5°$, $z = 155°$
[C] $x = 25°$, $y = 35°$, $z = 120°$
[D] $x = 12.5°$, $y = 60°$, $z = 155°$

Since the sides of the parallelogram are parallel the angle marked 25° and the angle marked x are alternate interior angles and are congruent and therefore equal in measure, x = 25°.
Since opposite angles of a parallelogram are congruent z = 120°.
Since consecutive angles in a parallelogram are supplementary, x + y + 120° = 180°.
We can substitute the know value for x, x = 25°.
The resulting equation is 25° + y + 120°= 180°.
The next step is to combine the constant terms, 145° + y = 180°.
Subtract 145° from each side of the equation to get y = 35°.
Now we know that the answer is C.

Example 3



Given: $UVWX$ is a parallelogram, $m\angle WXV = 17°$, $m\angle WVX = 29°$,

$XW = 41$, $UX = 24$, $UY = 15$

A. Find $m\angle WVU$.
Since m∠WXV = 17°, and ∠WXV and ∠XVU are alternate interior angles, m∠XVU = 17°.
By the angle sum theorem, m∠XVU + m∠XVW = m∠WVU
By substitution, 17° + 29° = m∠WVU
m∠WVU = 46°

B. Find *WV*.
One of the properties of parallelograms is the opposite sides are congruent and therefore equal in length. WV = XU = 24

C. Find $m\angle XUV$.
One of the properties of parallelograms is the consecutive angles are supplementary. Since the m∠WVU = 46°, m∠XUV + 46° = 180°, and m∠XUV = 134°

D. Find *UW*.
One of the properties of parallelograms is the diagonals bisect each other. When you apply this property UY = YW. Since the UY = 15, YW = 15. By segment addition, UY + YW = UW.
By substitution 15 + 15 = UW, and UW = 30.

Example 4

If the diagonals of a parallelogram are perpendicular, then the parallelogram is also what type of figure?

Knowing the properties of a parallelogram we know that the answer must be a rhombus.

Example 5

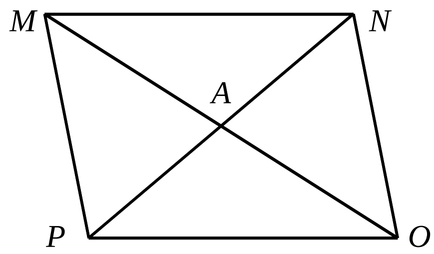
Consecutive angles in a parallelogram are always ________.

[A] vertical angles
[B] complementary angles
[C] supplementary angles
[D] congruent angles

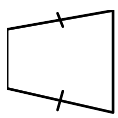
Knowing the properties of a parallelogram we know that the answer is C.

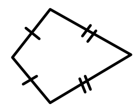
Practice

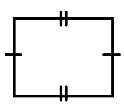
1. Find AM in the parallelogram if $PN = 6$ and $MO = 18$.

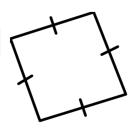


2. Choose the figure below that satisfies the definition of a kite.

[A] 

[B] 

[C] 

[D] 

Practice Answers

1. In a parallelogram the diagonals bisect each other, so AM will be half MO. MO = 9

2. B. A kite is a quadrilateral with two pairs of consecutive congruent sides.

Math Lesson Plan
Eligible Content M11.C.1.2.3
Identify and/or use properties of isosceles and equilateral triangles.

Objective Identify and use the properties of isosceles and equilateral triangles.

Introduction
These are two basic geometry vocabulary words. Students should know the meaning of these terms and be able to do some basic problems involving these types of triangles.

Skill Review

M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems

Definitions

Congruent figures have the same shape and size.

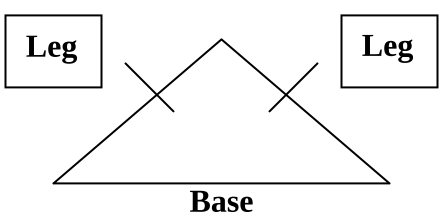
An **equilateral triangle** is a triangle with all three sides congruent.

An **isosceles triangle** is a triangle with at least two sides congruent. By definition an equilateral triangle is also isosceles.

A **scalene triangle** is a triangle with no sides congruent.

Lesson

In an *isosceles triangle*, the legs and base angles are always congruent.

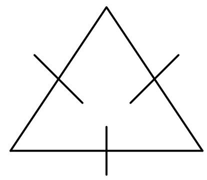


In an equilateral triangle all sides and all angles are congruent.

If you are given one angle of an isosceles triangle, you can find the other two missing angles using the fact that the two base angles are congruent and the sum of the angles of a triangle is 180°.

Also, if you are given the length of one leg then you can find the other leg, since the two are congruent. If you are given algebraic expressions for each leg you can set then equal to each other and solve the equation.

In an *equilateral triangle* all sides and all angles are congruent.

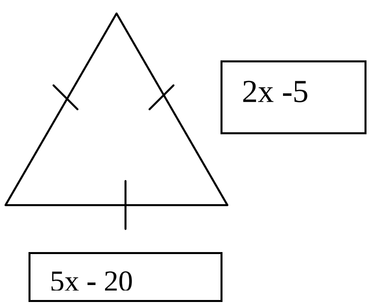


Since all of the angles of an equilateral triangle are congruent and the sum of the interior angles of a triangle is 180°, each of the angles is 60°.

x°+ x°+x°	**= 180°**	**Each angle is x° and the sum of the angles is 180°.**
3x°	**= 180°**	**Combine x terms.**
x°	**= 60°**	**Divide each side of the equation by 3.**

Examples and Work

Example 1
Find the value of x in the following problem. Then find the length of the side.



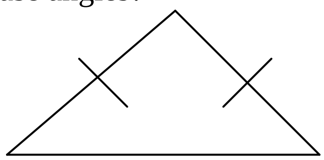
Since this is an equilateral triangle all the sides will be congruent and equal in length.

5x – 20	= 2x – 5	Set the side lengths equal.
3x - 20	= -5	Subtract 2x from each side.
3x	= 15	Add 20 to each side.
x	= 5	Divide each side by 3.

Substitute the x into each expression for the side length and simplify.
5(5) – 20 = 5
2(5) –5 = 5
The length of each side would be 5 units.

Example 2

If the vertex angle of the following isosceles triangle is 100°, then what are the measures of the base angles?



$x° + x° + 100° = 180°$	The sum of the angles of a triangle is 180°.
$2x° + 100° = 180°$	Combine x terms.
$2x° = 80°$	Subtract 100° from each side.
$x = 40$	Divide each side by 2°.

Each base angle would be 40°.

Example 3

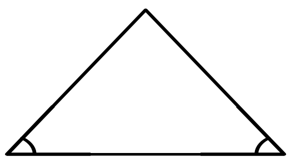
If the base angle of an isosceles triangle is 50°, then what are the measures of the other two angles?

If one base angle of an isosceles triangle is 50°, then the other base angle is also 50°. Adding the two base angles and subtracting that sum from 180 can find the vertex angle of the triangle°. In this case, 180° - (50° +50°) = 80°, so the vertex angle would be 80°.

Practice

1. Which of the following shows an equilateral triangle?

[A]



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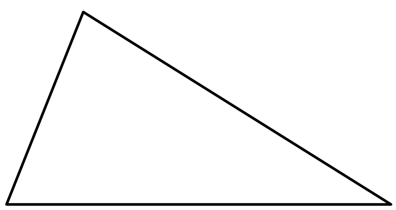
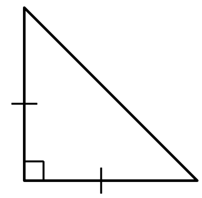
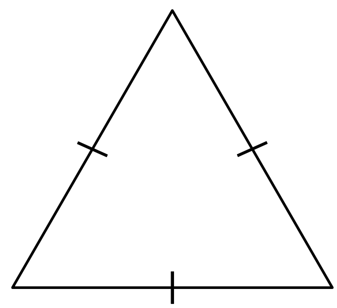


[C]



[D]



2. If the base angle of an isosceles triangle is 55°, what are the measures of the other angles?

Practice Answers

1. The answer is D since that is the only triangle with three congruent sides.

2. If one base angle of an isosceles triangle is 55°, then the other base angle is also 55°.
   Adding the two base angles and subtracting that sum from 180 can find the vertex angle of the triangle°.
   In this case, 180° - (55° +55°) = 70°, so the vertex angle would be 70°.

Math Lesson Plan
Eligible Content M11.C.1.3.1
Identify and /or use properties of congruent and similar polygons or solids.

Objectives: Use the properties of congruent and similar polygons or solids to solve problems.

Introduction: Tell students the day's lesson will be about polygons and solids. In particular, the lesson is about polygons or solids with the same size and shape or polygons and solids of similar size and shape. Sometimes certain distances and measurements that are not directly able to be measured in real life can be found using polygons and solids that are the same or similar size and shape. Blueprints and maps are just two examples where similar polygons are used.

Skill Review:

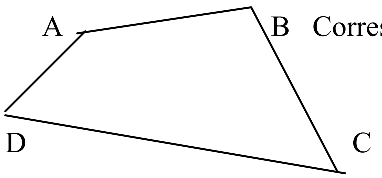
M11.A.2.1.3 Identify and/or use proportional relationships in problem solving settings.
M11.B.2.1.1 Measure and /or compare angles in degrees (up to 360˚).

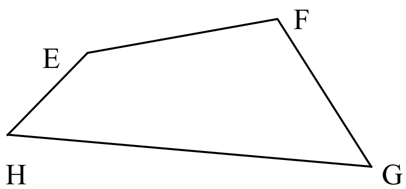
Definitions:

Congruent angles: Angles that are equal in measure.
Congruent figures: Figures that have exactly the same size and shape.
Congruent polygons: Polygons whose sides and angles can be placed in a correspondence such that corresponding sides are congruent and corresponding angles are congruent.
Congruent segments: Segments that are equal in measure.
Similar figures: Figures that have the same shape but not necessarily the same size.
Similar polygons: Polygons whose vertices can be paired in such a way that corresponding angles are congruent and corresponding sides are in proportion.
Similar solids: Two solids are similar if they have the same shape and all corresponding linear measures, such as heights and radii, are in proportion.
Proportion: A statement that two ratios are equal.

Lesson:

Ask students if they have ever read a blueprint or used a map when traveling. These are two examples where the properties of similar polygons and similar figures are used.
Explain to students that polygons are congruent if their corresponding sides are congruent (equal) and their corresponding angles are congruent (equal).
For example, in the following congruent polygons (quadrilaterals because they have 4 sides):





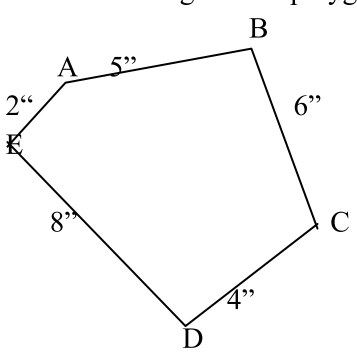
Corresponding sides are equal:

AB = EF
BC = FG
CD = GH
DA = HE

Corresponding angles are equal:

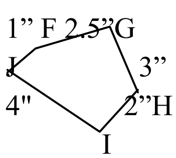
$\angle A = \angle E$
$\angle B = \angle F$
$\angle C = \angle G$
$\angle D = \angle H$

Explain to students that polygons are similar if their corresponding angles are congruent (equal) and their corresponding sides are proportional (equal ratios).
In the following similar polygons which are pentagons because they have five sides:



Corresponding angles are congruent (equal):

$\angle A = \angle F$
$\angle B = \angle G$
$\angle C = \angle H$
$\angle D = \angle I$
$\angle E = \angle J$



Corresponding sides are proportional:

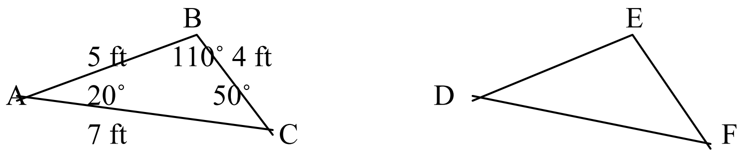
$$\frac{AB}{FG} = \frac{BC}{GH} = \frac{CD}{HI} = \frac{DE}{IJ} = \frac{EA}{JF}$$

$$\frac{5}{2.5} = \frac{6}{3} = \frac{4}{2} = \frac{8}{4} = \frac{2}{1}$$

Each ratio reduces to $\frac{2}{1}$.

Examples:

1. $\Delta ABC \cong \Delta DEF$



Find the lengths of DE, EF and FD

<u>Work</u>:

Because the corresponding sides of congruent triangles are congruent (equal):

DE = 5 ft

EF = 4 ft

FD = 7 ft

Find the measures of ∠D, ∠E, and ∠*F*:

Because corresponding angles of congruent triangles are congruent (equal):

∠D = 20˚

∠E = 110˚

∠F = 50˚

2. Find the height of the pole using shadows.

<u>Work</u>: Measure the shadow of the pole. Then have someone measure your shadow.

The triangle formed by the pole, its shadow, and a line segment from the end of the shadow to the top of the pole is similar to:

The triangle formed by you, your shadow, and a line segment from the end of your shadow to the top of your head.

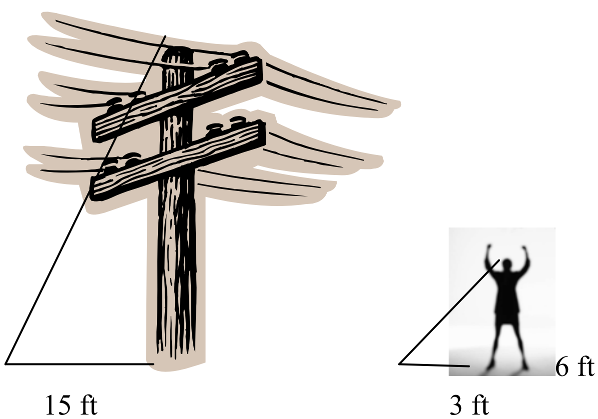
Since corresponding sides of similar polygons are proportional the following proportion can be used to find the height of the pole:

$$\frac{\text{Height of pole}}{\text{Shadow of pole}} = \frac{\text{Height of person}}{\text{Shadow of person}}$$

$$\frac{H}{15} = \frac{6}{3}$$

$$3H = 6 \times 15$$

Height of pole is 30 feet.



Practice:

1. A customer has a triangular flower bed outlined with treated wood. The dimensions of the triangle are 6 feet 8 inches, 4 feet 10 inches, and 9 feet 3 inches. You are asked to construct another flower bed congruent to the first one. What is the total length of the treated wood you will need to buy to construct the second congruent flower bed?

   (A) 19.21 feet

   (B) 20 feet 9 inches

   (C) 21.10 feet

   (D) 20.9 feet

2. You are given the blueprint for the floor plan of a three bedroom home. The scale ratio is 1 in. : 16 ft. If the family room measures 1 ¼ inches by ¾ inches on the blueprint, what are the dimensions of the family room?

Practice Answers:

1. (B)

Work:
Since corresponding sides of congruent triangles are congruent (equal in length), add:

6 feet 8 inches
4 feet 10 inches
<u>9 feet 3 inches</u>
19 feet 21 inches

Since 1 foot equals 12 inches, subtract 12 inches from 21 inches. Add this 1 foot to 19 feet.
The final answer is 20 feet 9 inches.

2. Since a blueprint and an actual room are similar figures, their dimensions are proportional. That means that their dimensions are equal ratios.

Work:

$$\frac{1 \text{ in}}{16 \text{ ft}} = \frac{1\,\frac{1}{4} \text{ in}}{\text{length}} \qquad\qquad \frac{1 \text{ in}}{16 \text{ ft}} = \frac{\frac{3}{4} \text{ in}}{\text{width}}$$

$$1\,(\text{I}) = 16\,(1\,\tfrac{1}{4}) \qquad\qquad 1\,(\text{w}) = 16\,(3/4)$$

$$1 = 20 \text{ feet.} \qquad\qquad \text{W} = 12 \text{ feet.}$$

The actual dimensions of the family room would be 20 feet by 12 feet.

Math Lesson Plan
Eligible Content M11.C.1.4.1
Find the measure of the side of right triangle using the Pythagorean Theorem

Objective: Use the Pythagorean Theorem to solve problems involving right triangles.

Introduction

Tell students that the day's lesson will be about triangles and ask students to define triangle (three sided polygon). Discuss the need to work with triangles in the construction process. Ask students for examples when triangles are used or apparent in construction, and in everyday life. Sometimes we need to determine the area of the triangle, and sometimes we need to know the length of the sides. We may not have all of the information, so we need to calculate what we don't have.

Skill Review

M11.A.1.1.1 Find the square root of an integer to the nearest tenth using a calculator.
M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems

Definitions

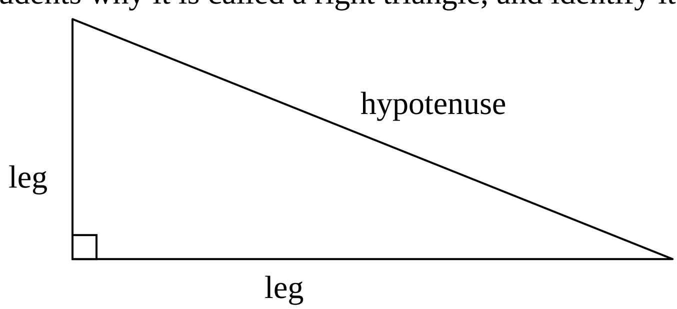
Right Triangle
A triangle with one right angle

Legs of a Right Triangle
The sides of the triangle that form the right angle

Hypotenuse
In a right triangle, the side that is opposite the right angle. The hypotenuse is always the longest side of the right triangle

Lesson

Ask students about the different types of triangles, and be sure to identify the right triangle. Ask students why it is called a right triangle, and identify its characteristics.



Pythagorean Theorem – If a triangle is a right triangle with legs lengths a and b and hypotenuse of length c, then $a^2 + b^2 = c^2$.

* This formula is included on the PSSA formula sheet

$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

Examples

Find the length of the hypotenuse of a right triangle whose legs measure 8m and 15m.

<u>Work</u>

$a^2 + b^2 = c^2$

$8^2 + 15^2 = c^2$

$64 + 225 = c^2$

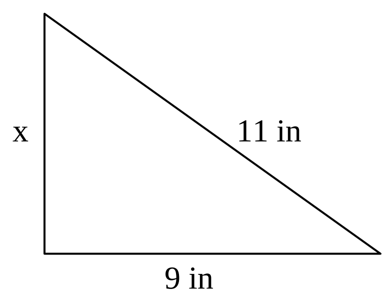
$289 = c^2$

$\sqrt{289} = c$

$17 = c$

The length of the hypotenuse is 17 m.

Find x to the nearest hundredth of an inch.



<u>Work</u>

$a^2 + b^2 = c^2$

$x^2 + 9^2 = 11^2$

$x^2 + 81 = 121$

$x^2 + 81 - 81 = 121 - 81$

$x^2 = 40$

$x = \sqrt{40}$

$x \approx 6.32$ in

June 2008

Practice

1. A carpenter wants to use a 12-ft. ladder to reach the top of a 10-ft wall. How far must the base of the ladder be from the base of the wall? Round the answer to the nearest hundredth.

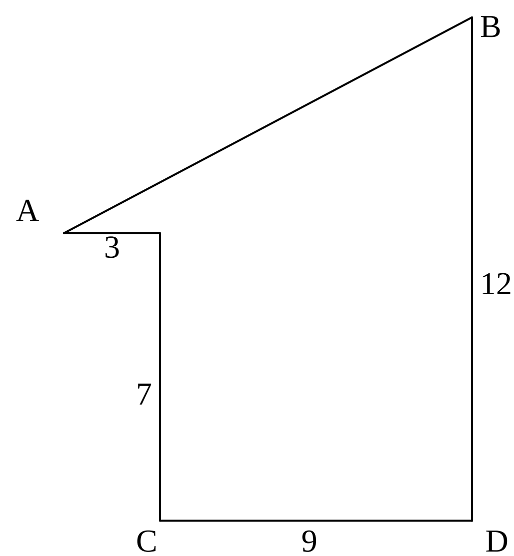
   (A) 15.62 feet

   (B) 10 feet

   (C) 6.63 feet

   (D) 1.41 feet

2. Find the length of segment AB.



Practice Answers

1. C

   The height of the wall represents one leg of the right triangle. The length of the ladder represents the hypotenuse of the triangle.

   $a^2 + b^2 = c^2$

   $x^2 + 10^2 = 12^2$

   $x^2 + 100 = 144$

   $x^2 + 100 - 100 = 144 - 100$

   $x^2 = 44$

   $x = \sqrt{44}$

   $x = 6.63$ feet

2. To find the length of the vertical leg, take 12 (the total height) and subtract 7 (the height of the rectangle. The find the length of the horizontal leg, take 9 (the length of the rectangle) and add 3 (the additional length of the triangle).

   The leg lengths of 5 and 12 will be used to calculate the length of the hypotenuse AB.

   $a^2 + b^2 = c^2$

   $5^2 + 12^2 = c^2$

   $25 + 144 = c^2$

   $169 = c^2$

   $\sqrt{169} = c$

   $13 = c$

   The length of the hypotenuse is 13 m.

Math Lesson Plan
Eligible Content M11.C.3.1.1
Calculate the distance and/or midpoint between 2 points on a number line or a coordinate plane (formulas provided on the reference sheet)

Objective: Solve problems using distance formula and midpoint formula.

Introduction

A formula can be used to calculate the distance between two points on the coordinate plane. A second formula can be used to determine the midpoint of the segment connecting any two points on the coordinate plane.

Skill Review

M11.A.1.1.1 Find the square root of an integer to the nearest tenth using a calculator.
M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems

Definitions

<u>Distance</u>
The absolute value of the difference between the coordinates of any two points. Distance is always positive.

<u>Midpoint</u>
The point that divides a segment into two congruent segments.

Lesson

Given two points on the coordinate plane (x_1, y_1) and (x_2, y_2), use the following formulas to calculate the distance between the points and the coordinates of the midpoint of the segment between the points.

Distance Formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Use the values given in the problem, and substitute these values into each formula. Both formulas are provided on the 11th Grade PSSA Formula sheet.

Examples

Find the distance between the points (2, −5) and (6, −10). Round the answer to the nearest tenth.

<u>Work</u> Use the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ where

$x_1 = 2$ $x_2 = 6$ $y_1 = -5$ $y_2 = -10$

Distance $= \sqrt{(6-2)+(-5-(-10))}$

$= \sqrt{(4)^2 + (5)^2}$

$= \sqrt{16+25}$

$= \sqrt{41}$

≈ 6.4

Find the midpoint of the segment connecting the points (0, −4) and (−3, 8)

<u>Work</u> Use the formula $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ where

$x_1 = 0$ $x_2 = -3$ $y_1 = -4$ $y_2 = 8$

Midpoint $= (\frac{0+-3}{2}, \frac{-4+8}{2})$

$= (\frac{-3}{2}, \frac{4}{2})$

$= (-1.5, 2)$ or $(\frac{-3}{2}, 2)$

Practice

1. What is the midpoint of the line segment connecting the points (−1, 5) and (3, 8)?

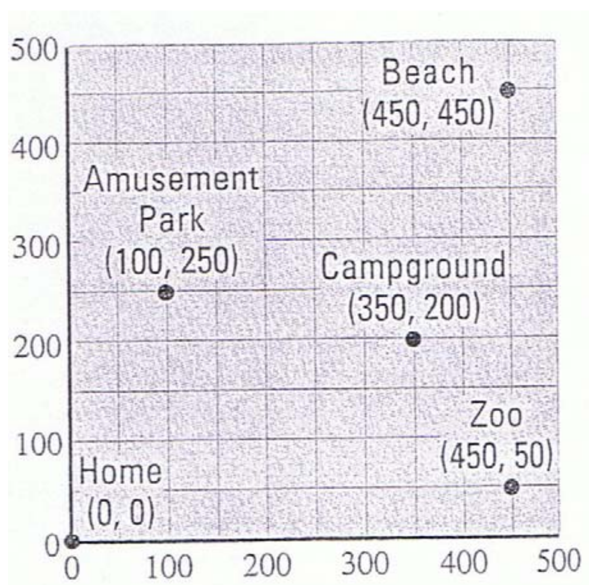
   (A) $(1, \frac{13}{2})$

   (B) $(2, \frac{13}{2})$

   (C) $(1, \frac{3}{2})$

   (D) $(2, \frac{3}{2})$

2. You are planning a family vacation. Each side of a square in the coordinate that is superimposed on the map represents 50 miles. You leave your home and go to the amusement park. After visiting the park, you go to the beach. Then you return home. How far did you travel?



Practice Answers

1. A

Midpoint $= \dfrac{-1+3}{2}, \dfrac{5+8}{2})$

$= (\dfrac{2}{2}, \dfrac{13}{2})$

$= (1, \dfrac{13}{2})$

2. The coordinates of the home location are (0, 0).

The coordinates of the amusement park are (100, 250).

The coordinates of the beach are (450, 450).

To find the total distance traveled, calculate the distance between each set of coordinates and total the distances.

Distance (home to park)

$= \sqrt{(0-100)^2+(0-250})^2$

$= \sqrt{(-100)^2+(-250)^2}$

$= \sqrt{10000+62500}$

$= \sqrt{72500}$

$\approx$ 269.3 mi

Distance (park to beach)

$= \sqrt{(100-450)^2+(250-400})^2$

$= \sqrt{(-350)^2+(-200})^2$

$= \sqrt{122500+40000}$

$= \sqrt{162500}$

$\approx$ 403.1 mi

Distance (beach to home)

$= \sqrt{(450-0)^2+(450-0})^2$

$= \sqrt{(450)^2+(450)^2}$

$= \sqrt{202500+202500}$

$= \sqrt{405000}$

$\approx$ 636.4 mi

Total mileage 1308.8 miles

Math Lesson Plan
Eligible Content M11.C.3.1.2
Relate slope to perpendicularity and/or parallelism (limit to linear algebraic expressions; slope formula provided on the reference sheet).

Objective: To find the slope of parallel and perpendicular lines.

Introduction

A formula can be used to calculate the slope of lines on the coordinate plane. The definitions and theorem of parallel lines and perpendicular lines will enable to find their slopes.

Skill Review

M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems
M11.D.2.1.3 Write, solve, and/or apply a linear equation
M11.d.3.2.3 Slope and y-intercept

Definitions

Slope of a line
The ratio of the vertical rise to the horizontal run between any two points on a line.

Slope m = $\frac{rise}{run}$ (the letter m is used to designate slope)

Parallel
Parallel lines are coplanar lines that do not intersect.

Perpendicular lines
Two lines that intersect to from right angles are called perpendicular lines.

Lesson

Slope can be calculated between two points on a line using the coordinates of the points. To calculate slope using the points (x_1, y_1) and (x_2, y_2), use the formula

$$slope \quad m = \frac{rise}{run} = \frac{change\ in\ y - cooridnates}{change\ in\ x - coordinates} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept form of an equation: $y = mx + b$

Equations of a lines between tow points: $y - y_1 = m(x - x_1)$

Use the values given in the problem, and substitute these values into each formula. These formulas are provided on the 11th Grade PSSA Formula sheet.

Two lines with the same slope and different y-intercepts are parallel. Conversely, two lines are parallel is they have the same slope and different y-intercepts.

Two lines are perpendicular if the product of their slopes is -1. Conversely, if the slopes of two lines are negative reciprocals of each other, then the lines are perpendicular.

Examples

Find the slope of a line parallel to the given line and a line perpendicular to the given line containing points A(-2, 5) and B(0, -1).

<u>Work</u>

Use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ where

$x_1 = -2$ $x_2 = 0$ $y_1 = 5$ $y_2 = -1$

Slope of original line $= \frac{-1-5}{0--2} = \frac{-6}{2} = 3$

Slope of a parallel line = 3

Slope of a perpendicular line $= -\frac{1}{3}$

Determine whether each pair of lines are *parallel*, *perpendicular*, or *neither*.

$$\begin{cases} 7x + 2y = 14 \\ 7y = 2x - 5 \end{cases}$$

<u>Work</u>

Determine the slope of each line: y = mx + b where m = slope

7x + 2y = 14	7y = 2x - 5
2y = -7x + 14	$y = \frac{2}{7}x - \frac{5}{7}$
$y = -\frac{7}{2}x + 7$	$m = \frac{2}{7}$
$m = -\frac{7}{2}$	

The slopes are opposite reciprocals, so the lines are perpendicular.

Practice

1. Determine whether each pair of lines are *parallel*, *perpendicular*, or *neither*.

$$\begin{cases} -5x + 3y = 2 \\ 3x - 5y = 15 \end{cases}$$

(A) parallel
(B) perpendicular
(C) neither
(D) both parallel and perpendicular

2. Find the slope of a line parallel to the given line and a line perpendicular to the give line containing the points M(8, 3) and N(-1, 5)

Practice Answers

1. C

$-5x + 3y = 2$	$3x - 5y = 15$
$3y = 5x + 2$	$-5y = -3x + 15$
$y = {}^{5}/_{3}x + {}^{2}/_{3}$	$m = {}^{3}/_{5}\,x - 3$
$m = {}^{5}/_{3}$	$m = {}^{3}/_{5}$

The slopes are not the same and they are not opposite reciprocals, so the lines are neither.

2. slope of line $m = \dfrac{5-3}{-1-8} = \dfrac{2}{-9}$

slope of line parallel = $-\dfrac{2}{9}$

slope of line perpendicular = $\dfrac{9}{2}$

Math Lesson Plan
Eligible Content M11.D.1.1.1
Demonstrate an understanding of patterns, relations and functions.

Objective: Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.

Introduction

Tell students that the day's lesson will be about learning to recognize and describe patterns that will allow them to make accurate predictions or generalizations. The students will represent the pattern algebraically or graphically.

Skill Review

None

Definitions

Pattern – an arrangement of numbers, pictures, and/or other forms of data.

Conjecture – an opinion or conclusion formed by recognizing a pattern in data. A conjecture can be true or false.

Sequence – a set of elements that are arranged in a particular pattern.

Lesson

Given a sequence to study, a pattern will be recognized in order to make a conclusion.

Procedure:

1. Observe the given data. The data could be the terms in a sequence, figures in a pattern, entries in a table, etc.

2. Look for and describe a pattern.

3. State a conclusion (conjecture) based on your observing the data and recognizing a pattern.

Examples

1. What is the next term in the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$?

<u>Work</u>

The pattern is one-half of the previous term; $1 \cdot \frac{1}{2} = \frac{1}{2}$, $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$.
Therefore, the next term in the sequence is $\frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$.

2. Analyze the following sequence and represent the pattern algebraically.
   3, 6, 12, 18,….

<u>Work</u>

Let 3 = x. 6 is 2 times x, 12 is 4 times x, 18 is 6 times x. Therefore, the algebraic pattern is x, 2x, 4x, 6x,…

3. Which of the following best describes the pattern 4, 8, 12, …?

   A. 1 + n, 4 + n, 8 + n, …

   B. $n^2, n^3, n^4, \ldots$

   C. n, 2n, 3n, …

   D. $n, \frac{n}{2}, \frac{n}{3}, \ldots$

(Pennsylvania Department of Education)

<u>Work</u>

C.

4 = n, 8 = 2 x 4 = 2n, 12 = 3 x 4 = 3n,…

Practice

1.

If this pattern of dot-figures is continued, how many dots will be in the 100th figure?

A. 100
B. 101
C. 199
D. 200
E. 201

(Pennsylvania Department of Education)

2. Sam made the following conjecture after testing his theory with several numbers: The square root of a number is always smaller than the number you start with." Is Sam's conjecture true? Why or why not?

Practice Answers

1. C

$1^{st} = n \quad = 1$
$2^{nd} = n + 2 = 3$
$3^{rd} = n + 4 = 5$
$4^{th} = n + 6 = 7$
.
.
.
$100^{th} = n + 198 = 199$

2. No.

Square roots like $\sqrt{100} = 10$, $\sqrt{81} = 9$, $\sqrt{16} = 4$, $\sqrt{169} = 13$, $\sqrt{4} = 2$ fit Sam's theory. However, $\sqrt{1} = 1$, and $\sqrt{\frac{1}{4}} = 1/2$. In these cases, the number you start with can be greater than or equal to the square root.

Math Lesson Plan
Eligible Content M11.D.1.1.2
Determine if a relation is a function given a set of points or a graph.

Objective: Identify functions.

Introduction

A set of ordered pairs is defined as a relation. Relations can be represented by a table of values, a set of ordered pairs, a mapping, or a graph. Certain relations are considered to be functions.

Skill Review

None needed

Definitions

<u>Relation</u>
Any set of ordered pairs.

<u>Function</u>
A rule that establishes a relationship between two quantities, the input and the output. There is exactly one output for each input.

Lesson

A function is a type of relation that has exactly one output for each input.. For example, in the set of ordered pairs below:

$$(1, 8) \quad (3, -5) \quad (2, 6) \quad (8, -10) \quad (5, -14)$$

Each input (x-coordinate) has exactly one output (y-coordinate).

In the table below, the input 3 has been mapped to two different outputs −7 and 2. This relation is not a function.

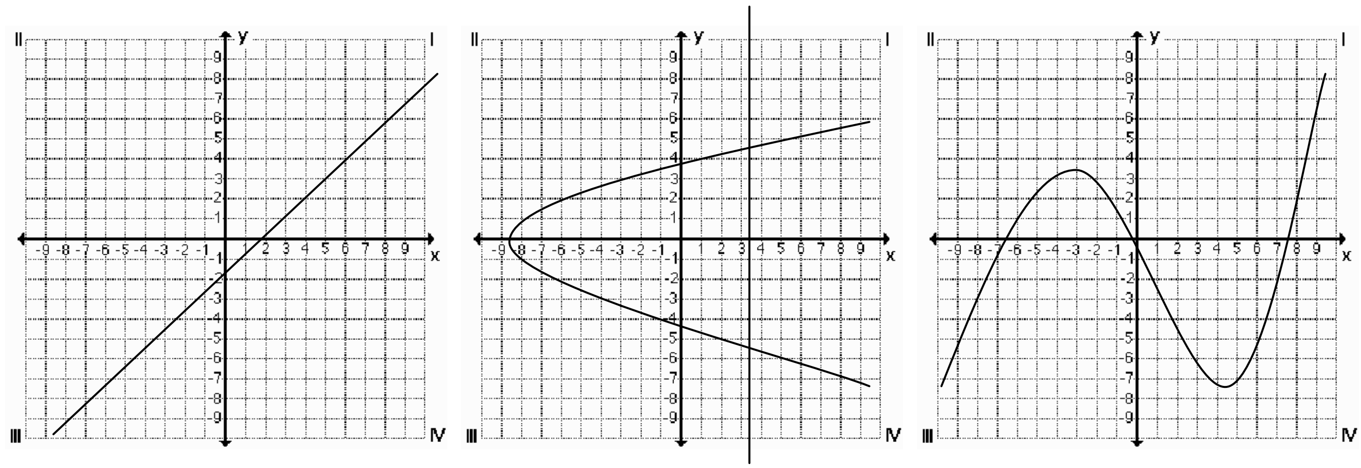
x	y
1	5
2	0
3	-7
3	2
4	8
5	-3

In the mapping below, the relation is a function because each input is mapped to exactly one output.

$$\begin{array}{c} 0 \rightarrow -4 \\ 1 \rightarrow \ \ 2 \\ -1 \rightarrow -2 \end{array}$$

When you graph a function, the input is given by the horizontal (x) axis and the output is given by the vertical (y) axis. To determine if the graph of a relation is a function, the graph must pass the vertical line test. A graph is a function if no vertical line intersects the graph at more than one point.

A. B. C.



Graph A passes the vertical line test because no vertical line intersects the graph more than once. Graph A is a function.

Graph B does not pass the vertical line test (see above). Graph B is not a function.

Graph C passes the vertical line test because no vertical line intersects the graph more than once. Graph C is a function.

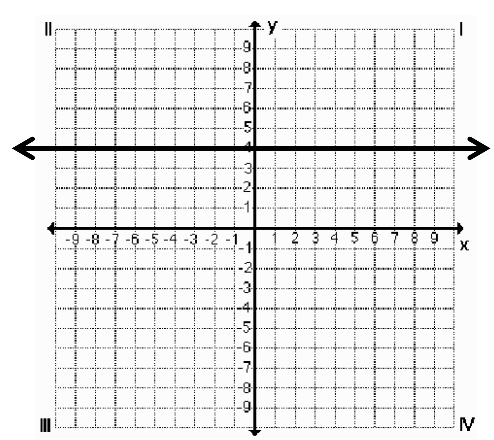
Examples

Does this set of ordered pairs represent a function? $(2, 4)$ $(5, -3)$ $(4, 6)$ $(8, -3)$ $(9, -12)$

Work — This set of ordered pairs does represent a function because none of the input values are repeated (it does not matter that the output of -3 was repeated).

Does this graph represent a function?

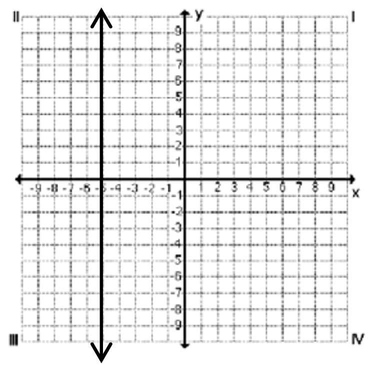
Work — Yes, the graph does pass the vertical line test.



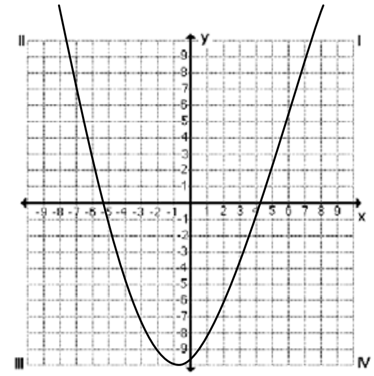
Practice

1. Which of the following graphs is a function?

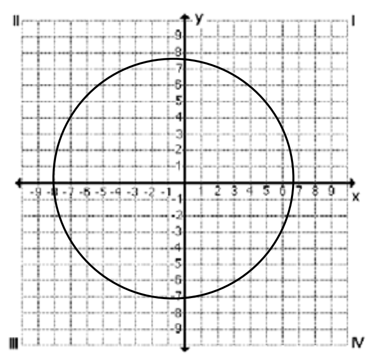
A.



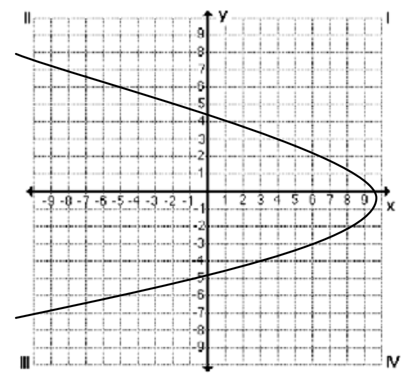
B.



C.



D.



2. The table shows the number of shots attempted and the number of shots made by 9 member of the Philadelphia 76ers in Game 1 of the 2001 NBA Finals.

Player	Shots attempted	Shots made
Bryon Russell	12	6
Karl Malone	25	9
Greg Foster	5	1
Jeff Hornacek	10	2
John Stockton	12	9
Howard Eisley	6	4
Chris Morris	6	3
Greg Ostertag	1	1
Shandon Anderson	5	3

Is this relation a function? Explain.

Practice Answers

1. B. This is the only graph that passes the vertical line test.

2. This relation is not a function because the input of 6 shots attempted is repeated.

Math Lesson Plan
Eligible Content M11.D.1.1.3
Identify the domain, range, or inverse of a relation (may be presented as ordered pairs or a table)

Objective: Given a relation, determine the domain, range, and/or inverse of the relation.

Introduction

In math ordered pairs are often used to describe locations and to record scientific data. Points on a plane are named so that each name refers to exactly one point. The name consists of two numbers, called coordinates of the point. By using two real number lines together, any set of ordered pairs can be graphed. These ordered pairs can be separated into input values and output values. This information can be used to identify the inverse of a relation.

Skill Review

None needed

Definitions

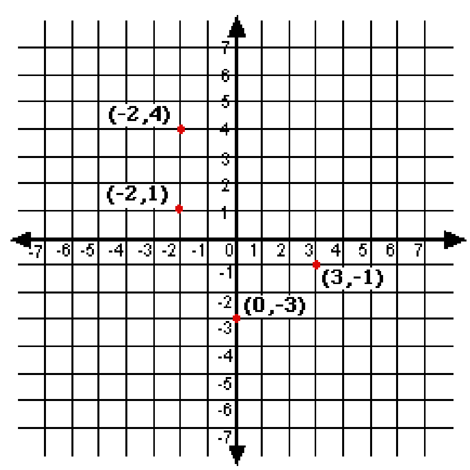
Domain
The set of all input values (typically x values).

Range
The set of all output values (typically y values).

Inverse
The relation that results from interchanging the first and second coordinates of each ordered pair in a relation represented by a set of ordered pairs.

Lesson

Ordered pairs are often used to describe locations. Points on a plane are named so that each name refers to exactly one point. The graph of each ordered pair (x,y) is a point in the coordinate plane. The name consists of two numbers, called coordinates of the point. The first number is the x-coordinate and the second number is the y-coordinate.



A relation is a set of ordered pairs. The domain is the set of first coordinates of the ordered pairs. The range is the set of the second coordinates of the ordered pairs.

The graph at the right consists of four points. It can be described by a set of ordered pairs, C = {(-2, 4), (-2, 1), (0, -3), (3, -1)}, which gives the coordinates of the points of the graph. Set C is called a relation.

For C, the set of all first coordinates {-2, 0, 3}, is called the domain of the relation. Notice that -2 is listed only once. The set of all second coordinates of C, {4, 1, -3, -1}, is called the range of the relation.

Each relation has an inverse. The inverse of any relation is obtained by switching the coordinates in each ordered pair of the relation.

Given C = {(3, 2), (4, -6), (3, -4), (-1, -6)}

The inverse of C = {(2, 3), (-6, 4), (-4, 3), (-6, -1)}

Examples

State the domain and range of the following relation. List the inverse.

{(2, –3), (4, 6), (3, –1), (6, 6), (2, 3)}

Work:

domain: {2, 3, 4, 6} the set of all input (x) values (do not list any number more than once)

range: {–3, –1, 3, 6} the set of all output (y) values (do not list any number more than once)

inverse: {(−3, 2), (6, 4), (−1, 3), (6, 6), (3, 2)} (reverse the order of each ordered pair)

State the domain and range of the following relation. List the inverse.

x	y
−3	4
4	3
2	0
−3	7
1	−3
2	−7

Work:

domain: {−3, 1, 2, 3, 4} the set of all input (x) values (do not list any number more than once)

range: {−7, −3, 0, 3, 4, 7} the set of all output (y) values (do not list any number more than once)

inverse: {(4, −3), (3, 4), (0, 2), (−3, 7), (1, −3)}

Practice

1. What set of ordered pairs has a range of $\{-2, 4, 6, 8, 11\}$?

   A. $\{(-2, 5), (4, 9), (6, 8), (8, 10), (11, 5)\}$

   B. $\{(1, 2), (8, 4), (1, 6), (0, 8), (12, 11)\}$

   C. $\{(3, -2), (-1, 4), (4, 6), (20, 8), (-3, 11)\}$

   D. $\{(-6, 2), (5, 2), (9, 7), (-5, 12), (13, 8)\}$

2. The Smith family is looking for a new home. They collected information about the houses available in town.

Price (in thousands of dollars)	Area (in square feet)
325	4400
89	2250
101.8	2950
348	4400
219.9	3300
269.5	3800
125.8	2800
219.5	3500

   A. Determine the domain of the relation.

   B. Determine the range of the relation.

   C. List the inverse of the relation.

Practice Answers

1. C

2. Domain {89, 101.8, 125.8, 219.5, 219.9, 269.5, 269.5, 325, 348}

   Range {2250, 2800, 2950, 3300, 3500, 3800, 4400}

   Inverse

4400	325
2250	89
2950	101.8
4400	348
3300	219.9
3800	269.5
2800	125.8
3500	219.5

Math Lesson Plan
Eligible Content M11.D.2.1.1
Solve compound inequalities and /or graph their solution sets on a number line (may include absolute inequalities).

Objective: Solve and graph compound inequalities on a number line.

Introduction:
Explain to students that today's lesson will be about inequalities. Measurements are not always exact. If a person is making a product, there will be a certain tolerance as far as the measurements are concerned. For example, machine parts may have a tolerance of ±1 millimeter. A part that is suppose to measure 80 millimeters could measure as much as 80 + 1 or 81 millimeters or as little as 80 – 1 or 79 millimeters and still be functional. These inequalities will be graphed on a number line.

Skill Review:

M11.A.3.1.1 Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).

Definitions:

Inequality: A statement of unequal expressions using the symbols "less than" (<) or " less than or equal to" (≤), or "greater than" (>) or "greater than or equal to" (≥).

Compound inequality: Two inequalities joined by *and* or by *or*.

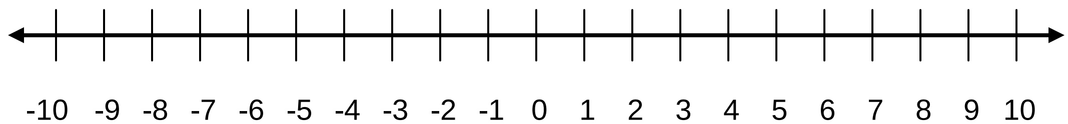
Conjunction: A compound inequality joined by *and*. Example: $-5 < t < 70$

Disjunction: A compound inequality joined by *or*.

Absolute value of a number: The distant from zero on a number line (Absolute value is never negative.) Absolute value is indicated by two vertical lines. Examples: $|5| = 5$ and $|-5| = 5$

Lesson:

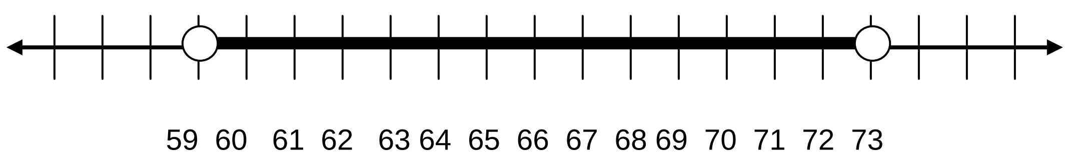
Ask students if they are familiar with the “less than” (<), “less than or equal to” (≤), “greater than” (>) and “greater than or equal to” (≥) symbols.
Also review a number line with students.



When graphing a conjunction compound inequality such as -5<t< 70 (t< 70 and t> -5), the number line will be “darkened” between the circles.
When graphing a disjunction compound inequality such as m<2 or m> 10, the number line will have arrows going in opposite directions. This will be demonstrated in the following examples:
Note: When multiplying or dividing an inequality by a negative number, the direction of the inequality changes. Example: -5 ≤ 2 becomes 5≥-2 when both sides of the inequality are multiplied be -1.

Examples:

1. Certain medicines, paints, etc. must be stored at appropriate temperatures.
   For example, if a label says store between 59˚ and 73˚ F, this can be represented as the compound inequality: t<73˚ and t>59˚. This compound inequality is called a <u>conjunction</u> because they are joined by “<u>and</u>”. This compound inequality is graphed on a number line with “***<u>open</u>***” circles at 59 and 73 because *t* ***<u>may not equal</u>*** 73˚ and 59˚.Then <u>“darken” the number line between</u> 59 and 73 to indicate the temperatures between 59 and 73 satisfy the compound inequality. This compound inequality can also be written as 59˚<t<73˚.



2. A compound inequality that is joined by "<u>or</u>" is called a <u>disjunction</u>.
   For example, suppose you are given the compound inequality: $\frac{X+3}{2} \le 1$ or $9x + 2 \ge 11$ Use basic rules of algebra to isolate x.

<u>Work</u>:

$\frac{x+3}{2} \le 1$ Multiply both sides of the inequality by 2.

$X + 3 \le 2$ Subtract 3 from both sides of the inequality.

$\underline{\;\;-3\;\;-3}$

$X \le -1$

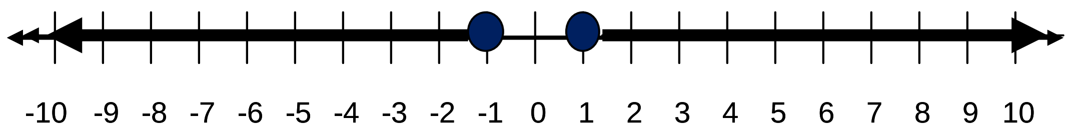
$9x + 2 \ge 11$ Subtract 2 from both sides.

$\underline{\;\;\;\;-2\;\;\;-2}$

$9x \ge 9$ Divide both sides by 9.

$X \ge 1$

Draw "***<u>closed</u>***" circles at -1 and 1 because x ***<u>may equal</u>*** these numbers.
Then draw an arrow to the right of 1 because x can be greater than 1 and draw an arrow to the left of -1 because x can be less than -1.



3. Solve $| 9x + 18 | \le 45$. Less than or equal to indicates a <u>conjunction</u> for absolute value.

Note: For an absolute value inequality, the positive and the ***<u>negative</u>*** of the number must be used. (Greater than or equal to would indicate a disjunction. It would be written as: $9x+18<-45$ or $9x+18>45$. The graph of $>$ or $\ge$ inequality would have arrows going in opposite directions.)

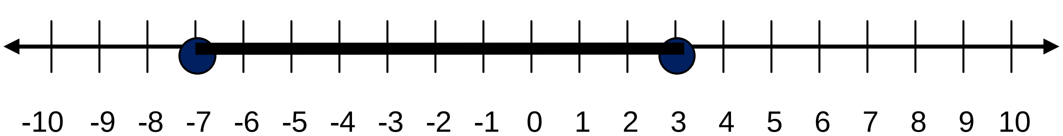
<u>Work:</u> Therefore $-45 \le 9x + 18 \le 45$ Subtract 18 from each section.

$\underline{-18 \qquad -18 \;\; -18}$

$-63 \le 9x \le 27$ Divide each expression by 9.

$-7 \le x \le 3$ Draw closed circles at -7 and 3 because x may equal -7 and 3.

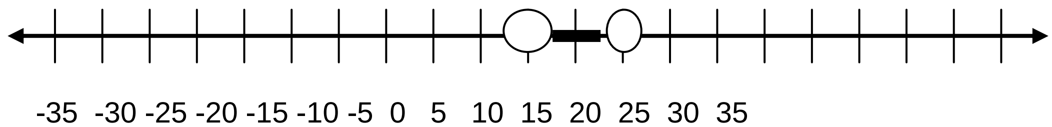
Then darken the number line to the left of 3 because x can be less than 3 and to the right of -7 because x can be greater than -7.



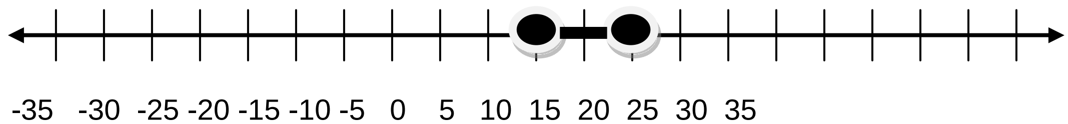
Practice:

1. If a certain product is to be stored upright between 15˚C and 25˚C, what is the compound inequality that represents these conditions where t stands for temperature?

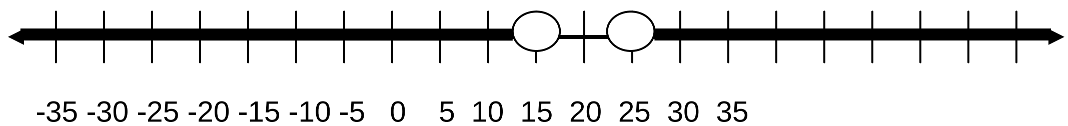
(A) 15˚C< t <25˚C



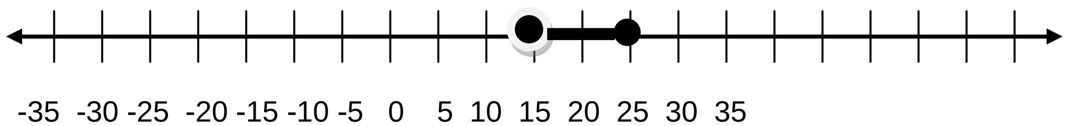
(B) t≤ 25˚C and t≥15˚C



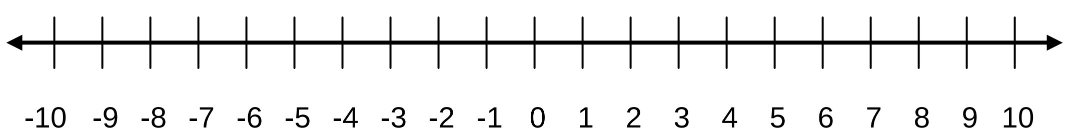
(C) t< 15˚C or t>25˚C



(D) t≤15˚C or t≥25˚C

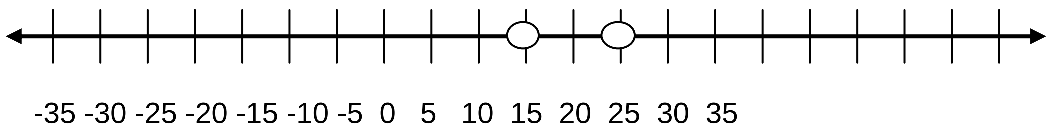


2. Given l 2x -4 l ≥ 2, solve the absolute value inequality and graph it.



Practice Answers:

1. (A) Because the temperature must be between 15˚C and 25˚C, t< 25˚C and t>15˚C or 15<t<25.
Since the temperature may not equal 15˚C and 25˚C, there would be open circles at 15 and 25.
In addition, this is a conjunction because it can be written as an "and" statement.
The graph of a conjunction consists of the number line "darkened" between 15 and 25.



2. l 2x-4 l ≥ 2.

Work:

Note: When working with absolute inequalities, the positive and ***negative*** number must be used.
This absolute value compound inequality can be written as:

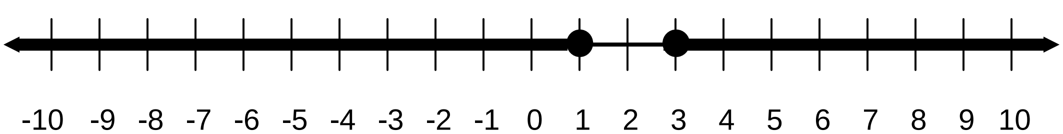
$2x - 4 \leq -2$ or $2x - 4 \geq 2$ Use the rules of algebra to solve for x.

+4 +4 +4 +4 Add 4 to each side of the inequalities.

$\frac{2x}{2} \leq \frac{2}{2}$ or $\frac{2x}{2} \geq \frac{6}{2}$ Divide each side of the inequalities by 2.

X ≤1 or x ≥ 3

Because x may equal 1 or 2, there would be closed circles at 1 and 3.
Since this is an "or" statement, this is a disjunction.
Therefore, arrows would be drawn in opposite directions from 1 and 3.



Math Lesson Plan
Eligible Content M11.D.2.1.2
Identify or graph functions, linear equations or linear inequalities on a coordinate plane.

Objective: Graph functions, linear equations or linear inequalities on a coordinate plane.

Introduction:

Tell students that many real-life situations can be represented by equations.
The graphs of these equations can show whether quantities such as costs, profits, temperature, etc. are increasing or decreasing. The graphs of inequalities can show if certain quantities are within an acceptable range.

Skill Review:

M11.A.3.1.1 Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).
M11.A.1.1.2 Determine if a relation is a function given a set of points or a graph.
M11.D.4.1.1 Match the graph of a given function to its table or equation.

Definitions:

Linear Function: A linear function is a function which can be written in the form f(x) = mx + b or y = mx +b where *m* and *b* are constants in the form of real numbers.
m is the slope of the graph of the line and ***b*** is the number where the graph of the line crosses the y-axis (*b* is called the y-intercept).

Linear Inequality: A linear inequality in two variables is an inequality that can be written in one of the following forms: $Ax + By < C$, $Ax + By \leq C$, $Ax + By > C$, $Ax + By \geq C$, where A, B, and C are real numbers.

Lesson:

Ask students if they have ever graphed lines or inequalities. Give them examples of graphs that you have encountered in your career.
Explain to students that equations such as $6x + 2y = 10$ and $y = -3x + 5$ are linear equations.
The graphs of these equations will be a straight line on a coordinate system.
Tell the students that inequalities such as $2x - 3y < 8$ and $-6x + 7y \geq 9$ are linear inequalities.
The graphs of these inequalities will be a straight line which is shaded on one side of the line.

Examples:

1. Here are three ways to graph the linear equation $y = 2x + 5$:

a. Let $x = 0$ and solve for y: $y = 2(0) + 5$, resulting in the point (0, 5).

Let $y = 0$ and solve for x: $0 = 2x + 5$, resulting in the point (-5/2, 0).

$\underline{-5 \qquad -5}$ Subtract 5 from both sides of the equation.

$-5 = 2x$ Divide both sides by 2.

$-5/2 = x$

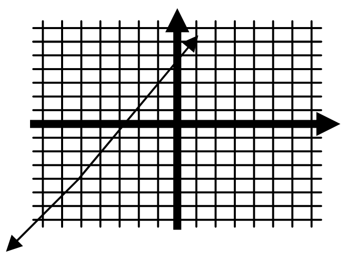
Then graph both points and connect them to form a line.

b. Make a table of values for x and y by picking values for x and substituting them into the equation to find y:

Chose x	-1	0	1
Find y	3	5	7

Then graph these points and connect them to form a line.

c. Use the fact that in $y = 2x + 5$, 5 is where the graph of the line crosses the x-axis and 2 is the slope. Therefore, count 2 squares up from 5 and 1 square to the right to find a second point on the graph. In all cases, the graph will be:

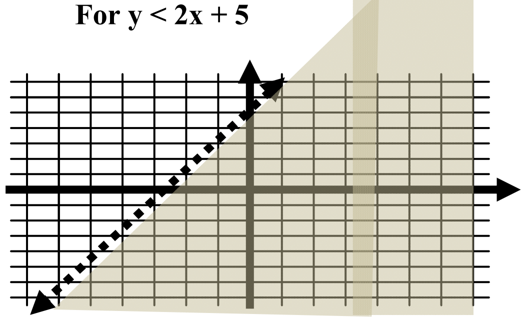


1. The graph of a linear inequality in two variables consists of a boundary line dividing the graph into two half-planes. The half-plane that is shaded contains the points that are solutions of the inequality.

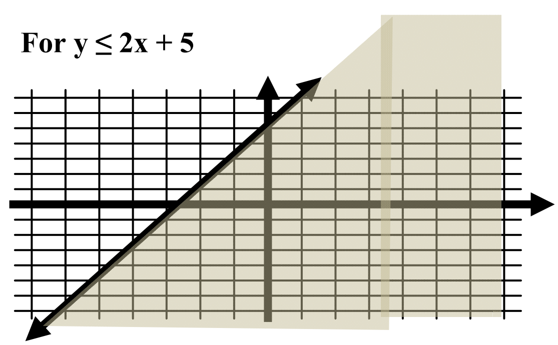
a. First graph the boundary line as outlined in example 1.
Use a dashed line for < or >. Use a solid line for ≤ or ≥.

b. Second, to decide which side of the boundary line to shade, test a point ***<u>not</u>*** on the boundary line to see whether it is a solution of the inequality.
Then shade the appropriate half-plane (one side of the line).

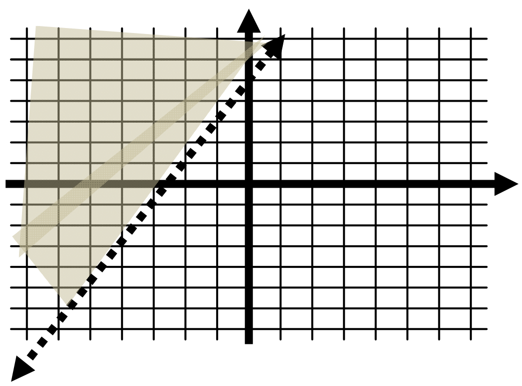
For $y < 2x + 5$



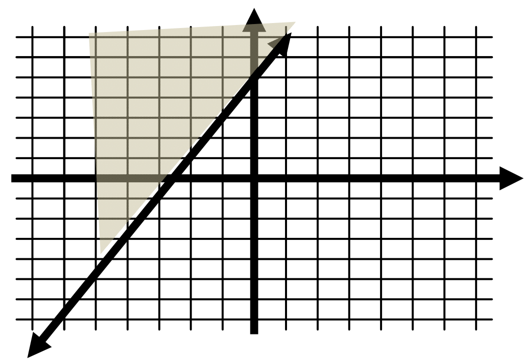
For $y \leq 2x + 5$



For $y > 2x + 5$

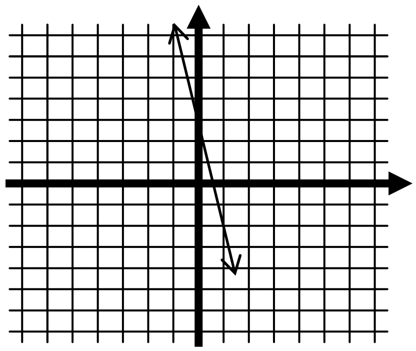


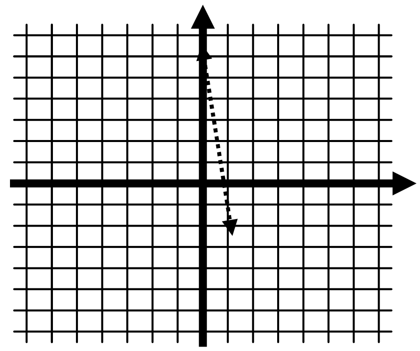
For $y \geq 2x + 5$

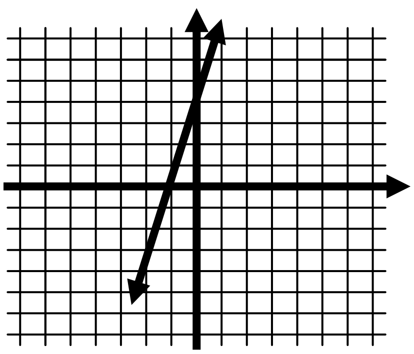


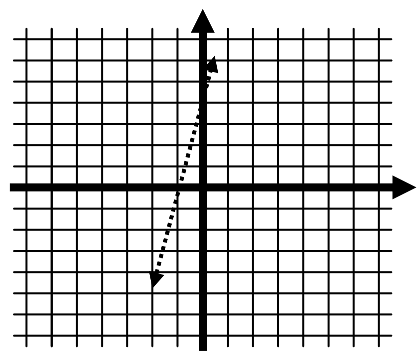
Practice:

1. The wholesale price, y, for a box of parts that you need is a function of the manufacturing cost per part, x. If there are 5 parts per box of parts that you need and the manufacturer includes a $4.00 markup per package, the wholesale price is y = 5x + 4, which graph represents the wholesale price?

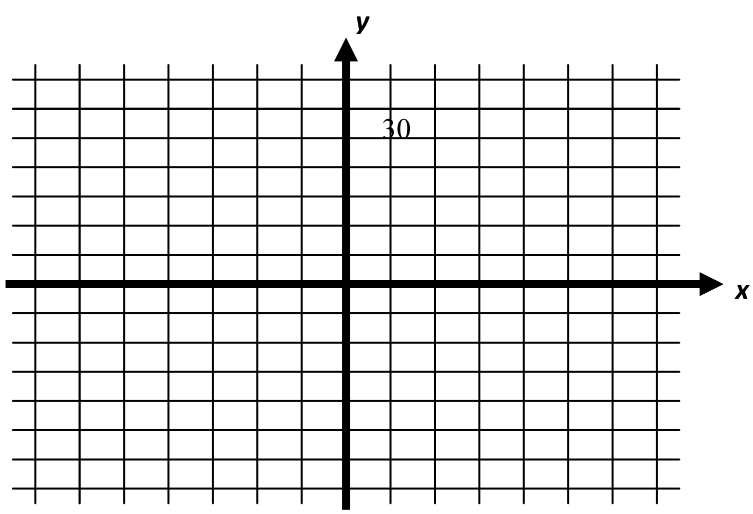
(A) 

(B) 

(C) 

(D) 

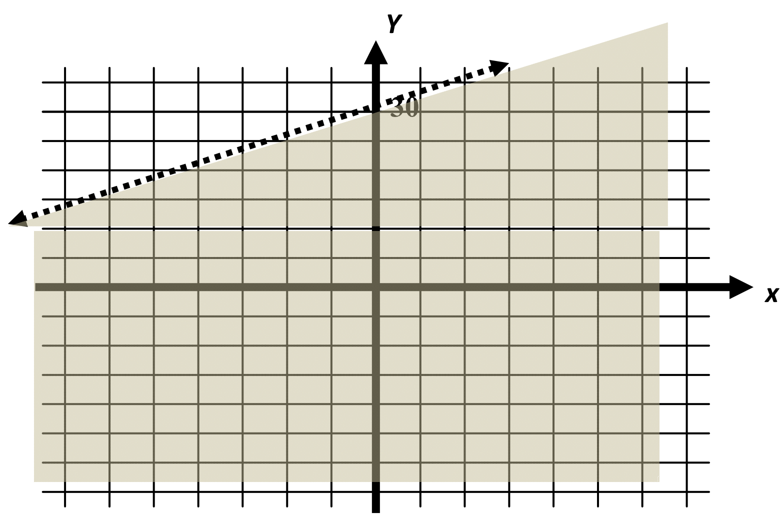
2. You want to open your own truck rental company. You do some research and find that the majority of truck rental companies in your area charge a flat fee of $29.99, plus $.49 for every mile driven. You want to charge less so that you can advertise your lower rate and get more business. Graph the equation, y = 29.99 + 0.49x where y is the cost of renting a truck from other rental companies and x is the number of miles driven. Shade the region where the amount you will charge must fall.



Practice Answers:

1. (C) For the linear equation, $y = 5x + 4$, the line should be solid which eliminates answers (B) and (D). The 4 tells where the line should cross the y-axis (this is called the y-intercept).
   The 5, which is the slope, tells you for every 5 squares up the line will move 1 square to the right. Since 5 is a positive number the line will go up from left to right which eliminates answers (A) and (B) because those lines go down from left to right.

2. Since 29.99 is the y-intercept (number where the line crosses the y-axis), count each square on the y-axis as 5. Since you want to charge less than the competition, make the line dashed and shade below the line. You are graphing $y < .49x + 29.99$. Because .49, or approximately ½, is the slope, the line rises one square for each two squares it goes horizontally.



Math Lesson Plan
Eligible Content M11.D.2.1.3
Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs

Berks Career & Technology Center

Objective: Write, solve and/or apply a linear equation (including problem situations)

Introduction In many real world problems, the variables are related so that both variables when graphed produce a linear relationship; straight line graph.

Skill Review
M11.D.2.1.2, M11.D.3

Definitions
None

Lesson

Perform the following examples.

Example1: A catering company will provide food for a party for \$15 per guest plus a fixed charge of \$200.

a. Write a linear equation representing the relationship between the number of guests, g, and the total cost, c.
b. What is the slope of the line? What does it represent?

Solution: a. Let g = number of guests
c = total cost
Therefore $c = \$15g + \$200$, which includes the given information

b. Since the equation in part a is written in slope-intercept form the slope=\$15 and represents the charge per guest.

Example 2: The formula for converting Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. Solve for C.

$F = \frac{9}{5}C + 32$	Formula for conversion from Celsius to Fahrenheit
$F - 32 = \frac{9}{5}C$	To isolate C, subtract 32 from each side
$\frac{5}{9}(F - 32) = C$	Multiply each side by $\frac{5}{9}$

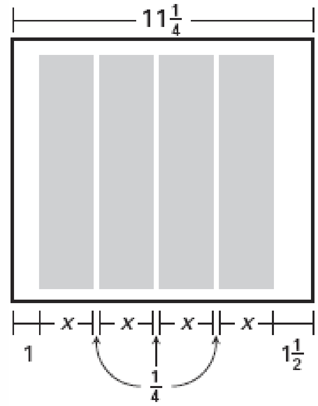
The original formula could be used to produce a linear graph of F as a function of C; F(C).
The solved formula could be used to produce a linear graph of C as a function of F; C(F).

Example 3.

The front page of your school newspaper is $11\frac{1}{4}$ inches wide. The left margin is 1 inch and the right margin is $1\frac{1}{2}$ inches. The space between the four columns is $\frac{1}{4}$ inch. Find the width of each column.

SOLUTION

The diagram shows that the page is made up of the width of the left margin, the width of the right margin, three spaces between the columns, and the four columns.



Verbal Model $\boxed{\text{Left margin}} + \boxed{\text{Right margin}} + 3 \cdot \boxed{\text{Space between columns}} + 4 \cdot \boxed{\text{Column width}} = \boxed{\text{Page width}}$

Labels	Left margin $= 1$	(inch)
	Right margin $= 1\frac{1}{2}$	(inches)
	Space between columns $= \frac{1}{4}$	(inch)
	Column width $= x$	(inches)
	Page width $= 11\frac{1}{4}$	(inches)

Algebraic Model $1 + 1\frac{1}{2} + 3\left(\frac{1}{4}\right) + 4x = 11\frac{1}{4}$

Solving for x, you find that each column can be 2 inches wide.

Practice

1. You are personalizing sweatshirts to sell at a fund raiser. Each sweatshirt requires $\frac{3}{4}h$ to paint and $1\frac{1}{2}h$ to dry. It will also require a total of $2\frac{3}{4}h$ to prepare the sweatshirts. If you have a total of 32 h to spend on the sweatshirts and you can work on only one at a time, how many can you make to sell? Write an equation to model the problem. Then solve the problem.

2. *Multiple Choice* A sales clerk's base salary is $21,000. The clerk earns a 6% commission on total sales. How much must the clerk sell to earn $30,000 total?
   A. $15,000
   B. $1,500
   C. $150
   D. $15
   E. $150,000

Practice Answers

1. Let n = number of sweatshirts.

$\frac{3}{4}n+1\frac{1}{2}n+2\frac{3}{4}=32$	Write the words in equation form
$2\frac{1}{4}n+2\frac{3}{4}=32$	Simplify
$\frac{9}{4}n+\frac{11}{4}=32$	Convert mixed numbers to improper fractions
$\frac{9}{4}n=\frac{117}{4}$	Subtract $\frac{11}{4}$ from each side and simplify
$n=13$	Multiply each side by $\frac{4}{9}$ and simplify

You can make 13 sweatshirts to sell.

2. Let t = total sales

And Total earnings = E = .06t+$21000	Write the words in equation form
$E-21000=.06t$	Subtract 21000 from each side to solve for t
$\frac{E-21000}{.06}=t$	Divide each side by .06
$\frac{30000-21000}{.06}=t$	Substitute 30000 for desired earnings
$t=\$150000$	Simplify

The sales clerk must have total sales of $150000 in order to earn $30000
Therefore the correct choice is

E. $150000

Math Lesson Plan
Eligible Content M11.D.2.1.4
Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs

Objective: Write and/or solve systems of equations using graphing, substitution and/or elimination (limit systems to 2 equations).

Introduction

When two or more linear equations are considered together, they are called a ***system of linear equations***, or more simply, a ***linear system***. Linear systems may be solved using any of three different methods; graphical method, substitution, and by linear combination (sometimes known as elimination).

Skill Review

M11.D.2.1.2, M11.D.2.1.3, M11.D.3.2.2, M11.D.3.2.3

Definitions

A **system of two linear equations** in two variables and consists of two equations, $Ax + By = C$ and $Dx + Ey = F$

A **solution** of a system of linear equations in two variables is an ordered pair (x, y) that satisfies *both* equations.

The Substitution Method
Step 1: Solve one of the equations for one of its variables.
Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.
Step 3: Substitute the value from Step 2 into the revised equation from Step 1 and solve.

A **linear combination** of two equations is an equation obtained by adding one of the equations (or a multiple of one of the equations) to the other equation.

The Linear Combination Method
Step 1: Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.
Step 2: Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.
Step 3: Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable

Lesson

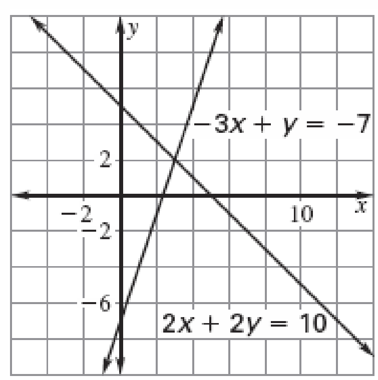
The following linear system will be solved by all three methods – Examples 1, 2, & 3.

$$-3x + y = -7 \qquad \text{Equation 1}$$
$$2x + 2y = 10 \qquad \text{Equation 2}$$

Example 1.

Solve the linear system graphically. Check the solution algebraically.

Solve each equation for 'y' putting each equation into slope-intercept form.



y = 3x – 7	Slope: 3; y-intercept: -7	Equ 1
y = - x + 5	Slope: - 1 ; y-intercept: 5	Equ 2

Graph each equation. The lines appear to intersect at (3, 2) * See the above graph.

To check (3, 2) as a solution algebraically, substitute 3 for x and 2 for y in each original equation.

Equation 1	Equation 2
-3x + y = - 7	2x + 2y = 10
-3(3) + (2) ? – 7	2(3) + 2(2) ? 10
- 7 = - 7	10 = 10

Because (3, 2) is a solution of each equation on the linear system, it is a solution of the linear system.

Keep in mind that a solution found by the graphical method is an approximate answer only and should always be checked for accuracy.

Example 2.

Solve the original linear system by the substitution method.
Look at both of the linear equations and decide which variable in which equation is easiest to solve for.
It would appear that the y-variable in Equation 1 is the easiest.

$y = 3x - 7$	Revised Equation 1

Substitute this result from Equ 1 into Equation 2 and solve for the x

$2x + 2y = 10$	Write Equation 2.
$2x + 2(3x - 7) = 10$	Substitute the revised Equ 1 for the y-variable.
$2x + 6x - 14 = 10$	Distribute
$8x - 14 = 10$	Simplify
$8x = 24$	Add 14 to each side
$x = 3$	Solve for x, divide each side by 8

To find the value of y, substitute 3 for x in the revised Equ 1

$y = 3x - 7$	Write Revised Equation 1
$y = 3(3) - 7$	Substitute 3 for x
$y = 2$	Simplify

The solution is (3, 2). Same as in example 1

Example 3.

Solve the original linear system by the linear combination (elimination) method.
The equations should both be written in standard form with like terms in columns.
Look at both of the linear equations and decide which variable would be easiest to eliminate by multiplying and producing opposite coefficients.
It would appear that the y-variable is the easiest to eliminate.

$-3x + y = -7$	Multiply Equ 1 by 2	$-6x + 2y = -14$	
$2x + 2y = 10$	Multiply Equ 2 by (-1)	$\underline{-2x - 2y = -10}$	
		$-8x \quad = -24$	Add the equations
		$x = 3$	Solve for x.

Substitute this result into the equation of choice say Equation 2

$2(3) + 2y = 10$	Substitute into Equ 2
$6 + 2y = 10$	Simplify
$2y = 4$	Subtract 6 from each side.
$y = 2$	Divide each side by 2

The solution is (3, 2). Same as in Example 1 and Example 2

Practice

1. Solve the following linear system by either substitution or linear combination.

$11x + 6y = 1$	Equation 1
$3x + 2y = -3$	Equation 2

2. ***Multiple Choice*** Which ordered pair is a solution of the following system of linear equations?

$$x + 2y = -1$$
$$2x - y = 13$$

A. (5, 3)
B. (-3, -5)
C. (5, -3)
D. (-5, 3)
E. (3, 5)

Practice Answers

1. Solve the original linear system by the linear combination (elimination) method.
Look at both of the linear equations and decide which variable would be easiest to eliminate by multiplying and producing opposite coefficients.
It would appear that the y-variable is the easiest to eliminate

$11x + 6y = 1$	Multiply Equ 1 by (-1)	$-11x - 6y = -1$	
$3x + 2y = -3$	Multiply Equ 2 by 3	$\underline{9x + 6y = -9}$	
		$-2x = -10$	Add the equations
		$x = 5$	Solve for x.

Substitute this result into the equation of choice say Equation 2

$3(5) + 2y = -3$	Substitute into Equ 2
$15 + 2y = -3$	Simplify
$2y = -18$	Subtract 15 from each side.
$y = -9$	Divide each side by 2

The solution is (5, -9).

2. Substitute each of the selections into both equations.

Selection C. (5, -3) produces a true statement for both equations

Math Lesson Plan
Eligible Content M11.D.2.1.5
Solve quadratic equations using factoring (integers only – not including completing the square or the Quadratic Formula).

Objective: Find solutions to quadratic equations using factoring only.

Introduction

Tell students that the day's lesson will be about learning how to find solutions for quadratic equations. We will be doing this by factoring only. All problems will be factorable.

Skill Review

M11.A.1.2.1
M11.D.2.2.2

Definitions

Standard form of a quadratic equation
The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$. There will be two solutions for every quadratic equation.

Zero-Product Property
If $ab = 0$, then $a = 0$ or $b = 0$.

Lesson

To solve a quadratic equation by factoring:

Procedure:

1 – Write the equation in standard form.
2 – Factor the equation.
3 – Set each factor equal to 0 and solve.

Examples

Solve the equation $80x = 16x^2$ by factoring.

Work

1 – Write the equation in standard form:

$16x^2 – 80x = 0$

2 – Factor the equation:

$16x (x – 5) = 0$

3 – Set each factor equal to 0 and solve:

$16x = 0 \qquad x – 5 = 0$

$x = 0 \qquad x = 5$

The solutions are $x = 0, 5$

Solve the equation $x^2 + 7x = 18$ by factoring.

Work

1 – Write the equation in standard form.

$x^2 + 7x - 18 = 0$

2 – Factor the equation:

$(x – 2)(x + 9) = 0$

3 – Set each factor equal to 0 and solve.

$x – 2 = 0 \qquad x + 9 = 0$

$x = 2 \qquad x = - 9$

The solutions are $x = - 9, 2$

Practice

1. Solve the equation $2x – x^2 = - 35$ by factoring.

   (A) $x = 5, 7$

   (B) $x = -5, 7$

   (C) $x = - 7, 5$

   (D) $x = - 7, - 5$

2. Find the x-intercepts of the graph of the function $f(x) = 32x + 16x^2$. (Hint: the x-intercepts are the values of x such that $f(x) = 0$).

Practice Answers

1. B

Standard form is $x^2 - 2x - 35 = 0$
Factors are $(x - 7)(x + 5) = 0$

$x - 7 = 0$	$x + 5 = 0$
$x = 7$	$x = -5$

Solutions are $x = -5, 7$
Therefore the answer is B.

2. Set $32x + 16x^2 = 0$. Write in standard form and solve the equation to find the x-intercepts.
Standard form: $16x^2 + 32x = 0$
Factor: $16x(x + 2) = 0$

Set each factor equal to 0 and solve:	$16x = 0$	$x + 2 = 0$
	$x = 0$	$x = -2$

The solutions are $x = -2, 0$.

Math Lesson Plan
Eligible Content M11.D.2.2.1
Add, Subtract and/or multiply polynomial expressions (express answers in simplest form – nothing larger than a binomial multiplied by a trinomial).

Berks Career & Technology Center

Objective: Combine like terms of polynomials using addition, subtraction, and multiplication.

Introduction

Inform students that many things in life can be modeled using mathematics, especially in the areas of finance. Stock market trends, profit and loss summaries, and other economic calculations can all be modeled with polynomials using some variable. In order to manipulate these models, it is important to know how to combine and simplify polynomial expressions.

Skill Review

M11.A.2.2.1 Simplify/evaluate expressions involving positive and negative exponents, roots, and/or absolute value.

M11.A.2.2.2 Simplify/evaluate expressions involving multiplying with exponents, powers of powers, and powers of products.

Definitions

<u>Polynomial:</u>

A function in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$

Example: $f(x) = x^3 + 2x^2 - 4$

<u>Terms:</u>

The terms of a polynomial are the parts separated by addition.

Example: In the polynomial above, the terms would be x^3, $2x^2$, and -4

- Remember that the polynomial could be written as $f(x) = x^3 + 2x^2 + (-4)$, which is why -4 is the last term.

<u>Like Terms:</u>

The terms of a polynomial that contain variables with the same exponent.

Example: $3x^2$ and $7x^2$ are like terms

Lesson

First, show students an example of a polynomial. Define terms of a polynomial for them and have the students break down the polynomial into separate terms. Note to students that all of the terms have the same variable, and that the only thing that varies between the terms is the exponent associated with the variable.

To add two polynomials is simply a matter of identifying and combining *Like Terms*, which should be defined for students using the example above.

Example (Adding Polynomials)

$(9x^3 - 2x + 1) + (5x^2 + 12x - 4) =$ **?**

Note that $9x^3 - 2x + 1$ and $5x^2 + 12x - 4$ are two different polynomials that are being added. To simplify this expression, remove the parentheses and write one long expression, ordering the terms so that the term with the highest exponent appears first, followed by the term with next highest exponent, and so forth. Place terms with the same exponents next to each other so that they can be easily combined.

So, $(9x^3 - 2x + 1) + (5x^2 + 12x - 4) = 9x^3 + 5x^2 - 2x + 12x + 1 - 4$

In this expression, -2*x* and 12*x* are like terms, and 1 and -4 are like terms. To combine them, simply add the coefficients of the terms.

So, $9x^3 + 5x^2 - 2x + 12x + 1 - 4 = 9x^3 + 5x^2 + 10x - 3$

This expression is now simplified.

Example (Subtracting Polynomials)

$(2x^2 + 3x) - (3x^2 + x - 4) =$ **?**

To simplify this, change the sign of every term in the second polynomial to the opposite sign (positives become negative and negatives become positive). Now you can add the polynomials instead of subtracting them.

$(2x^2 + 3x) - (3x^2 + x - 4) = (2x^2 + 3x) + (-3x^2 - x + 4)$

Now, add the two polynomials using the procedure described in the first example.

$(2x^2 + 3x) + (-3x^2 - x + 4) = -3x^2 + 2x^2 + 3x - x + 4 = -x^2 + 2x + 4$

This expression is now simplified.

Example (Multiplying Polynomials)

$(x-3)(3x^2-2x-4)=$ **?**

To simplify this expression, multiply the entire first polynomial by *each* of the terms in the second expression.

So, $(x-3)(3x^2-2x-4)=(x-3)3x^2-(x-3)2x-(x-3)4$

Now, distribute.

So, $(x-3)3x^2-(x-3)2x-(x-3)4 = 3x^3-9x^2-2x^2+6x-4x+12$

Lastly, combine like terms.

So, $3x^3-9x^2-2x^2+6x-4x+12 = 3x^3-11x^2+2x+12$

The expression is now simplified.

Practice

Simplify each expression.

1. $(8x^2+1)+(3x^2-2)$

A)	$5x^2-3$	B)	$11x^2+3$
C)	$11x^2-1$	D)	$8x^2+3x^2-2+1$

2. $(3x^3+10x+5)-(x^3-4x+6)$

A)	$2x^3+14x-1$	B)	$2x^3+6x+11$
C)	$3x^3-40x+30$	D)	$3x^3-x^2+14x+11$

3. $(x+3)(x^2-4x+9)$

A)	$x^2-3x+12$	B)	$x^3-x^2-3x+27$
C)	$x^3+7x^2+21x+27$	D)	$-x^2+7x-6$

Open Ended Question: Answer the question, providing all work necessary to arrive at your solution.

4. Over a span of time from 2001 to the present, the gross income for a particular company can be modeled by I = $246t^2+788t+159$ and the expenses for the same company can be modeled by E = $174t^2+254t+131$, where t is the number of years since 2001. Write a model for the *net* income of the company. (Hint: Net income is determined by subtracting expenses from the gross income.)

Practice Answers

1. Answer: C $(8x^2+1)+(3x^2-2)$

$= 8x^2+3x^2+1-2$

$= 11x^2-1$

2. Answer: A $(3x^3+10x+5)-(x^3-4x+6)$

$=(3x^3+10x+5)+(-x^3+4x-6)$

$= 3x^3-x^3+10x+4x+5-6$

$= 2x^3+14x-1$

3. Answer: B $(x+3)(x^2-4x+9)$

$=(x+3)x^2-(x+3)4x+(x+3)9$

$= x^3+3x^2-4x^2-12x+9x+27$

$= x^3-x^2-3x+27$

4. Solution: $(246t^2+788t+159)$ - $(174t^2+254t+131)$

$=(246t^2+788t+159)+(-174t^2-254t-131)$

$= 246t^2-174t^2+788t-254t+159-131$

$= 72t^2+534t+28$

Math Lesson Plan
Eligible Content M11.D.2.2.2
Factor algebraic expressions, including difference of squares and trinomials (trinomials limited to the form $ax^2 + bx + c$ where a is not equal to 0).

Berks Career & Technology Center

Objective: Factor algebraic expressions using either difference of squares or trinomial methods.

Introduction

Tell students that the day's lesson will be about learning how to factor algebraic expressions by using difference of squares and trinomial methods. This will be used later to solve quadratic equations.

Skill Review

M11.A.1.2.1

Definitions

<u>Factor</u>
When factoring a polynomial, the polynomial must be written as the product of two or more other polynomials. The factored form of $2x^2 + 5x + 3$ is $(2x + 3)(x + 1)$. $(2x + 3)$ and $(x + 1)$ are the factors of $2x^2 + 5x + 3$.

Lesson

In factoring, the product is given and we must find the factors. When these factors are multiplied, the product is the original polynomial. Two types of factoring are difference of squares and trinomials.

Difference of Squares Factoring:

Procedure:

1. The polynomial must have only two terms, both of which are perfect squares, with a subtraction sign between them. $a^2 – b^2$
2. Find the square root of both terms. $\sqrt{a^2} = a$ and $\sqrt{b^2} = b$
3. Factors will be $(a + b)(a – b)$. Example $x^2 – 49 = (x + 7)(x – 7)$

Trinomial Factoring Procedure:

Procedure:

1 – Case 1: $x^2 \pm bx + c$

If last sign is "+", signs in the factors will be the same. They will be whatever the first sign is. Now look for two numbers whose product is "c" and whose sum is "b". Example $x^2 + 5x + 6 = (x + 2)(x + 3)$.

2 – Case 2: $x^2 \pm bx - c$

If last sign is "-", signs in the factors will be different (one + and one -). Now look for two numbers whose product is "c" and whose difference is "b". The larger of these numbers is placed with the sign of the "b" term. Example $x^2 + 5x - 6 =$
$(x + 6)(x - 1)$.

3 – Case 3: $ax^2 \pm bx \pm c$

Use rules for cases 1 and 2 to help determine signs. Now you need two numbers whose product is "a" and two other numbers whose product is "c". Try all combinations until you find the one that gives you the correct middle term.
Example $6x^2 + 17x - 10 = (2x - 1)(3x + 10)$.

Examples

Factor $9x^2 - 64y^2$

Work

Given polynomial is a difference of squares.
$a = \sqrt{9x^2} = 3x$, $b = \sqrt{64y^2} = 8y$
$9x^2 - 64y^2 = (3x + 8y)(3x - 8y)$

Factor $x^2 + 13x + 36$

Work

Given polynomial is a trinomial in case 1 form.
Numbers that multiply to 36 are: 1and 36, 2 and 18, 3 and 12, 4 and 9, or 6 and 6.
Choosing from these pairs, only 4 and 9 add up to 13.
$x^2 + 13x + 36 = (x + 4)(x + 9)$

Factor $x^2 - 5x - 14$

<u>Work</u>

Given polynomial is a trinomial in case 2 form.
Numbers that multiply to 14 are: 1 and 14 or 2 and 7.
Choosing from these pairs, only 2 and 7 have a difference of 5. Since the middle term is negative, the bigger number (7) has to be negative.
$x^2 - 5x - 14 = (x + 2)(x - 7)$

Factor $18x^2 - 27x + 10$

<u>Work</u>

Given polynomial is a trinomial in case 3 form.
Numbers that multiply to 18 are: 1 and 18, 2 and 9, or 3 and 6.
Numbers that multiply to 10 are: 1 and 10 or 2 and 5.
Keep trying all combination until the middle term adds up to 27.
$18x^2 - 27x + 10 = (3x - 2)(6x - 5)$

Practice

1. Factor $12x^2 - 28x - 49$

   (A) $(2x - 7)(6x + 7)$

   (B) $(2x + 7)(6x + 7)$

   (C) $(2x - 7)(6x - 7)$

   (D) $(2x + 7)(6x - 7)$

2. Factor $4x^2 - 100$

Practice Answers

1. A

Given polynomial is a trinomial in case 3 form.
Numbers that multiply to 12 are: 1 and 12, 2 and 6, or 3 and 4. Numbers that multiply to 49 are: 1 and 49 or 7 and 7.
Keep trying all combinations until the middle terms have a difference of 28.
$12x^2 - 28x - 49 = (2x - 7)(6x + 7)$ which is choice A.

2. Given polynomial is a difference of squares.
$a = \sqrt{4x^2} = 2x$ and $b = \sqrt{100} = 10$
$4x^2 - 100 = (2x + 10)(2x - 10)$

Math Lesson Plan
Eligible Content M11.D.2.2.3
Simplify algebraic fractions

Objective: Simplify algebraic fractions to their lowest form.

Introduction
Tell students that the day's lesson will be about learning how to simplify algebraic fractions so that they are in their lowest form. This is the same as reducing a fraction.

Skill Review

M11.A.1.2.1
M11.D.2.2.2

Definitions

Algebraic fraction
In an algebraic fraction, the numerator and denominator will contain polynomials. These are also referred to as rational expressions.

Lesson

To simplify an algebraic fraction:

Procedure:

1. Factor all numerators and denominators, if possible.
2. Cancel any common factors.
3. Write remaining factor as a fraction.

Examples

Simplify $\dfrac{12x^2y^4}{8xy^2}$

Work

1 – Numerator and denominator are already in factored form.

2 – GCF for the numerator and denominator is $4xy^2$.

$$\frac{4xy^2(3xy^2)}{4xy^2(2)}$$

3 – $4xy^2$ cancels so the answer is $\dfrac{3xy^2}{2}$

Simplify $\dfrac{x^2-25}{6x^2+29x-5}$

Work

1 – Factor numerator and denominator:

$$\frac{(x+5)(x-5)}{(x+5)(6x-1)}$$

2 – Cancel the common factor of (x + 5):

3 – Answer is $\frac{(x-5)}{(6x-1)}$

Practice

1. Simplify $\frac{2x^2+x-3}{4x+6}$

(A) $\frac{2x^2+x-3}{4x+6}$

(B) $\frac{(2x+3)(x-1)}{2(2x+3)}$

(C) $\frac{(2x-3)(x+1)}{2(2x+3)}$

(D) $\frac{(x-1)}{2}$

2. Simplify $\frac{x^2+10x+25}{x^2+9x+20}$

Practice Answers

1. D

Factor: $\frac{(2x+3)(x-1)}{2(2x+3)}$

Cancel common factor of (2x + 3)

Remaining fraction is $\frac{(x-1)}{2}$

Therefore the answer is D.

2. Factor the numerator and denominator: $\frac{(x+5)(x+5)}{(x+4)(x+5)}$

Cancel common factor of (x + 5)

Answer is $\frac{(x+5)}{(x+4)}$

Math Lesson Plan
Assessment Anchor M11.D.3.1.1
Identify, describe and/or use constant or varying rates of change

Objectives:

- Identify constant rates of change in situations and model such situations algebraically
- Identify exponential growth or decay in situations and model such situations algebraically

Prerequisite Skills

M11A.2.2.1 Simplify/evaluate expressions involving positive and negative exponents
M11.A.2.1.1 Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations)

Definitions

Rate of change: The ratio of the change in one variable (*y*) to the change in another variable (*x*).

Constant: A fixed amount. The term "*a constant rate of change*" refers to a rate of change that remains the same at all times. Algebraically, these situations can be modeled by the equation $y = cx$, where x and y are quantities of each variable, and c is the rate of change.

Exponential growth or decay: A situation in which the rate of change over a period of time (t) remains proportional to a current quantity of a variable. Algebraically, the value of a variable y after t amount of time can be modeled by the equation $y = ab^{\frac{t}{r}}$, where a is the initial quantity, and b is the rate of change in r amount of time.

Introduction

Inform your students that today you have a challenge for them. Tell them they are investing $100.00 into a savings account where they will keep it for the next 20 years. They have two account options from which to choose. One account adds $1 to the account annually. Another account adds 1% of the account's balance annually.

Ask your students which account would make more money the first year. Students should note that each account will give $1 in the first year, so amount made is the same.

Now, ask your students which account will make more money the second year. Students should answer that the first account will make $1, as it always does. However, the second account will add 1% of the account's balance. The balance of the account is $101, so 1% (or 0.01) of that would be: 0.01*$101=$1.01

So, the second account would make more money the second year, and would continue to accrue more money each year after that.
Inform students that these two types of accounts represent two different rates of change. Give students a definition of a rate of change, and inform students that in this savings account example, the two variables being compared are the amount of money added to the account and the amount of time that passes.

The account which adds a flat rate of $1 per year is an example of a *constant rate of change*, as it always stays a fixed amount. Give this definition to students. The account that adds 1% of the account balance annually is an example of *exponential growth*, as the rate of change of the account changes is proportional to the current value of the account. Give the definition of exponential growth/decay to students.

Lesson

In this lesson, students will learn how to model situations involving rates of change algebraically, and use those models to predict future quantities. The first objective students will master is how to identify and use constant rates of change.

Example 1: Constant Rates of Change

In a hardware production plant, a machine produces 200 screws per minute. If a box of screws contains 350 screws,

a) How many minutes will it take to fill a single box?

b) How many minutes will it take fill a pallet of 50 boxes?

Solution:
Note that the amount of screws produced is a fixed amount, so this is an example of a constant rate of change.

a) The total amount of screws produced at any time can be modeled by the equation

$$y = c \cdot x$$

where y is the amount of screws produced, c rate at which the screws are produced and x is the number of minutes. These types of problems are covered in M11.A.2.1.1.

A single box is 350 screws, so y = 350. The rate of change is 200 screws per minute, therefore, c = 200.

$y = c \cdot x$	Algebraic model
$350 = 200 \cdot x$	Substituting values
$\frac{350}{200} = \frac{200 \cdot x}{200}$	Divide both sides by 200 to isolate x
$1.75 = x$	

So, one box can be produced in 1.75 minutes, or one minute and 45 seconds. ($0.75 \cdot 60 = 45$ seconds)

b) In order to solve this problem, the total number of screws to be produced must first be found. If there 250 screws in a box, and 50 boxes to be produced, then the total amount needed is $250 \cdot 50 = 12{,}500$. So now, y = 12,500.

$y = c \cdot x$	Algebraic model
$12500 = 200 \cdot x$	Substituting values
$\frac{12500}{200} = \frac{200 \cdot x}{200}$	Divide both sides by 200 to isolate xi
$62.5 = x$	

So, an entire pallet could be produced in 62.5 minutes, or one hour, two minutes, and thirty seconds.

<u>Example 2: Exponential Rates of Change (Annual Interest)</u>

You invest $9,000 into a fund that gains grows at a rate of 1.054 (5.4% times the current value of the account) per year. What will the value of the account be in seven years?

Solution
In this problem, the change in the value of the account varies proportionally with the current value of the account, so this problem can be modeled using exponential growth.

Remember, the value of a variable y after t amount of time can be modeled by the equation $y = ab^{\frac{t}{r}}$, where a is the initial quantity, and b is the rate of change in r amount of time.

In this example, you initially invest $9,000, so a = 9000.
The rate of change is 1.054 in one year, so b = 1.054 and r = 1.
Lastly, the money is invested over 7 years, so t = 7.

$y = ab^{\frac{t}{r}}$	Algebraic model
$y = 9000(1.054)^{\frac{7}{1}}$	Substituting values
$y = 13{,}005.49$	

So, the account will be worth $13,005.49 seven years from now.
(Review M11A.2.2.1 for help evaluating exponential expressions)

Practice

1. The distance from Philadelphia to Pittsburg is approximately 255 miles. If you drive on the Pennsylvania Turnpike at a constant speed of 75 miles per hour, how long would you expect to travel to get from one city to the other?

   A) 3 hours, 24 minutes
   B) 3 hours, 40 minutes
   C) 4 hours, 10 minutes
   D) 4 hours, 40 minutes
   E) 19,125 hours

2. The population of Reading, Pennsylvania, is approximately 81,200. If this population increases at a rate of 5% (1.05) per year, what would you expect the approximate population of the city to be in 10 years?

   A) 100,000
   B) 110,000
   C) 120,000
   D) 130,000
   E) 150,000

3. A bacteria culture's population triples every 43 minutes. If there are 215,743,167 bacteria in the population at 8:00 am, approximately how many ***billion*** bacteria will be in the population at 11:30 am the same day?

Practice Answers

1. A

Constant rate of change c = 75, y = 255

$y = c \cdot x$	Algebraic model
$255 = 75 \cdot x$	Substituting values
$\frac{255}{75} = \frac{75 \cdot x}{75}$	Solve for x

$3.4 = x$

** Your answer is 3 hours and 0.4(60) = 24 minutes

2. D

Exponential growth a = 81,200; b=1.05; t = 10; r = 1

$y = ab^{\frac{t}{r}}$	Algebraic model
$y = 812000(1.05)^{\frac{10}{1}}$	Substitute values
$y = 132266.2$	Evaluate

3. Answer: 46 billion

This is an exponential growth problem, as the amount that the population increases is proportional to the initial population of the culture. Note that the total time elapsed in this problem is from 8:00 am to 11:30 am. 11.5 – 8 = 3.5 hours. However, since the bacteria population triples every 43 minutes, it is necessary to convert 3.5 hours to minutes to find the total time t.

t = 3.5(60) = 210 minutes
a = 215,743,167
b = 3 (Note that the bacteria population *triples*, so the rate of change is 3)
r = 43

$y = ab^{\frac{t}{r}}$	Algebraic model
$y = 215743167(3)^{\frac{210}{43}}$	Substitute values
y = 46,138,567,615	Evaluate

Math Lesson Plan
Eligible Content M11.D.3.1.2
Analyze change in various contexts

Objective: Determine how a change in one variable relates to a change in a second variable (e.g., y = 4/x. If x doubles, what happens to y?).

Introduction

Tell students that today's lesson will be about how a change in one variable affects a second variable in direct variations and inverse variations.

Skill Review

M11.D.3.1.1

Definitions

Direct Variation – y varies directly as x or y is directly proportional to x, if there is a nonzero constant k such that y = kx.

Constant of Variation, k, (also called the constant of proportionality) – equals the rate of change for data that describe the variation.

Inverse Variation – y varies inversely as x, or y is inversely proportional to x, if there is a nonzero constant k such that y = k/x.

Lesson

When the first variable varies directly with the second, it is a direct variation, i.e., y = kx. When the first variable varies inversely with the second variable, it is an inverse variation, i.e., y = k/x.

Procedure:

1. Check the wording of the problem to determine if it is direct or inverse variation.

2. If the problem asks for the constant of variation, then you must solve for k. If it is a direct variation, then k = y/x. If it is an inverse variation, then k = yx.

3. If the problem asks for an equation, then you must find k first. Then replace k in the direct or indirect variation.

Examples

1. If x is doubled in the inverse variation, $y = 40/x$, what happens to y?

Work

The right side of the equation must be multiplied by ½ , so we would have to multiply the right side by ½. Therefore, y is halved.

2. Determine whether x and y show direct variation, inverse variation, or neither.

x	31	20	17	12
y	217	140	119	84

Work

The constant of variation k, is equal to y/x when it is a direct variation and yx when it is an inverse variation.

Therefore:

Direct variation is $k = y/x$.

$y/x = 217/31 = 7$
$y/x = 140/20 = 7$
$y/x = 119/17 = 7$
$y/x = 84/12 = 7$

Inverse variation is $k = yx$

$yx = 217(31) = 6727$
$yx = 140(20) = 2800$
$yx = 119(17) = 2023$
$yx = 84(12) = 1008$

k must be the same for each set of data. Therefore the data shows a direct variation.

3. The area, A, of a circle is proportional to the square of its radius. When the radius is ten feet the area of the circle is approximately 314 square feet. What is the radius of a circle if the area is approximately 157 square feet?

Work
The direct variation would be $a = \pi r^2$. When $r = 10$, Area $= \pi(10)^2 \approx 314$ square feet.
When the area is 157, the equation is: $157 = \pi r^2$. So $r^2 \approx 157/\pi$, and $r^2 \approx 50$, and $r \approx 7$.

4. The time it takes you to hear thunder varies directly with your distance from the lightning. If you are 2 miles from where lightning strikes, you will hear thunder about 10 seconds after you see the lightning. Write an equation for the relationship between time and distance.

Define: x = your distance in miles from the lightning
y = the number of seconds between seeing lightning and hearing thunder

Relate: The time varies with the distance.
When $x = 2$, $y = 10$

Write:

$$y = kx$$
$$10 = k(2)$$
$$\frac{10}{2} = \frac{k(2)}{2}$$
$$5 = k$$
$$y = 5x$$

The equation $y = 5x$ relates the distance x in miles you are from lightening to the time y in seconds it takes you to hear the thunder.

(Prentice Hall)

Practice

1. The value of x varies inversely with y, and $x = 2$ when $y = 5$. What is x when $y = 10$?

   A. 0
   B. 4
   C. 5
   D. 7
   E. 1

2. If we increase x in the inverse variation, $y = 8/x$, describe the change in y.

3. The weight that an object exerts on a scale varies directly with the mass of the object. If a bowling ball has a mass of 6 kg, the scale reads 59 N. Write an equation for the relationship between weight and mass.

(Prentice Hall)

Practice Answers

1. E

This is an inverse variation so x = k/y, and k = xy. With the given values substitute 2 for x and 5 for y, so k = (2)(5) = 10. Again, k = xy, so substitute 10 for k and 10 for y. This gives 10 =(x)(10). Solve the equation for x by dividing both sides by 10. Therefore, x = 1.

2 As x increases, y decreases.

y = 8/x. If x = 2, the y = 4. (y = 8/2)
If x = 4, then y = 2. (y = 8/4)
If x = 8, then y = 1. (y = 8/8)

3. Define: x = the mass in kg
y = the weight the object exerts on the scale

Relate: The weight (in Newtons) varies with the mass (in kg)
x = 6 kg
y = 59 N

Write:
$$y = kx$$
$$\frac{59}{6} = \frac{k(6)}{6}$$
$$k = 9.8$$

The equation is y = 9.8x.

Math Lesson Plan
Eligible Content M11.D.3.2.1
Apply the formula for the slope of a line to solve problems

Objective: Find the slope of a line.

Introduction

Many examples of slope can be seen around you. The steepness of a ski slope is a one example. A slope that is easy has a relatively small slope, while one that is particularly steep has a greater slope.

Skill Review

M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems

Definitions

<u>Slope of a line</u>
The ratio of the vertical rise to the horizontal run between any two points on a line

Slope $m = \frac{rise}{run}$ (the letter m is used to designate slope)

Lesson

Slope can be calculated between two points on a line using the coordinates of the points. To calculate slope using the points (x_1, y_1) and (x_2, y_2), use the formula

$$\text{slope} \quad m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope can also be calculated when the lengths of the rise and run are known. Using the formula listed above, replace the rise and run with their corresponding lengths.

Examples

Find the slope of the line that passes through the points (1, 0) and (–3, 2).

Work

Let $(x_1, y_1) = (1, 0)$ and $(x_2, y_2) = (-3, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-3 - 1} = \frac{2}{-4} = -\frac{1}{2}$$

The slope of the line is $-\frac{1}{2}$

A ramp is placed next to a loading dock. The length of the ramp is 8 feet. The height of the ramp is 1.5 feet. What is the slope of the ramp?

Work

$$m = \frac{rise}{run} = \frac{1.5\, feet}{8\, feet} = \frac{15}{80} = \frac{3}{16}$$

(multiply numerator and denominator by 10 to remove decimal from problem)

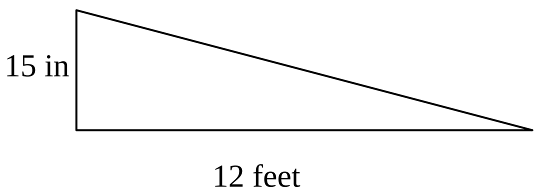
The slope of the ramp is $\frac{3}{16}$

Practice

1. Find the slope of the line passing through the points (1, 2) and (2, 1)

   A. 1
   B. −2
   C. 2
   D. −1

2. The uniform Federal Accessibility Standards outlines the standards for buildings to make them accessible to handicapped persons. One standard is the slope of ramp be less than or equal to $\frac{1}{12}$.



   A. Does the ramp shown meet the standard?

   B. If the ramp covers a horizontal distance of 20 feet, will the slope meet the standard?

   C. To equal a slope of $\frac{1}{12}$, how far must the ramp extend horizontally?

Practice Answers

1. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-2}{2-1} = \dfrac{-1}{1} = -1$

Answer: D

2. A. To calculate the slope of the ramp, first change the units of the horizontal distance to inches so that both units are the same.

12 feet = 144 inches.

Slope of ramp $m = \dfrac{rise}{run} = \dfrac{15}{144} = \dfrac{5}{48} \approx 0.104$

Standard slope $m = \dfrac{rise}{run} = \dfrac{1}{12} \approx 0.083$

The slope of the ramp is greater than the slope of the standard ramp.

The ramp does not meet the standard.

B. Change the new horizontal distance to inches.

20 feet = 240 inches

Slope of new ramp $m = \dfrac{rise}{run} = \dfrac{15}{240} = \dfrac{1}{16} = 0.625$

Standard slope $m = \dfrac{rise}{run} = \dfrac{1}{12} \approx 0.083$ (from part A)

The slope of the ramp is less than the slope of the standard ramp.

The ramp does meet the standard.

C. Since the slope of the standard is $\dfrac{1}{12}$, set up a proportion using 15 inches as the height.

$\dfrac{1}{12} = \dfrac{15}{x}$ 1x = 180

x = 180 inches change inches to feet

The ramp must extend 15 feet horizontally.

Math Lesson Plan
Eligible Content MlI.D.3.2.3
Compute the slope and/or y-intercept represented by a linear equation or graph.

Objective: To find the slope and/or y-intercept given a linear equation or graph of a line.

Introduction

Today you will be able to find the slope and/or y-intercept of a line given either a linear equation or a graph of the line. A graph is a visual that can describe a trend such as an increase or decrease. The method that will be used to find the slope and/or y-intercept given the equation of a line is the Slope-Intercept Form of the Equation of The Line.

Skill Review

M11.D.2.1.3 Write, solve and/or apply a linear equation.
M11.D.3.2.2 Given the graph of a line, write or identify the linear equation in slope-intercept form.

Definitions

Slope

Slope $m = \dfrac{\text{vertical rise}}{\text{horizontal run}}$ between any two points on the line.

y-Intercept
The y-coordinate of a point where the graph of a line crosses the y-axis. Example: If a line crosses the y-axis at the point (0,2), the y-intercept is 2.

Linear Equation
A linear equation in x and y is an equation that can be written in the form $Ax + By = C$ where A and B are numbers that are not both zero. Example: $3x + 2y = 5$.

Lesson

In order to find the slope and/or y-intercept given any linear equation, a student must first write this linear equation in the

SLOPE-INTERCEPT FORM OF THE EQUATION OF **A** LINE

slope…… ……y-intercept

$$\underset{\text{slope}}{y = \overset{\downarrow}{m}x + \overset{\downarrow}{b}}$$

$y = mx + b$ where m is the slope and b is the y-intercept.

Examples

A. Equations already written in slope-intercept form.

Find the **slope** m and **y-intercept** b of the following linear equations.

Equation	slope m	y-intercept b
1. $y = 2x + 3$	2	3
2. $y = -x - 6$	-1	-6
3. $y = \frac{1}{2}x$	½	0
You try:		
4. $y = 3x + 1$	_____	_____
5. $y = 0.75x - 7$	_____	_____
6. $y = 5$	_____	_____

B. Equations **not** written in slope-intercept form.

*Rewrite the equation in slope-intercept form by solving the equation for y. Identify the slope m and y-intercept b.

Equation	Slope-intercept form	slope m	y-intercept b
1. $2x + y = -3$	$y = -2x - 3$	-2	-3
2. $10x - 5y = 50$	$y = 2x - 10$	2	-10

1. Solve for y :

$$\begin{array}{rcl} 2x + y &=& -3 \\ \underline{-2x } && \underline{-2x} \\ 0 + y &=& -3 - 2x \end{array}$$

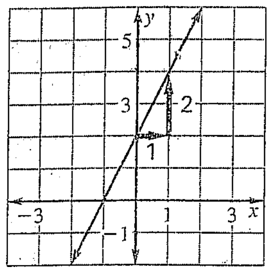
2. Solve for y:

$$\begin{array}{rcl} 10x - 5y &=& 50 \\ \underline{-10x } && \underline{-10x} \\ 0 \quad \frac{-5y}{-5} &=& \frac{50 - 10x}{-5 \quad -5} \end{array}$$

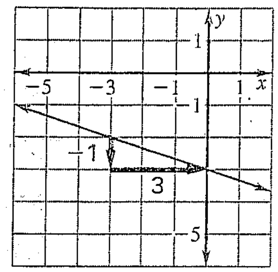
$$y = -10 + 2x$$

C. Given a **graph** of a linear equation, find the **slope m** and **y-intercept b.**

1. slope m = 2 ; y-intercept b = 2

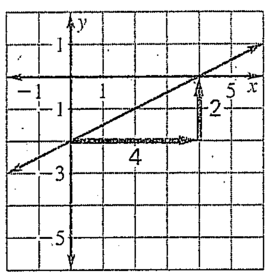


2. slope m = $\frac{-1}{3}$; y-intercept = -3

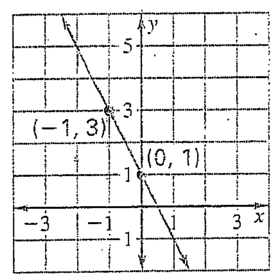


You try:

3. slope m = _____ ; y-intercept b = _____



4. slope m = _____ ; y-intercept b = _____



Practice

Find the slope m and the y-intercept b of the following linear equations.

1. $y = 5x - 1$

(A) slope m = -5; y-intercept b = -1

(B) slope m = 5 ; y-intercept b = -1

(C) slope m = -1 ; y-intercept b = 5

(D) slope m = 1 ; y-intercept b = -5

2. $-3x + y = 8$

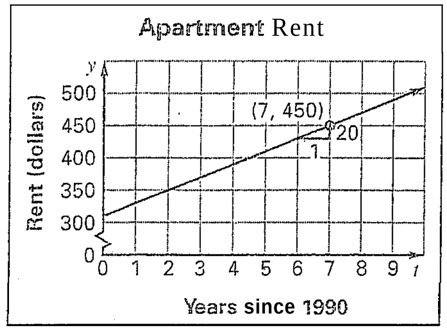
(A) slope m = 3 ; y-intercept = 8

(B) slope m = -3 ; y-intercept = 8

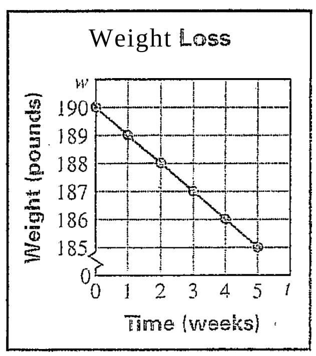
(C) slope m = 8 ; y-intercept = -3

(D) slope m = -8 ; y-intercept = 3

3. Between 1990 and 2000, the monthly rent for a one-bedroom apartment increased by $20 per year. In 1997, the rent was $450 per month. Find the slope m and y-intercept b using the graph provided.



4. The graph below represents the weight loss of a wrestler as he prepares for the state meet. Find the slope of the line and the w-intercept and explain what each represents.



Answers

A. Examples that you try :

	slope m	y-intercept b
4.	3	1
5.	0.75	- 7
6.	0	5

C. Examples that you try :

	slope m	y-intercept b
3.	$\frac{2}{4} = \frac{1}{2}$	- 2
4.	- 2	1

Practice

1. B

2. A Rewrite $-3x + y = 8$ in slope-intercept form first.

$$\begin{array}{rcrcl} -3x & + & y & = & 8 \\ +3x & & & & +3x \\ \hline 0 & + & y & = & 8 + 3x \end{array}$$

So, slope-intercept form is $y = 3x + 8$.

3. Slope m = $\frac{\$20}{1 \text{ year}}$ = \$20 ; y-intercept b = \$310 (approximately)

4. Slope m = $\frac{-1}{1}$ = -1 ; represents how much weight (pounds) the wrestler loses each week.

   w-intercept = 190 ; represents the wrestler's starting weight in pounds. Notice the trend that the wrestler loses weight each week. (a decrease)

Math Lesson Plan
Eligible Content M11.D.4.1.1
Match the graph of a given function to its table or equation.

Objective: Identify graphs of functions represented with equations and tables.

Introduction

Functions can represent **real-life** relationships between two quantities such as a restaurant menu with food items listed with the price or a person's gross paycheck based on total hours worked. In a function, there is exactly one output value for a given input value. Functions can be represented in words, in symbols, with a table, and with a graph. You will be able to observe and analyze a function's graph in order to make future predictions.

Skill Review

M11.A.3.1.1 Simplify/evaluate expressions using the order of operations to solve problems.
M11.D.1.1.1 Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.

Definitions

Function
A rule that establishes a relationship between two quantities, called the **input** and the **output.** For each input, there is exactly one output.

Input
A value in the domain of the function.

Output
A value in the range of a function.

Input-Output Table
A table used to describe a function by listing the outputs for several different inputs.

Lesson

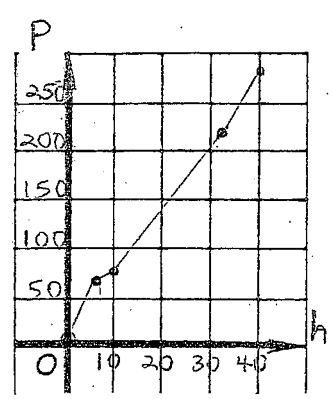
When working with functions that are represented with an input-output table, each value in the first column - the input is paired with a value in the second column - the output. In a function, the input values make up the domain of the function and the output values make up the range of the function. For each input, there is exactly one output.

Examples

1. Input-Output Table for the function represented by the equation $P = 7.15 \cdot h$ where P represents total pay and h represents hours worked. To calculate total pay P, you multiply $7.15 times h, the hours worked.

Input h	Output P
Hours worked (h)	Total Pay (P)
8	$57.20
10	$71.50
32	$228.80
40	$286.00

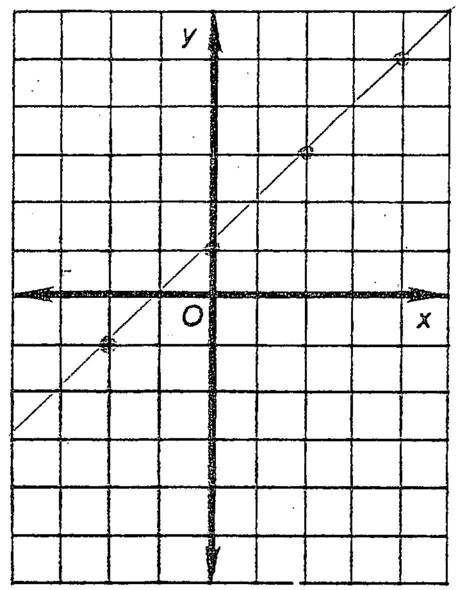
2. Graph of the function represented by the equation $P = 7.15\ h$.



In order to identify the input-output table that corresponds to the graph of a function, it is necessary to write the ordered pairs of the points on the graph. Next, compare your ordered pairs to the values of the table. Using the graph below, the ordered pairs are (-2,-1), (0,1), (2,3), and (4,5). Notice that these ordered pairs match the values in the table. Therefore, this table matches the graph of this function.

3.

Input x	output
-2	-1
0	1
2	3
4	5

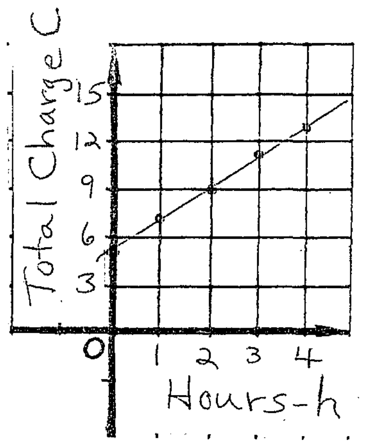


4. When given the equation of a function, you need to substitute numbers for the input values of the equation and calculate the resulting output values to make an input-output table. For example, given the equation $C = 5 + 2h$ where C is the total charge to park your vehicle at the airport and h is the number of hours your vehicle is parked with an initial charge of $5 per vehicle. Then graph the results of the input-output table or if given a graph match the points of the graph to the table.

Input-Output Table

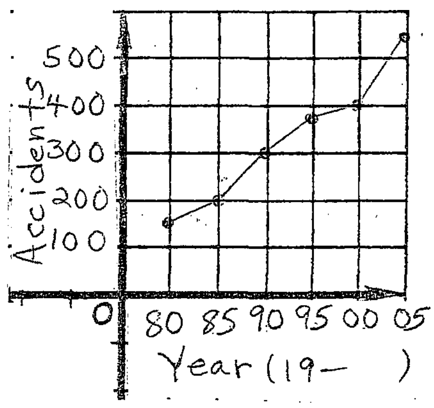
Input h	Output C
1	7
2	9
3	11
4	13

INPUT	FUNCTION (equation)	OUTPUT
h = 1	$C = 5 + 2(1)$	7
h = 2	$C = 5 + 2(2)$	9
h = 3	$C = 5 + 2(3)$	11
h = 4	$C = 5 + 2(4)$	13



Practice

1. The graph shows the number of traffic accidents the police department in one town recorded for the 25 year period from 1980 - 2005. Which input-output table is best represented by the graph?



A.

Input	Output
150	1980
200	1985
300	1990
375	1995
400	2000
550	2005

B.

Input 1980	Output 125
1985	200
1990	300
1995	350
2000	400
2005	525

C.

Input	Output
1980	150
1985	200
1990	300
1995	375
2000	400
2005	550

D.

Input	Output
1980	100
1985	205
1990	300
1995	390
2000	400
2005	500

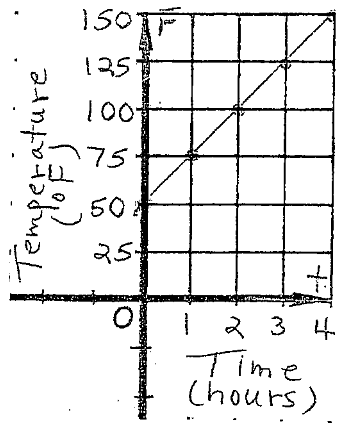
2. Which function equation is represented by the graph?

A. $F = 50 + 25t$

B. $F = 25 + t$

C. $F = 25 + 50t$

D. $F = 25t$



Answers

1. C

Write the ordered pairs of the points represented on the graph. They are (1980,150), (1985,200), (1990,300), (1995,375), (2000,400), (2005,550) which matches the input-output table of answer C.

2. A

Substitute input values t into the function equation to calculate the output values F. Use the numbers along the horizontal axis as your input values. Start with the first equation $F = 50 + 25t$ and continue until the table matches the ordered pairs of the points of the graph.

Input-Output Table

Input t	Output F
0	50
1	75
2	100
3	125
4	150

INPUT	FUNCTION (equation)	FUNCTION
$t = 0$	$F = 50 + 25(0)$	50
$t = 1$	$F = 50 + 25(1)$	75
$t = 2$	$F = 50 + 25(2)$	100
$t = 3$	$F = 50 + 25(3)$	125
$t = 4$	$F = 50 + 25(4)$	150

Referring to the graph, observe that the ordered pairs are (0,50), (1,75), (2,100), (3,125), and (4,150).

Math Lesson Plan
Eligible Content M11.E.1.1.1
Formulate or answer questions that can be addressed with data and/or organize, display, interpret or analyze data.

Objective: Create and/or use appropriate graphical representations of data, including box-and-whisker plots, stem-and-leaf plots or scatter plots.

Introduction

Sometimes a better representation of a set of data than the median, mean and mode is given by quartiles. Three numbers are utilized to systematically divide the set of data into four equal parts' or quarters (quartiles). The lower quartile is the median (middle) of the lower half; the median is the middle number of the entire set of data; and the upper quartile is the median (middle) of the upper half.

Skill Review

M11.E.2.1.1 Calculate measures of central tendency (median)

Definitions

A **<u>stem-and-leaf plot</u>** shows how data are distributed.

A **<u>box-and-whisker plot</u>** divides data into four parts. The *median* divides that data into a lower half and an upper half. The **<u>median</u>** of a set of data is the middle value when the values are written in increasing order. The median of the lower half of the values is called the **<u>lower quartile</u>**. The median of the upper half is called the **<u>upper quartile</u>.**

The **<u>range</u>** of a set of data is the difference between the greatest and the least value.

The **<u>interquartile range (IQR)</u>** of a set of a data is a measure of the spread of the middle 50% of the data. The IQR is the difference between the upper and lower quartiles.

Lesson

The above definitions should be given to the students followed by the next two examples.
When asked to find the median and mode in addition to the mean the following steps should be taken in solving the problem:

Making a stem-and-leaf plot:

1. Identify the smallest and largest data values in the set.
2. Determine the value to use for the *stem.*
3. Write the *stems* in a column from least to greatest.
4. Draw a vertical line to the right of the stems.
5. Write the *leaves* in increasing order to the right of their stems.
6. Write an explanation (key) for the data.

After outlining the above steps proceed to Example 1.

Making a Box-and-Whisker Plot

1. Write the data in increasing order
2. Find the median
    a. If there is an odd number of data values, the median is the middle value.
    b. If there is an even number of data values, you must average the two middle values to get the median.
3. Find the upper and lower quartiles.
4. Draw a number line with equal spacing.
5. Label the number line according to your data values.
6. Plot the median, lower quartile, upper quartile, and extremes (the smallest and largest data values) on the same horizontal line.
7. Draw a "box" (rectangle) around the quartiles.
8. Draw the segment in the box where the median is located.
9. Draw the "whiskers" to the extreme values.

After outlining the above steps proceed to Example 2.

Examples

Example 1: The travel times (in minutes) for 14 students on a school bus are 15, 12, 8, 22, 17, 6, 13, 24, 11, 27, 7, 3, 12, and 14. Display the data using a stem-and-leaf plot.

Solution: The times vary from 3 to 27, so let the ***stems*** be the ten's digits from 0 to 2. Let the ***leaves*** be the one's digits. Make a key that shows how to interpret the digits.

Stem	Leaves
0	3 6 7 8
1	1 2 2 3 4 5 7
2	2 4 7

Key: 2 | 7 = 27

Example 2: The scores on a test in your science class are 85, 90, 72, 95, 93, 87, 88, 80, 78, 100, 96, 92, 86, 95, 94, and 88. Display the data using a box-and-whisker plot.

Solution: Write the data in increasing order. Find the median and the quartiles.

Median: $\dfrac{88+90}{2} = 89$

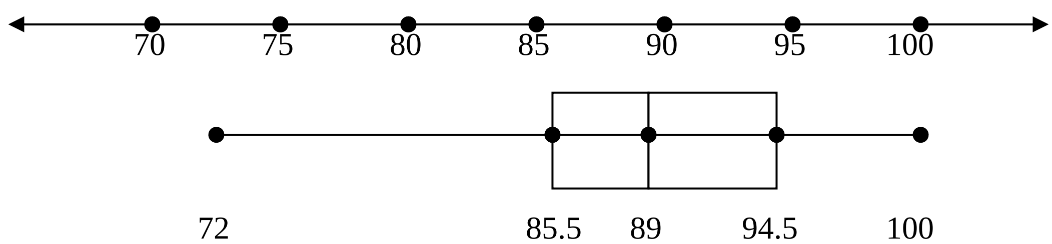
Lower half (8 values) Upper half (8 values)

$\underbrace{72 \quad 78 \quad 80 \quad 85 \quad 86 \quad 87 \quad 88 \quad 88}_{} \quad \underbrace{90 \quad 92 \quad 93 \quad 94 \quad 95 \quad 95 \quad 96 \quad 100}_{}$

Lower quartile: $\dfrac{85+86}{2} = 85.5$ **Upper quartile:** $\dfrac{94+95}{2} = 94.5$

Range: 100 – 72 = 28 **IQR:** 94.5 – 85.5 = 9

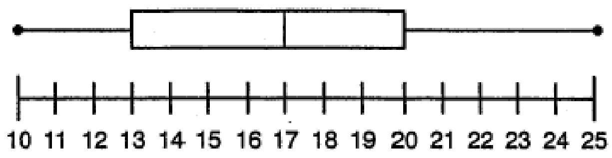
Use a number line to draw the box-and-whisker plot.



Practice

1.

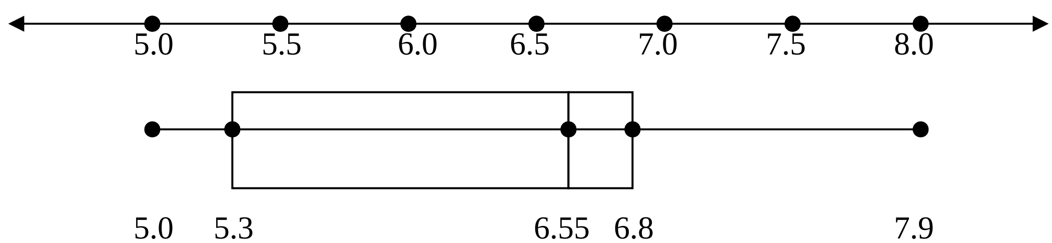
Look at the box-and-whisker plot below. What range of values contains 50% of the data?



A. 10 - 20
B. 13 - 25
C. 17 - 25
D. 17 - 20

**** SPECIAL NOTE – THE ABOVE PROBLEM IS TAKEN DIRECTLY FROM THE PDE WEBSITE.***

2. Use the box-and whisker plot below which shows the average wind speeds (in miles per hour) in California locations in January.



A) What is the median wind speed?

B) What are the upper and lower quartiles of the data set?

C) What is the range?

D) What is the interquartile range (IQR)?

Practice Answers

1. By definition the median is the middle value of the data set. Since the range 17 – 25 covers the range of values from the middle value to the end of the upper quartile it obviously contains half (50%) of the values in the data set.

   Answer: C

2. A) 6.55 mi/hr

   B) Lower Quartile = 5.3 mi/hr
   Upper Quartile = 6.8 mi/hr

   C) 7.9 – 5.0 = 2.9 mi/hr

   D) 6.8 – 5.3 = 1.5 mi/hr

Math Lesson Plan
Eligible Content M11.E.1.1.2
Analyze data and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, or scatter plots)

Objective: Use statistical plots to answer questions.

Introduction

Data is often presented in the form of a plot. In this lesson, you will learn to interpret three different types of plots: box-and-whisker plots, stem-and-leaf plots, and scatter plots. Each plot provides distinct information regarding the spread of data, and you will learn to analyze the information provided.

Skill Review

M11.E.1.1.1 Create and/or use appropriate graphical representations of data, including box-and-whisker plots, stem-and-leaf plots, or scatter plots.

Definitions

<u>Box-and-whisker plot</u>
A visual display that uses the median, the lower and upper quartiles, and the least and greatest values of a set of data.

<u>Stem-and-leaf plot</u>
Stem-and-leaf plots are a method for showing the frequency with which certain classes of values occur.

<u>Scatter plot</u>
A coordinate graph containing points that represent a set of ordered pairs; used to analyze relationships between two real-life quantities.

<u>Mean</u>
The average of set of data that is calculated by dividing the sum of the data by the number of items in the set.

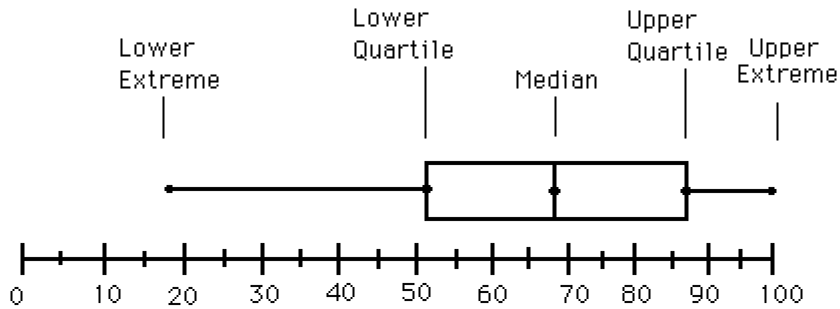
<u>Median</u>
The middle value when data are arranged in numerical order.

<u>Mode</u>
The number that occurs most often in a set of data.

<u>Quartile</u>
The first quartile is the median of the lower half of the data. The third quartile is the median of the upper half of the data. The second quartile is the median of the entire set of data.

Lesson

<u>Box-and-whisker plot</u>



A box-and-whisker plot is used to analyze the dispersion of data, in other words, the way data is spread out or grouped together. From a box-and-whisker plot, you can determine the least value, the first (lower) quartile, the median, the third (upper) quartile, and the greatest value. These values separate the data into 4 groups, each with the same number of values. Half (50%) of the set of data falls within the box of the plot. Each whisker contains ¼ (25%) of the data. Multiple box-and-whisker plots can be used to compare different sets of data.

<u>Stem-and-leaf plot</u>

A stem-and-leaf plot is used to display frequency and distribution of data. Thus plot is different from the box-and-whisker plot because it displays all values in the set. Stem-and-leaf plots can be used to located outliers (values in a set that are much larger or much smaller than the majority of values). It can also locate clusters (groups of values grouped closely together). Because all values in the set are listed on the plot, it is also possible to calculate the mean, median, mode and range of the value.

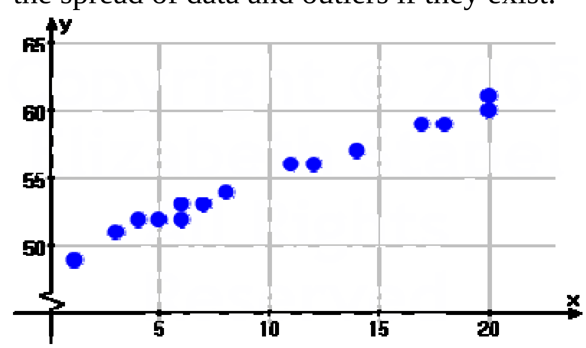
Test grades

stem	leaf
4	2
5	
6	7
7	3 8
8	2 4 6
9	1 4 9

<u>Scatter plot</u>

A scatter plot is used to graph ordered pairs of numbers. The ordered pair typically consists of two real world values listed as an ordered pair and plotted along the x-axis and y-axis. The points on the graph remain unconnected. Scatter plots are primarily used to determine the correlation between two values. This type of graph can also show the spread of data and outiers if they exist.

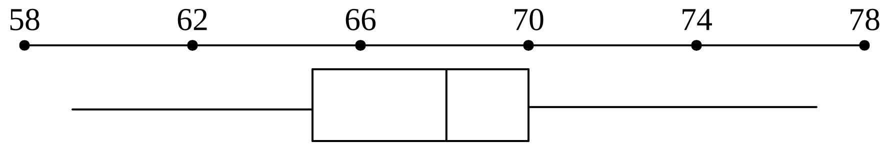
The correlation of a scatter plot is determined by the slope of the line through the majority of the points. If a line through the points is increasing from left to right, the plot has a positive correlation. If a line through the point is decreasing from left to right, the plot has a negative correlation. If no line can be drawn because the points are spread all over the graph, the plot has no correlation. The graph at the right shows a positive correlation between the values.



Examples

Use the box-and-whisker plot to answer the questions that follow.

<u>Height of 100 11th grade students (in inches)</u>



What is the median height? 68 in

How tall is the shortest student? The tallest student? 59 in, 77 in

What range of values contains the middle 50% of students? 65 in – 70 in

How tall are the top 25% of students? 70 in – 77 in

Use the stem-and-leaf plot to answer the questions that follow.

Test grades

stem	leaf
4	2
5	
6	7
7	3 8
8	2 4 6
9	1 4 9

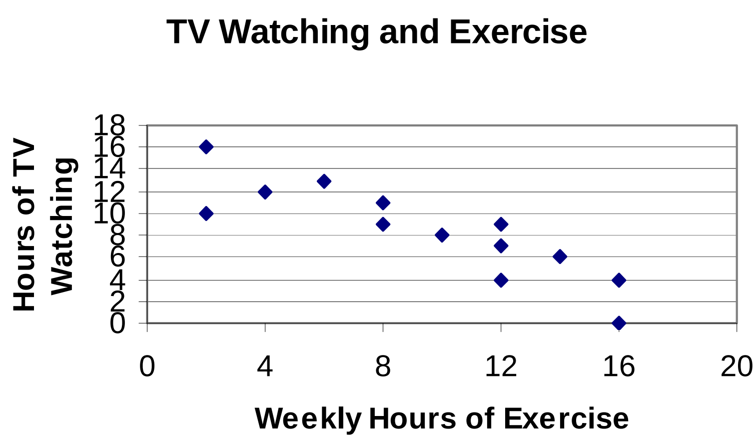
What percentage of student earned an A on the test?

3 A's; 10 total $\frac{3}{10} = .3 = 30\%$

Does an outlier exist? If so, what is that grade?

Yes, 42%

Use the scatter plot to answer the following questions.



Is there a correlation between the data?

Yes, negative

Predict the weekly number of hours of television watching of a student who exercises 9 hours per week.

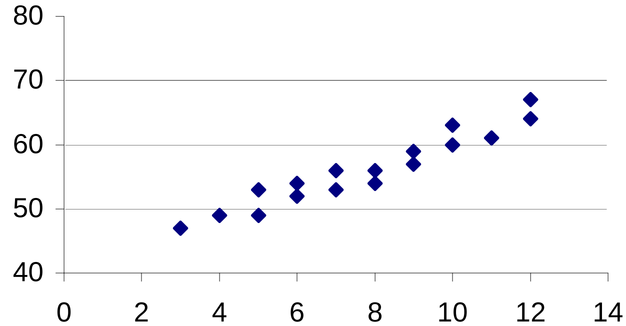
9 or 10 hours

June 2008

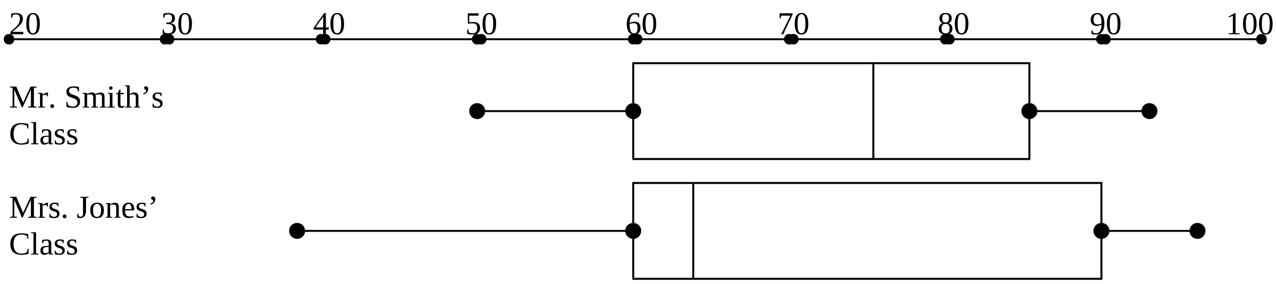
Practice

1. The graph displays the age and height of a group of children. How many students were included in the set of data?

   A. 10
   B. 11
   C. 16
   D. 17



2. The box-and-whisker plot below displays test information for two algebra classes. Use the plots to answer the questions that follow?



   A. Which class had the highest score?

   B. Which class had the lowest score?

   C. For which class is the range of the middle 50% of the scores greater?

   D. For which class are the highest scores clustered more closely?

Practice Answers

1. Count the number of points on the graph, there were 17 students included.

   Answer: C

2. A. Which class had the highest score?

      Jones (point farthest to the right)

   B. Which class had the lowest score?

      Jones (point farthest to the left)

   C. For which class is the range of the middle 50% of the scores greater?

      The interquartile range for Smith is approximately 85 – 60 = 25
      The interquartile range for Jones is approximately 89 – 60 = 29

      The bigger interquartile range - Jones

   D. For which class are the highest scores clustered more closely?

      The right whisker is smaller for Jones

Math Lesson Plan
Eligible Content M11.E.2.1.1
Measures of Central Tendency. Determine the mean, mode and median of a set of data.

Objective: Find the mean, mode and/or median of a set of data given or represented on a table or graph.

Introduction

There are many occasions in real life when it is necessary to determine behavior or specifically the tendency within a set of data. For a set of data, the three measures of central tendency may nearly be identical or may vary widely. Determining which of the three measures is most useful depends on the context in which the data are being used. For example, a teacher may give a test and want to determine the mean, mode and median to evaluate the validity of the test or the overall performance of the students taking the test. A number of scores may be used to determine a player's (such as a bowler) consistency and/or ranking when compared with other players of the same game.

Skill Review

None

Definitions

A **measure of central tendency** is a number or piece of data used to represent a typical value for a data set. The *mean*, the *median*, and the *mode* are three commonly used measures of central tendency.

The ***mean***, or **average**, of *n* numbers is the sum of the numbers divided by *n*. The mean is represented by $\bar{x}$, which is read as "*x*-bar". For a set of data $x_1, x_2, \ldots, x_n$ the mean is

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}.$$

The ***median*** of *n* numbers is the middle number when the numbers are written in numerical order. If *n* is even, the median is the average of the two middle numbers.

The ***mode*** of *n* numbers is the number or numbers that occur most frequently. There can be no mode, one mode, or many modes.

Lesson

The above definitions should be given to the students followed by the next two examples. When asked to find the median and mode in addition to the mean the following steps should be taken in solving the problem:

Step 1: Rearrange the data elements in numerical order.
Step 2: Find the mean (average) for the data elements by adding all the elements and dividing by the total number of elements.
Step 3: Find the median: The middle number if the total number of elements is odd and the average of the two middle elements if the total number of elements is even.
Step 4: Determine the most frequently occurring element – the mode. In some cases there will be no mode, in some cases only one element will occur the most frequent, and in some cases more than one element will occur the same number of most frequent times.

Examples

Example 1: The lengths (in minutes) of 13 movies are given below. Find the mean, median, and mode(s) of the data.

90, 102, 120, 180, 90, 85, 90, 137, 120, 145, 97, 93, 120

Solution: *Mean*: to find the mean, add the 13 numbers and divide by 13.

$$\frac{90+102+120+180+90+85+90+137+120+145+97+93+120}{13} = \frac{1469}{13} = 113\,\text{min}utes$$

Median: To find the median, first write the data in numerical order.

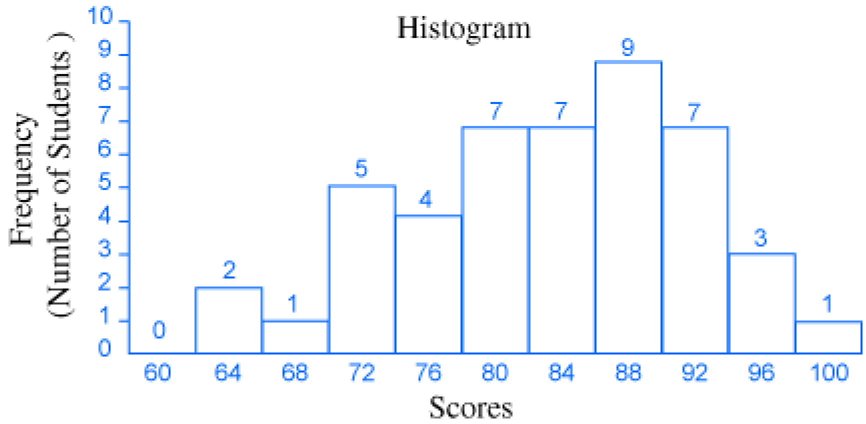
$$\underbrace{85, 90, 90, 90, 93, 97,}_{\text{6 values}}\ \underset{\textbf{Median}}{\mathbf{102}},\ \underbrace{120, 120, 120, 137, 145, 180}_{\text{6 values}}$$

The median is the middle value: 102 minutes

Modes:The modes are the values that occur most frequently:
90 minutes and 120 minutes.

Example 2:

The multiple-choice test scores of the students in Mr. Smith's chemistry class are recorded in the following histogram.



Find the mean, median, and mode of the data in the above histogram.

Solution**:**

Step 1: Rearrange the data elements.

64, 64, 68, 72, 72, 72, 72, 72, 76, 76, 76, 76, 80, 80, 80, 80, 80, 80, 80, 84, 84, 84, 84, 84, 84, 84, 88, 88, 88, 88, 88, 88, 88, 88, 88, 92, 92, 92, 92, 92, 92, 92, 96, 96, 96, 100

Step 2: Find the mean.

$$\text{mean} = \frac{(64\text{x}2) + (68\text{x}1) + (72\text{x}5) + (76\ \text{x}4) + (80\text{x}7) + (84\text{x}7) + (88\text{x}9) + (92\text{x}7) + (96\text{x}3) + (100\text{x}1)}{2 + 1 + 5 + 4 + 7 + 7 + 9 + 7 + 3 + 1}$$

$$\text{mean} = \frac{128 + 68 + 360 + 304 + 560 + 588 + 792 + 644 + 288 + 100}{46} \approx 83.3$$

Step 3: Find the median.

The sample size is even, for there are 46 data elements.
The median is the average value of the twenty-third and the twenty-fourth elements.

median = (84+84)/2 = 84

Step 4: Find the mode.

The score 64 occurs twice.
The score 68 occurs once.
The score 72 occurs five times.
The score 76 occurs four times.
The score 80 occurs seven times.
The score 84 occurs seven times.
The score 88 occurs nine times.
The score 92 occurs seven times.
The score 96 occurs three times.
The score 100 occurs once.

mode = 88

Practice

1. Your class wants to know the most typical age of a student in your class. It collects the ages of all students and the teacher. Which measure(s) of central tendency would give the best typical student age?

   A. mean
   B. median
   C. mode
   D. mean or median
   E. median or mode

2. A particular bowler has the following scores:

   110, 125, 126, 126, 128, 130, 131, 131, 132, 135

   A. Calculate the bowler's mean score.

   B. Calculate the bowler's median score.

   C. Calculate the mode.

   D. Which measure of central tendency is the best representation of the bowler's score?

Practice Answers

1. Since the teacher's age was included in the data set the average age will be raised dramatically. Usually most students in the same class will be of the same age (give or take one or two years) therefore the median or the mode would probably best represent a student's typical age.

   Answer: E

2. A. $$\frac{110+125+126+126+128+130+131+131+132+135}{10}=\frac{1274}{10}=127.4$$

   B. $$\underbrace{110,\ 125,\ 126,\ 126}_{\text{4 values}},\ 128,\ 130,\ \underbrace{131,\ 131,\ 132,\ 135}_{\text{4 values}}$$

   Median = $\frac{128+130}{2}=129$

   C. The data set has 2 modes: 126 and 131

   D. The mean is the best representation of the bowler's score (performance).

Math Lesson Plan
Eligible Content M11.E.2.1.2
Calculate and/or interpret the range, quartiles, and interquartile range of the data.

Objective: Calculate measures of dispersion.

Introduction

From the lesson on measures of central tendency, you learned the median is the middle value of a set of data; however, it does not show how the data is dispersed. Consider the following sets of data.

$\{-1, 0, 1\}$
$\{-1, 0, 1000000\}$
$\{-10000000000000000, 0, 100\}$

The number 0 is the median of each set. However, each set contains a different dispersion of numbers.

Skill Review

M11.E.2.1.1 Calculate measures of central tendency (median).

Definitions

Quartile
The first quartile is the median of the lower half of the data. The third quartile is the median of the upper half of the data. The second quartile is the median of the entire set of data.

Range
The difference between the greatest and least values in a set.

Interquartile Range
The difference between the values of the third and first quartile.

Lesson

The dispersion of a set of data is the way the value are clumped together or spread out. The range of data is calculated by finding the difference between the greatest and least values in the set.

The list of grades from a carpentry exam is: 89, 51, 90, 95, 100, 40, 70, 75, 78, 88, 81, 82, 100, 81, 90, 92, 81, 97

To calculate the range of data subtract the lowest score from the highest.

Range = 100 – 40 = 60

The quartiles are calculated by first splitting the data into two sets. Use the median value of the set to determine the numbers in the in the lower half of the data and the numbers in the upper half of the data. To find the median, the numbers must be arranged in numerical order.
Carpentry exam grades

40 51 70 75 78 81 81 81 82 88 89 90 90 92 95 97 100 100

There is no exact middle value because the set has an even number of values. The two numbers located in the middle of the set are:

40 51 70 75 78 81 81 81 82 88 89 90 90 92 95 97 100 100

To find the middle value, find the mean of the two middle numbers in the set.

Median = $\frac{82+86}{2}$ = 84

Using the median as the middle value, the set of numbers can be divided into two.

The lower half of the numbers	40	51	70	75	78	81	81	81	82
The higher half of the numbers	88	89	90	90	92	95	97	100	100

Each of these subsets of the original set has a median as well. The median of the first half of the numbers is the first or lower quartile. The median of the second half of the numbers is the third or upper quartile.

First Quartile 78

Third Quartile 92

The interquartile range is defined as the difference between the upper and lower quartiles. To find the interquartile range, subtract the first quartile from the third quartile.

Interquartile Range = 92 – 78 = 14

Practice

1. Find the interquartile range for the following set of data.

   124, 31, 169, 128, 129, 99, 149, 125, 175, 127, 219

   A. 44
   B. 125
   C. 129
   D. 169

2. The ages of the chess players invited to participate in a tournament are

   19, 25, 27, 22, 13, 24, 29, 21, 22, 55, 18, 46, 23, 17, 23

   Determine the range of ages, the median age, the lower quartile, the upper quartile and the interquartile range.

1. A After placing the numbers numerically the set looks like the following.

99 124 125 127 128 129 131 149 169 175 215

First quartile (125) Median (129) Third quartile (169)

Interquartile range = 169 – 125 = 44

2. Numerical order 13, 17, 18, 19, 21, 22, 22, 23, 23, 24, 25, 27, 29, 46, 55

Range 55 – 13 = 42

Median 23

First Quartile 19

Third Quartile 27

Interquartile Range 8

Math Lesson Plan
Eligible Content M11.E.2.1.3
Describe how outliers affect measures of central tendency.

Objective: Analyze the affect of an outlier on a set of data.

Introduction

Consider the following set of test scores:

65, 70, 75, 76, 77, 80, 82, 84, 86, 89, 90, 92, 93, 95, 97

This set of scores has a mean of 83.4 and a median of 84.

By replacing the lowest score in the set with a score of 0 and keeping the other scores the same, the set of test scores becomes:

0, 70, 75, 76, 77, 80, 82, 84, 86, 89, 90, 92, 93, 95, 97

After the change, the set of scores has a mean of 79 and a median of 84.

How has the change affected the measures of central tendency?

Skill Review

M11.E.2.1.1 Calculate or select the appropriate measure of central tendency (mean, median, or mode) of a set of data given or represented on a table, line plot, or stem-and-leaf plot.

Definitions

<u>Mean</u>
The average of set of data that is calculated by dividing the sum of the data by the number of items in the set.

<u>Median</u>
The middle value when data are arranged in numerical order.

<u>Mode</u>
The number that occurs most often in a set of data.

<u>Outlier</u>
An extreme value in a set of data. This number is much less than or much greater than the remaining numbers in the set.

Lesson

An outlier is defined as a value much greater or much less than any other number in a set of values. Outliers affect each of three measures of central tendency differently. Let's look at each measure individually.

Mean

The mean is calculated by finding the total of the numbers in a set of data and dividing the total by the numbers of values in the set. The sum of the values will certainly be affected by a number much less than or much greater than the other numbers in the set. We can say with certainty that the mean is affected by an extreme outlier.

Median

Extreme outliers do not affect the median as much as they do the mean. In the problem posed in the introduction to the lesson, the median was not affected at all by the change in the sets. Therefore, we can say the median is a more reliable tool to use when comparing sets of data with extreme outliers.

Mode

The mode is the number (or numbers) that occur most often in a set of data. The mode is not affected by extreme outliers; however, the mode is not tremendously useful in comparing sets of data due to the fact that there may be no mode or even multiple modes in a given set.

Example

A teacher gave a final exam to her Algebra class. The scores for the class are as follows:

67, 89, 91, 26, 88, 65, 78, 74, 75, 92, 95, 80, 62, 75

Determine the measures of central tendency. Which of these measures most accurately reflects the students' performance on the exam?

Work:

Place numbers in order from least to greatest.

26 62 62 65 67 74 79 81 82 88 89 91 92 95

Mean	75.3	Removing outlier (26)	Mean	79
Median	80		Median	81
Mode	62		Mode	62

The mode is least reflective of the overall student performance. There is little meaning to the fact that 2 students earned the same score. The mean is the most affected by the outlier; therefore, this is not representative of the class as a whole. In this case, the median is the more accurate representation of student performance. The median was affected very slightly by the outlier. The median shows the teacher the value at which half of the students performed better and half worse than that score.

Practice

1. The average prices of lunch at ten local high schools are as follows:

   $1.75, $1.80, $1.25, $2.00, $1.75, $1.90, $2.00, $1.95, $1.75, $2.00

   What is the mean lunch price after removing the outlier from the set?

   A. $1.69
   B. $1.81
   C. $1.82
   D. $1.88

2. A realtor publishes the average price of the homes she sells. In the last year, she has sold homes with values of:

$150,000	$400,000	$500,000
$425,000	$390,000	$476,000
$475,000	$498,000	$390,000

   Which measure of central tendency (mean, median, or mode) should she use as her "average", knowing the realtor wants to be seen as a high-end home seller?

Practice Answers

1. After removing $1.25 from the list as the outlier, the sum is 16.9. Dividing 16.9 by 9, the mean lunch price is $1.88

   Answer: D

2. The three measure of central tendency for this set of data are:

$150,000	$400,000	$500,000
$425,000	$390,000	$476,000
$475,000	$498,000	$390,000

150,000
390,000
390,000
400,000
425,000
475,000
476,000
498,000
500,000

Mean $411,556

Median $425,000

Mode $390,000

The realtor should choose the median because it is the measure with the highest value, which will meet her needs for advertising

Math Lesson Plan
Eligible Content M11.E.3.1.1
Find probabilities for independent, dependent or compound events and represent as a fraction, decimal or percent.

Objective: Find probabilities for compound events.

Introduction

Tell students that the day's lesson will be about learning how to extend their knowledge of probability to include compound events. These events can be independent or dependent based on the type of problem they are faced with.

Skill Review

M11.E.3.1.2

Definitions

Compound events
Compound events consist of two or more events.

Independent events
If the outcome of one event does not affect the outcome of the other event, the events are said to be independent.

Dependent events
If the outcome of one event affects the outcome of the other event, the events are said to be dependent.

Lesson

Theorem: If two events are independent, then the probability of both events occurring is found as follows:

$$P(A \text{ and } B) = P(A) \bullet P(B)$$

Theorem: Suppose two events, A and B, are dependent. Then the probability of both events occurring is found as follows:

$$P(A \text{ and } B) = P(A) \bullet P(B \text{ following } A)$$

When you make two or more selections while replacing any previous selection, you are working with independent events. If you do not replace each prior selection, then the number of outcomes changes each time. In this case, you are working with dependent events since whatever selection was made in the prior times affects what your next selection will be.

Remember when finding probability that your answer can be written as a fraction, a decimal or a percent.

Examples

Suppose a number cube (die) is rolled twice. What is the probability that an odd number will occur both times?

Work

Since you are rolling the same die two times, the events are independent of each other.

P(rolling two odd numbers) = P(first roll odd) • P(second roll odd)

Outcomes: 1, 2, 3, 4, 5, 6
Odd numbers: 1, 3, 5
P(odd) = $\frac{3}{6} = \frac{1}{2}$
P(rolling two odd numbers) = $\frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$ = 0.25 = 25%

A jar contains three red balls, two white balls, and one green ball. What is the probability of picking two white balls if the first ball is not replaced?

Work

Since you are not replacing the first ball, the number of outcomes changes for the second try. Therefore the events are dependent.

P(two white balls) = P(1st ball white) • P(second ball white)

P(1^{st} ball white) = $\frac{2}{6} = \frac{1}{3}$
Since the first ball selected was white and not replaced,
P(2^{nd} white) = $\frac{1}{5}$
P(two whites) = $\frac{1}{3} \bullet \frac{1}{5} = \frac{1}{15}$ = .07 = 7%

Practice

1. A club has 25 members, 20 boys and 5 girls. Two members are selected at random to serve as president and vice president. What is the probability that both will be girls?

    (A) $\frac{1}{5}$

    (B) $\frac{1}{25}$

    (C) $\frac{1}{30}$

    (D) $\frac{1}{4}$

2. What is the probability of selecting an eight followed by a nine from a regular deck of 52 cards if the first card is replaced before the second card is drawn?

Practice Answers

1. C

Events are dependent since the first selection change the number of outcomes for the second selection.

P(two girls) = P(1^{st} girl) • P(second girl)

$$P(1^{st} \text{ girl}) = \frac{5}{25} = \frac{1}{5}$$

$$P(\text{second girl}) = \frac{4}{24} = \frac{1}{6}$$

$$P(\text{two girls}) = \frac{1}{5} \bullet \frac{1}{6} = \frac{1}{30}$$

2. Since the first card is being replaced before the second card is drawn, the events are independent.

P(eight then nine) = P(eight) • P(nine)

$$P(\text{eight}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{nine}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{eight then nine}) = \frac{1}{13} \bullet \frac{1}{13} = \frac{1}{169}$$

Math Lesson Plan
Eligible Content M11.E.3.1.2
Find, convert, and/or compare the probability and/or odds of a simple event.

Objective: Compare odds and probability.

Introduction

Probability and Odds are used to define a specific situation is separate. If you toss a coin, it may come down heads or tails. The two possibilities are equally likely, assuming the coin is fair. The probability of it landing on tails is ½ because tails is one of the equally likely possibilities. Today you will learn how the odds of an event are written.

Skill Review

None

Definitions

<u>Probability</u>

The probability of an event is defined as $p(e) = \dfrac{\text{chances for an event happening}}{\text{total possible chances}}$

<u>Odds</u>

The odds of an event happening are written as a ratio. chances for: chances against

Lesson

The odds in favor of an event happening are the ratio of the probability that the event will occur to the probability that the event will not occur. The reciprocal of this ration represents the odds against the event occurring.

For example, a jar contains 7 red marbles and 3 green marbles.

The odds of choosing a red marble would be defined as the ratio of red marble to all other marbles. There are 7 red and 3 not red, so the odds would be 7 to 3 or 7:3.

The odds against choosing a red marble would be the reciprocal of the event. There are 3 not red, 7 red. The odds of this event happening would be 3 to 7 or 3:7.

If the ratio can be reduced, like a ratio of 25 to 10, it must be done. (5 to 2)

To work between probability and odds, identify the number in the total population, the number in favor of the event happening, and the number not in favor of the event happening. These three numbers can be used to define probability and/or odds for any event.

Example

The odds are 6 to 5 that a randomly chosen student in a certain group intends to go to college. What is the probability that a student in the group does not intend to go to college?

Work	Total number of students	6 + 5 = 11
	Number of students intending to go to college	6
	Number of students not intending to go to college	5

$$\text{Probability} = \frac{\text{Number not going to college}}{\text{Total number of students}}$$

$$= \frac{5}{11}$$

In a survey of 276 people, 23 reported that they were left-handed. What are the odds that a randomly chosen person is right handed?

Work	Total number of people	276
	Number of left handed people	23
	Number of right handed people	276 – 23 = 253
	Odds right handed : not right handed	253 to 23
	Reduce to lowest terms	11 to 1

Practice

1. The chance of rain is ¾. What are the odds against rain occurring?

   A. 3 to 1
   B. 1 to 3
   C. 1 to 4
   D. 3 to 4

2. Use the table below to answer the questions.

Grade Level	9	10	11	12
Male	y	5	5	6
Female	4	10	x	4

The table displays the gender and grade distribution of the students in the Ski Club.

A. If the probability of a student in 11th grade being male is $\frac{5}{11}$, find the value of x.

B. If the odds of a student in 9th grade being female are 4 to 8, find the value of y.

C. List the probability that a 10th grade ski club member is a female.

D. List the odds that a 12th grade ski club member is not female.

Practice Answers

1,	Total chances	4
	Chances of rain occurring	3
	Chances of rain not occurring	1
	Odds rain not occurring: rain occurring	1 to 3
	Answer B	

2.	A.	5 male students, 11 total students.	11 – 5 = 6	x = 6
	B.	4 female, 8 not female		y = 8
	C.	$\frac{10}{15} = \frac{2}{3}$		
	D.	6 not female, 4 female	6 to 4	3 to 2

Math Lesson Plan
Eligible Content M11.E.3.2.1
Determine the number of permutations and/or combinations or apply the fundamental counting principal.

Objective: Find the numbers of permutations or combinations of a set.

Introduction

There are many occasions in real life when it is necessary to determine the number of possibilities. For example, a builder offers 6 different homes with a choice of 7 exterior color schemes and 4 kitchen options. How many different homes can the builder place in the development? Maybe a teacher provides you with the following option in their test. She states that you many choose any two of the nine essays to complete. How many different combinations are available?

Skill Review

M11.A.3.1.1 Simplify/Evaluate expression using the order of operations to solve problems.

Definitions

Factorial
$n! = n\,(n - 1)(n - 2)...(2)(1)$ example $6! = (6)(5)(4)(3)(2)(1) = 720$

Fundamental Counting Principle
This principle can be used to count the number of ways two or more events can happen in succession.

Permutation
An arrangement of items in a particular order.

Combination
A set of items in no particular order.

Lesson

The fundamental counting principle should be used when deciding how many ways things can happen in succession. For example, using the model home above you would do the following to determine the number of different homes available.

Total # of different homes = (6 home styles) × (7 exterior colors) × (4 kitchens)
= 168 different homes

Permutations and combinations are used when determining the number of arrangements or choices available from a set of items. Permutations are used when the order of the items matters to the problem. Combinations are used when the items do not have to be in any particular order. An example of a permutation would be: How many ways can a president, secretary and treasurer be elected from a class of 20 students? In this problem, the students are placed in specific offices indicating the order matters. A similar example of a combination would be: How many ways can a committee of three students be elected from a class of 20 students? In the problem, the order the students are chosen does not matter.

<u>Formulas</u> (both are provided for the students on the PSSA Grade 11 Formula sheet)

Permutations: $P(n,r) = \dfrac{n!}{(n-r)!}$

n = number of items in set
r = numbers of items to be chosen

Combinations: $C(n,r) = \dfrac{n!}{r!(n-r)!}$

To calculate factorial problems, enter the number followed by the ! (factorial) button on the calculator. Then press enter.

Examples

The cafeteria is serving a special luncheon. Each student has a choice of the following:

Entrée	ham, steak, or chicken
Side	baked potato, mashed potatoes, rice, fries
Vegetable	squash or corn
Dessert	chocolate cake, apple pie, crème brulee

How many different meals can be made from these offerings?

Work

Total # of meals $= (3 \text{ meat}) \times (4 \text{ sides}) \times (2 \text{ veg}) \times (3 \text{ desserts})$

$= 72$ different meal options

Find the value of the following expression. $\dfrac{8!}{3!(8-3)!}$

Work $\dfrac{8!}{3!(5)!} = \dfrac{40320}{6 \times 120} = \dfrac{40320}{720} = 56$

Nine teams enter a tournament. How many arrangements of third, second, and first place are possible?

Work This is an example of a permutation because the order matters.

Total # of arrangements $= \dfrac{n!}{(n-r)!}$ where n = 9 and r = 3

$= \dfrac{9!}{(9-3)!}$

$= \dfrac{9!}{(6)!}$

$= \dfrac{362{,}800}{720}$

= 504 arrangements

Five essay questions appear on a test. You are supposed to choose three to answer. In how many ways can this be done?

Work This is an example of a combination because the order does not matter.

Total # of combinations $= \dfrac{n!}{r!(n-r!)}$ where n = 5 and r = 3

$= \dfrac{5!}{3!(5-3!)}$

$= \dfrac{5!}{3!(2!)}$

$= \dfrac{120}{6(2)}$

= 10

Practice

1. There are five dancers standing in a horizontal row on stage. In how many different ways can the director place the dancers in this line?

   A. 5
   B. 25
   C. 120
   D. 125

2. Police use photographs of various facial features to help witnesses identify suspects. One basic identification kit contains 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheeks.

   A. The developer of the identification kit claims that it can produce billions of different faces. Is this claim correct?

   B. A witness can clearly remember the hairline and the eyes and eyebrows of a suspect. How many different faces can be produced with this information?

Practice Answers

1. Total # of arrangements $= \dfrac{n!}{(n-r)!}$ where n = 5 and r = 5

$$= \frac{5!}{(5-5)!}$$

$$= \frac{5!}{(0)!}$$

$$= \frac{120}{1}$$ **Hint: 0! = 1**

= 120 arrangements

Answer: C

2. A. Number of faces = 195 × 99 × 89 × 105 × 74 = 13,349,986,650

The developer’s claim is correct since the kit can produce more than 13 billion faces.

B. Because the witness clearly remembers the hairline and the eyes and eyebrows, there is only 1 choice for each of these features.

Number of faces = 1 × 1 × 89 × 105 × 74 = 691,530.

Math Lesson Plan
Eligible Content M11.E.4.1.1
Estimate or calculate to make predictions based on a circle, line, bar graph or given situation

Objective: Approximate the best fitting line for a set of data.

Introduction

When you solve problems, you must first decide whether you need an estimate or an exact answer. Problems that ask about how much, about how many, or do you have enough can often be answered by using estimation. Even when problems require an exact answer, you can estimate before you start your calculations in order to get a "ballpark" figure or after to check your answer to see if it makes sense.

Skill Review

M11.A.3.1.1 Simplify/evaluate expressions using the order of operations to solve problems.

Definitions

<u>Bar graph</u>
A means of displaying statistical information in which horizontal or vertical bars are used to compare quantities.

<u>Circle graph</u>
A means of displaying data where items are represented as parts of the whole circle.

<u>Line graph</u>
A means of displaying data using points and line segments to show changes in data over periods of time.

Lesson

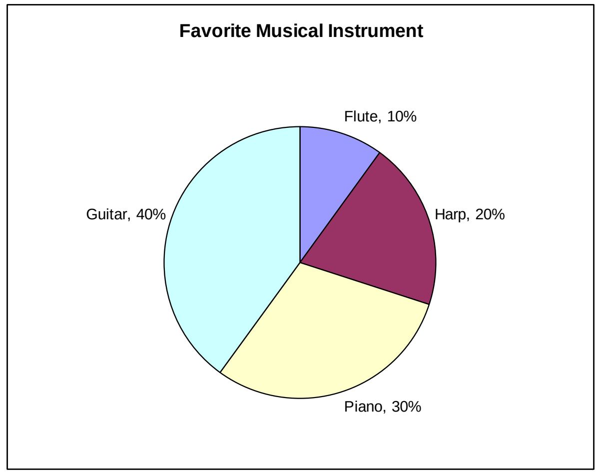
Different types of estimation techniques can be used depending on the problem and the degree of accuracy needed. Rounding in the quickest and most efficient way to estimate. When solving a problem asking for an estimate, it is always possible to determine as exact answer if you prefer.

Example

If the number of students whose favorite musical instrument is a harp is 35, how many students were surveyed?

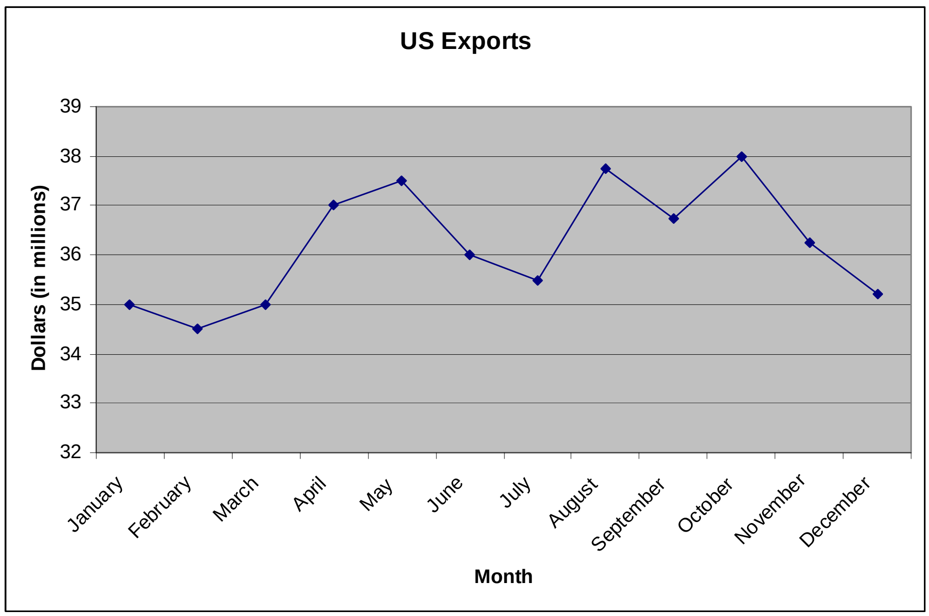
To solve this problem, notice that harp is the favorite of 20% or 1/5 of the students. To calculate the approximate answer, multiply 35 by 5.

The answer is 175.



Using the line graph at right, between which two months was the increase in permits issued greatest?

To answer this question, it is unnecessary to calculate the actual difference between each month. By looking at the graph, you can determine the steepest jump, therefore the largest increase happens between March and April.



Practice

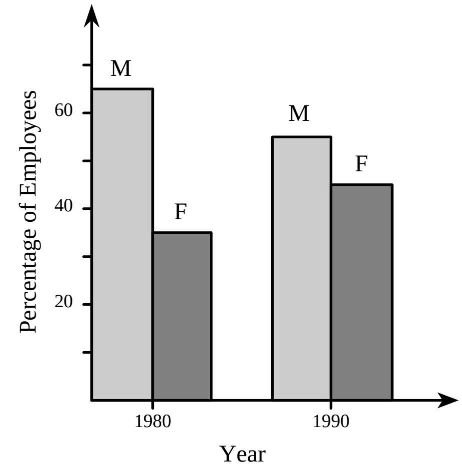
1. The percentages of males and females working for a company in 1980 and 1990 are shown n the bar graph. What is the increase in the percentage of females employed for the ten-year period?

    A. 10%

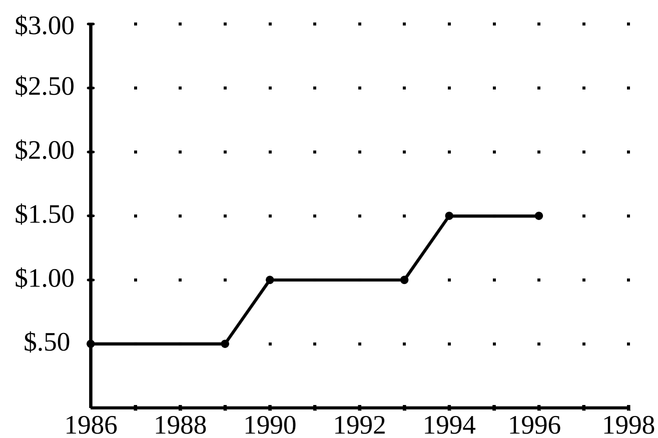
    B. 15%

    C. 20%

    D. 25%



2. The hourly parking fees for the local airport from 1986 through 1996 are shown on the line graph. Using this information, predict what the hourly parking fee will be for 1997.



Practice Answers

1. A. The increase is approximately 10%

2. The graph is displaying a patter of staying level for 4 years, then increasing by .50 for the next 4 years. The parking fee in 1997 using this pattern would remain at $1.50

Math Lesson Plan
Eligible Content M11.E.4.1.2
Use probability to predict outcomes

Objective: Use probability to determine possible outcomes.

Introduction

Probability is the likelihood that an event will happen. A probability can be used to predict the number of times something will happen. This type of probability is used often in the world of sports with items like batting average in baseball and percentage of completed throws in football.

Skill Review

M11.E.3.1.1 Find probabilities for independent, dependent, or compound events and represent as fractions, decimal or percent.

Definitions

Lesson

Probability can be used to estimate the number of times an event will occur in the future. For example, a baseball player has had 25 hits in 100 at bats. The probability the next at bat will be a hit is $\frac{25}{100} = \frac{1}{4}$, 0.25 or 25%. This probability can be used to estimate the number of hits this batter will have in an entire season. If the batter has approximately 700 at bats in a season, multiply the probability by the expected number of at bats.

0.25 (probability) $\times$ 700 (expected at bats) = 175 hits expected this season.

This same process takes place regardless of the form of the probability.

$\frac{1}{4} \times 700 = 175$ hits expected

$25\% \times 700 = 175$ hits expected.

You can predict outcomes with either theoretical or experimental probability.

Example

Approximately 9% of people are left-handed. Your high school has 997 students. Estimate the number of left handed students in your school.

Work: $9\% \times 997 = 89.73$

Approximately 90 students will be left-handed.

Practice

1. From shipment of 500 batteries, a sample of 25 was selected at random and tested. If 2 batteries in the sample were found to be dead, how many dead batteries would be expected in the entire shipment?

   A. 10
   B. 20
   C. 30
   D. 40
   E. 50

2. You rolled a die 125 times. The frequency of each number is listed in the table below.

Number	1	2	3	4	5	6
Frequency	14	21	40	16	28	6

   A. What is the experimental probability the die landed on a 3?

   B. What is the theoretical probability the die will land on a 3?

   C. Use the experimental and theoretical probabilities from parts A and B to determine how many 3's would be expected when a die is rolled 450 times based on your experimental probability and its theoretical probability?

Practice Answers

1. $\frac{2}{25} \times 500 = 40$ batteries

   Answer: D

2. A. Experimental Probability $\frac{40}{125} = \frac{8}{25}$

   B. Theoretical Probability $\frac{1}{6}$

   C. Estimation based on experimental probability $\frac{8}{25} \times 450 = 144$ times a 3 will be rolled.

   Estimation based on theoretical probability $\frac{1}{6} \times 450 = 75$ times a 3 will be rolled.

Math Lesson Plan
Eligible Content M11.E.4.2.1
Draw, find, and/or write an equation for a line of best fit for a scatter plot

Objective: Approximate the best fitting line for a set of data.

Introduction

When data show a positive or negative correlation, you can approximate the data with a line. Once a line has been drawn, the equation of the line can be calculated. The line of best fit (and the resulting equation) can be used to estimate values on a scatter plot.

Skill Review

M11.D.3.2.2 Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.E.1.1.1 Create and/or use appropriate graphical representations of data, including box-and-stem-and-leaf plots, or scatter plots.
M11.E.1.1.2 Analyze data and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, an scatter plots).

Definitions

Scatter plot
A set of unconnected points on a graph.

Lesson

Once a scatter plot has been drawn and a correlation (positive or negative) has been established, a line should be sketched to follow the pattern established by the set of points. There should be as many points above the line as below once the line has been drawn. After sketching the line, choose any two points that exist on the line. These points do not have to be points on the original scatter plot. Any points on the line will help establish the equation of the line. Using the points you have chosen, use the slope formula and point-slope formula to determine the equation of the line.

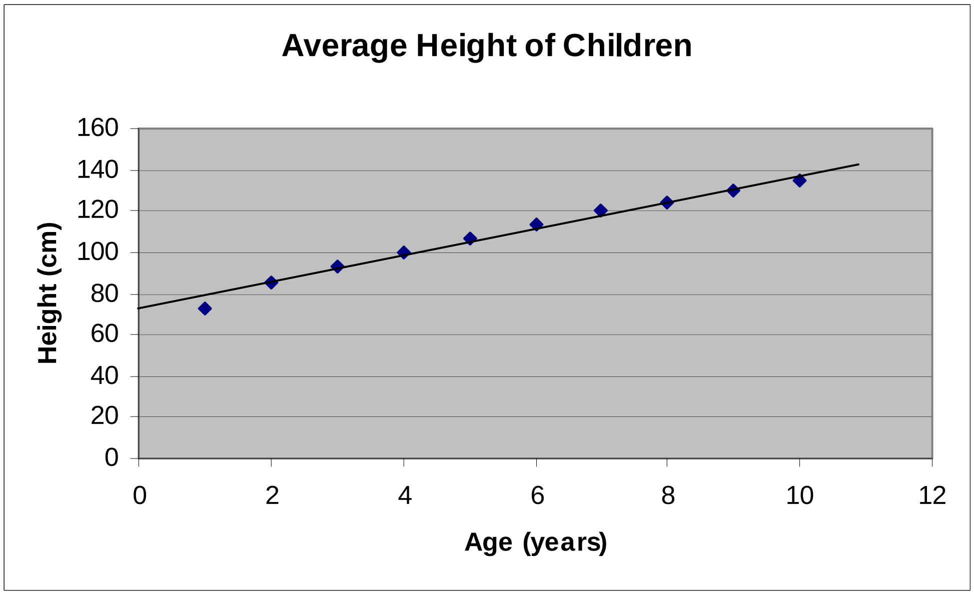
Example

The table below gives the average height of children for ages 1 – 10. Draw a scatter plot of the data and approximate the best-fitting line for the data.

Age (years)	1	2	3	4	5	6	7	8	9	10
Height (cm)	73	85	93	100	107	113	120	124	130	135

The graph below is a scatter plot of the data in the table.

A line of best fit was drawn through the points on the scatter plot. Some of the points fall above the line, some fall below the line.
Two of the points fall on the line. The points (3, 93) and (8, 124) will be used to establish the equation of the line



Step 1 Calculate the slope between the chosen points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{124 - 93}{8 - 3} = \frac{31}{5} = 6.2$$

In the case of a line of best fit, always calculate slope as a decimal

Step 2 Use slope and one of the chosen points in the point-slope formula

$y - y_1 = m(x - x_1)$ where m is the slope calculated is step 1 and (x_1, y_1) is the point chosen

$y - 93 = 6.1(x - 3)$ substitute values for m and (x_1, y_1)

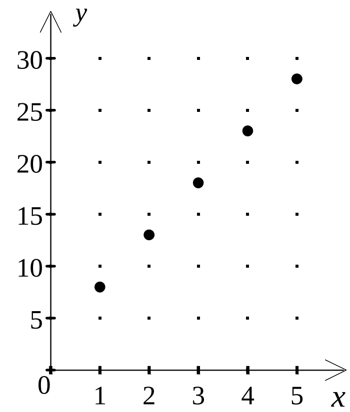
$y - 93 = 6.1x - 18.3$ distribute 6.1

$y = 6.1x + 74.7$ add 93 to both sides of the equation (solve for y)

The line of best fit for the scatter plot draw above is $y = 6.1x + 74.7$

Practice

1. Which equation represents the scatter plot?



A. $y = 3 - 5x$

B. $y = 5x - 3$

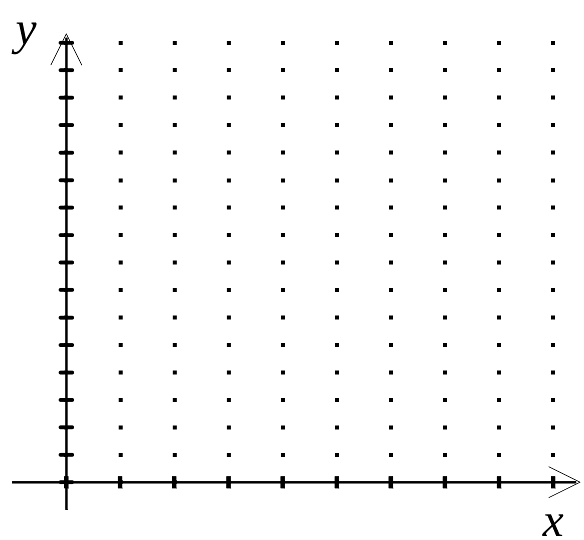
C. $y = 5x + 3$

D. $y = 5 - 5x$

2. For the following data:

x	1	2	3	4	5	6	7	8
y	1.75	4.1	4.95	7	8.15	11.1	11.95	14

A. Make a scatter plot of the data.

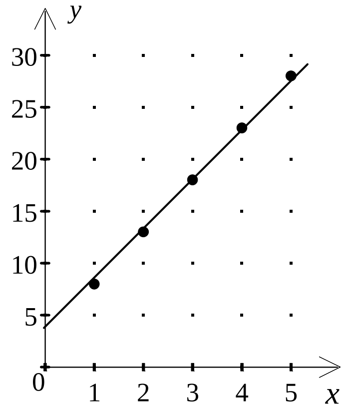


B. Approximate the best fitting line for the data.

C. Find an equation of your line of best fit.

Practice Answers

1.



Points on the line of best fit (2, 13) and (3, 18)

Slope between the points $m = \frac{18-13}{3-2} = \frac{5}{1} = 5$

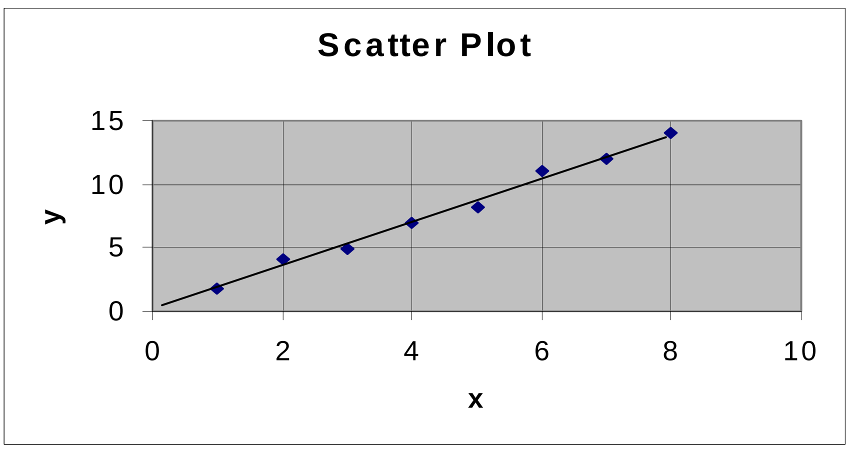
Point –slope formula $y - 13 = 5(x - 2)$

Distribute 5 $y - 13 = 5x - 10$

Add 13 to both sides $y = 5x + 3$

Answer: C

2. A.



B. line of best fit drawn on scatter plot

C. Two points on the line of best fit (4, 7) and (7, 11.95)

Students may choose different points than in this example. However, the final equation must be similar to the equation written using these points.

Slope between the points $m = \frac{11.95 - 7}{7 - 4} = \frac{3.95}{3} = 1.32$

Point –slope formula $y - 7 = 1.32(x - 4)$

Distribute 5 $y - 7 = 1.32x - 5.28$

Add 7 to both sides $y = 1.32x + 1.72$ This is the equation of the line of best fit.

Math Lesson Plan
Eligible Content M11.E.4.2.2
Develop and evaluate inferences and predictions or draw conclusions based on data or data displays

Objective: Make predictions using equations or graphs of best-fit lines of scatter plots.

Introduction

Many times the points in a scatter plot show a general trend. That trend could be approximated by a best-fit line drawn to include the most number of data points. The equation of this best-fit line can be used to predict outcomes outside the graphed data points.

Skill Review

M11.D.3.2.2 Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.E.1.1.1 Create and/or use appropriate graphical representations of data, including box-and-stem-and-leaf plots, or scatter plots.
M11.E.1.1.2 Analyze data and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, an scatter plots).
M11.E.4.2.1 Draw, find, and/or write an equation for a line of best fit for a scatter plot.

Definitions

A **scatter plot** is a graph used to determine whether there is a relationship between paired data.

If *y* tends to increase as *x* increases, then there is a **positive correlation.**

If *y* tends to decrease as *x* increases, then there is a **negative correlation.**

If the points show no linear pattern, then there is **relatively no correlation.**

The **best-fit line** is approximated by sketching the line that best fits the points, with as many points above the line as below it.

The line of best fit is often called the **regression line** and its equation is called the **regression equation.**

Lesson

Given a set of paired data graphed on a coordinate plane determine whether or not there appears to be a trend in the graph's behavior.

Step 1. Sketch the line that best fits the points, with as many points above the line as below it.

Step 2. Now, estimate the coordinates of two points on the line, not necessarily data points. Use these points to find an equation of the line.

Step 3. Find the slope of the line.

Step 4. Use point-slope form.

Step 5. Solve for y. This will yield the slope-intercept (y = mx + b) form of the equation.

Step 6. Use the new equation to make predictions by substituting x values into the equation.

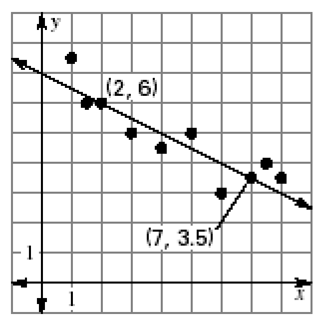
Examples

Example 1:

Fitting a Line to Data

Approximate the best-fitting line for the data in the table.

x	1	1.5	2	3	4	5	6	7	7.5	8
y	7.5	6	6	5	4.5	5	3	3.5	4	3.5



SOLUTION

To begin, draw a scatter plot of the data. Then sketch the line that best fits the points, with as many points above the line as below it.

Now, estimate the coordinates of two points on the line, not necessarily data points. Use these points to find an equation of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.5 - 6}{7 - 2} = \frac{-2.5}{5} = -\frac{1}{2}$	Find the slope of the line.
$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 6 = -\frac{1}{2}(x - 2)$	Substitute for m, x, and y_1.
$y - 6 = -\frac{1}{2}x + 1$	Distributive property
$y = -\frac{1}{2}x + 7$	Solve for y.

The above equation could now be utilized to predict values of y for selected values of x that lie outside of the given ordered pair data set.

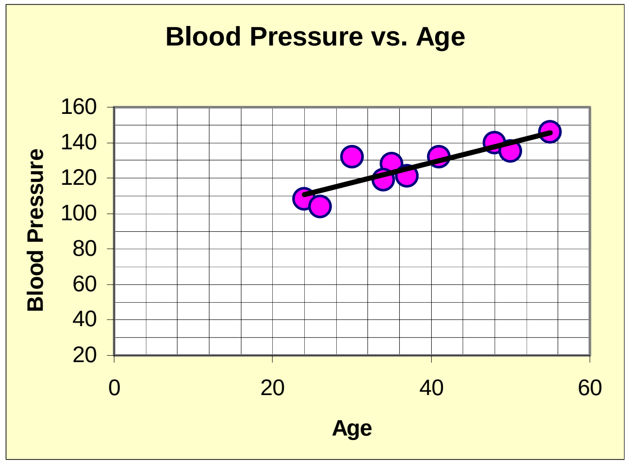
Example 2: Using the given data chart and accompanying graph of age versus blood pressure

A) Calculate the equation of the line of regression.

B) Calculate the approximate blood pressure for a 60 year old.

C) Calculate the approximate blood pressure for a 20 year old.

Age	Blood Pressure
35	128
24	108
48	140
50	135
34	119
55	146
30	132
26	104
41	132
37	121



Solution:

A) Use the two endpoints of the regression line shown on the graph (24,108) and (55,146).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{146 - 108}{55 - 24} = \frac{38}{31} \cong 1.23$$ Find the slope of the line

$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 108 = 1.23(x - 24)$	Substitute the appropriate values into point-slope form
$y - 108 = 1.23x - 29.52$	Distributive property
$y = 1.23x + 78.48$	Solve for y; slope-intercept form

Answer: $y = 1.23x + 78.48$; where y = blood pressure, x = age of patient

B) Use the regression line calculated in part A, substitute x = 60

$y = 1.23(60) + 78.48 = 152.28$

Answer: a 60 year old would be expected to have a blood pressure of 152

C) Use the regression line calculated in part A, substitute x = 20

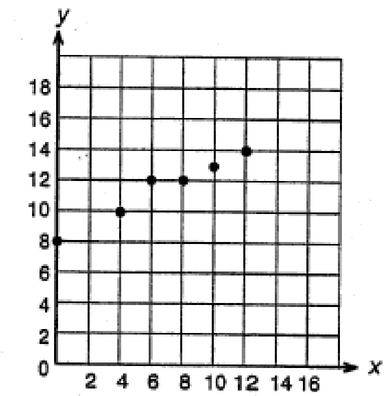
$y = 1.23(20) + 78.48 = 103.08$

Answer: a 60 year old would be expected to have a blood pressure of 103

Practice

1.

Which of the given equations most closely represents the line of best fit for the scatter plot given below?



A. $y = 2x + 8$

B. $y = \frac{1}{2}x + 8$

C. $y = -\frac{1}{2}x + 8$

D. $y = -2x + 8$

2.

The annual maintenance cost of an appliance is given by the regression equation $y = 12.5x + 19.2$, where y represents the total maintenance cost and x represents the age of the appliance in years. Rounded to the nearest dollar, what is the expected maintenance cost of a 14-year-old appliance?

Practice Answers

1. The graph shown has an obvious positive correlation. Only one data point appears to be off the best fit line that could be drawn. The best fit line would cross the y-axis at 8 (the point (0,8)) and the slope could very easily be calculated using the formula/definition of slope

$$slope = \frac{rise}{run} = \frac{1}{2}$$

$y - y_1 = m(x - x_1)$	Use point-slope form.
$y - 8 = ½(x - 0)$	Substitute the appropriate values into point-slope form
$y = ½\ x + 8$	Solve for y; slope-intercept form

Answer. B

2.

y = 12.5(14) + 19.2	Substitute the value x = 14 into the given regression equation
y = 175 = 19.2 = $194.20	Simplify
y = $194	annual maintenance cost on a 14 year old appliance rounded to the nearest dollar.