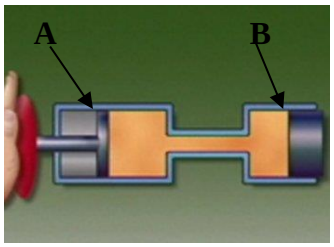
Use Pascal's Law = Use reasoning to solve equations and justify the solution method

Program POS Task: Demonstrate knowledge of fluid power systems.

Program Associated Vocabulary:
PRESSURE, FORCE, AREA OF A PISTON, LOAD, BORE (DIAMETER)

Program Formulas and Procedures:
FORMULA: Force output of cylinder during extension is the typical application of the F = P x A formula, where F=Force, P=Pressure and A=Area.

Example 1: If the hand pushing the plunger = 50 psi of pressure and the Diameter of Piston A=3" and the Diameter of Piston B=6", what is the Force (F_2) output?



$$F_2 = P \times A_2$$
$$F_2 = 50 \times (\pi 1.5^2)$$
$$F_2 = 50 \times (7.069)$$
$$F_2 = 353.4 \text{ lbs.}$$

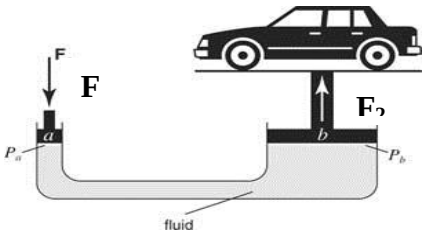
Example 2: In this example, if an auto repair shop has a compressor that produces 100 psi of compressed air and acts on piston A (2"d) what is the Force (F_2) produced by piston B (22" d)? The car weighs 4500 lbs. Does the system pictured as described produce enough Force (F_2) to raise the vehicle?

$$100 \text{ psi} = \frac{F_2}{A_2} = \frac{F_2}{\pi 11^2} = \frac{F_2}{380}$$

$$100 \times 380 = \frac{F_2}{380} \times \frac{380}{1}$$ Multiply both sides by 380.

$$F_2 = 38,000 \text{lbs.}$$

Enough to raise the car!



PA Core Standard: CC.2.2.HS.D.9

Description: Use reasoning to solve equations and justify the solution method.

Math Associated Vocabulary:
INVERSE, RECIPROCAL, PROPORTION, CROSS MULTIPLICATION, RATIO, CONSTANT, RADIUS

Formulas and Procedures:
Pascal's Law: Pressure applied to an enclosed fluid is transmitted, undiminished to every portion of the fluid and the walls of the containing vessel. Although the pressure is the same within the enclosure, it is extended over a much larger area, multiplying the force that moves a piston. The force of a small cylinder must be exerted over a much large distance—a small force exerted over a large distance is substituted for a large force over a small distance**.**

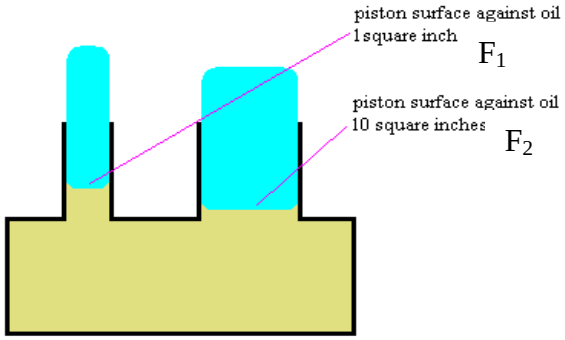
Example: Hydraulic lift principle. The surface area of the right piston is 10 times greater than the left one. The resulting force is 10 times larger. In this example, if 1 lb. of Force (F_1) is applied to the 1 in. sq. piston, then 10 lbs. of Force (F_2) is applied to the 10 in. sq. piston. The pressures remain the same.

$$F = P \times \text{Area}, \quad P = \frac{F}{A}$$

Check your work...

$$P = \frac{\text{Force}(F_1)}{\text{Area}}, \quad P = \frac{1}{1}, \quad P = 1$$

$$P = \frac{\text{Force}(F_2)}{\text{Area}}, \quad p = \frac{10}{10}, \quad P = 1$$



Formula for Pascal's Law:

$$P(F_1) = \frac{\text{Force}(F_2)}{\text{Area}}$$

$$A = \frac{F_2}{P}$$

$$F_2 = P \times A$$

Instructor's Script - Comparing and Contrasting

Understanding direct vs. inverse proportions can be very useful when students are making quick decisions as to how a change in a system may affect the result. For example, knowing that fuel mileage and miles driven are directly proportional means that an increase in miles driven with a single tank of fuel means the fuel mileage must also increase.

Within technical applications, proportional math comes from problems based on a given formula where a value is held steady and two other values are allowed to adjust.

It's important that students understand the distinction between direct and inverse. Direct indicates that the 2 values are allowed to change in the proportion and will go up together or go down together. An inverse proportion indicates the 2 values will change in opposite directions (one higher, one lower).

When working with a proportional problem with 2 ratios you can eliminate common factors on the top or bottom of both ratios (i.e. fractions on either side).

Common Mistakes Made By Students

When students compare direct and inverse proportional relationships, they may become confused and have difficulty differentiating one from the other. One way to keep them straight is to share this problem: $F_1 = 50$ lbs. $Radius_1 = 1$ $Radius_2 = 4$

1. Set up one pair of values on the same line, e.g. $\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad = \frac{50}{\pi 1^2} = \frac{F_2}{\pi 4^2} \quad = \frac{50}{3.14} = \frac{F_2}{50.27}$

   $$\text{Cross Multiply } \frac{50 \times 50.27}{3.14} = 800.47 \text{ lbs.}$$

2. Cross multiply (50 times 50.27) and divide by A_1, but first determine if you have to invert one ratio.

3. If you have to invert one ratio, then it is an inverse proportion.

4. If need be, set up the problem and do it both ways to see which answer makes sense? It must be an inverse proportion.

CTE Instructor's Extended Discussion

Mathematics is such an integral part of our work that many times we use math unknowingly. It is important that technical instructors demonstrate to students how math concepts are relevant in their technical training and that the math is presented in a way which shows a relationship to the math which CTE students use in their academic school settings. Each teacher must reach a comfort level necessary for teaching the math concepts and formulas. Using a program's tasks, technical teachers can highlight many examples of this math concept with students.

Problems	Career and Technical Math Concepts	Solutions
1. You have a cylinder with a bore of 5.0 inches and a pressure of 100 psi. What would the force output of the cylinder be?		
2. The gauge reading on the outlet of a hydraulic fitting is 750 psi. If the force (F_1) on the fluid is 250 lbs., what is the diameter and area of the piston? Use the formula: $A = \frac{F_2}{P}$ $r = \sqrt{\frac{Area}{\pi}}$		
3. A hydraulic press has an input cylinder 1 inch in diameter and an output cylinder 6 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 10 pounds is applied to the input piston.		

Problems	Related, Generic Math Concepts	Solutions
4. A car master cylinder has a bore of 1.25 inches and a pressure of 500 psi. What would the force output be at the wheel cylinders?		
5. The output force (F_2) of a fire truck's pumper is 1750 psi. If the pressure (F_1) on the water is 200 psi, what is the diameter and area of the piston?		
6. A hydraulic jack has an input cylinder 2 inches in diameter with an output cylinder 4 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 50 pounds is applied to the input piston.		

Problems	PA Core Math Look	Solutions
7. Diameter = 2.5"; F_1 = 350psi. Find F_2.		
8. F_1 = 25,000 lbs.; P = 350 psi; find the diameter and area of the piston.		
9. Input cylinder = 7" diameter; output cylinder = 15" diameter. Find F_2 when 800 lbs. is applied to the input cylinder.		

Problems	Career and Technical Math Concepts	Solutions
1. You have a cylinder with a bore of 5.0 inches and a pressure of 100 psi. What would the force output of the cylinder be?		F = P x A F = 100 psi x (π 2.5^2) F = 100 x (19.63) F = 1,963 lbs.
2. The gauge reading on the outlet of a hydraulic fitting (P) is 750 psi. If the force (F_1) on the fluid is 250 lbs., what is the diameter and area of the piston? Use the formula: $A = \frac{F_2}{P}$ $r = \sqrt{\frac{Area}{\pi}}$		Area = Force/Pressure Area = 750 lbs./250 lbs./in^2 Area of piston = 3 in^2 Radius = $\sqrt{3/\pi}$ = .98 inches Diameter = 2 x radius = 2 x .98 = 1.96 inches (diameter)
3. A hydraulic press has an input cylinder 1 inch in diameter and an output cylinder 6 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 10 pounds is applied to the input piston.		$F_1/A_1 = F_2/A_2$ $10/(\pi.5^2) = F_2/(\pi 3^2)$ $10/.79 = F_2/28.27$ F_2 = 357.85 lbs.
Problems	**Related, Generic Math Concepts**	**Solutions**
4. A car master cylinder has a bore of 1.25 inches and a pressure of 500 psi. What would the force output be at the wheel cylinders?		F = P x A F = 500 x ($\pi.625^2$)in^2 F = 613.59 lbs.
5. The output force (F_1) of a fire truck's pumper is 1750 lbs. If the pressure (P) on the water is 200 psi, what is the diameter and area of the piston?		$A = \frac{1750}{200}$ $A = 8.75in^2$ $r = \sqrt{\frac{8.75}{\pi}}$ r = 1.67 D = r x 2 D = 3.34
6. A hydraulic jack has an input cylinder 2 inches in diameter with an output cylinder 4 inches in diameter. Find the force (F_2) exerted by the output piston when a force of 50 pounds is applied to the input piston.		$F_1/A_1 = F_2/A_2$ $50/(\pi 1^2) = F_2/(\pi 2^2)in^2$ $50/3.14 = F_2/12.57$ F_2 = 200.16 lbs.
Problems	**PA Core Math Look**	**Solutions**
7. Diameter = 2.5"; F_1 = 350psi. Find F_2.		F = P x A F = 350 psi x ($\pi 1.25^2$) F = 350 x (4.9) F = 1,715 lbs.
8. F_1 = 25,000 lbs.; P = 350 psi; find the diameter and area of the piston.		$A = \frac{25,000}{350}$ $A = 71.4in^2$ $r = \sqrt{\frac{71.4}{\pi}}$ r = 4.77 D = r2 D = 9.54
9. Input cylinder = 7" diameter; output cylinder = 15" diameter. Find F_2 when 800 lbs. is applied to the input cylinder.		$F_1/A_1 = F_2/A_2$ 800 lbs./($\pi 3.5^2$)in^2 = F_2/($\pi 7.5^2$)in^2 800/38.48 = F_2/176.71 F_2 = 3,673.8 lbs.